

# Sorting

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Notes

Sorting → Arrangement of objects in a particular order wrt specific parameter.

2   8   1   3    $\xrightarrow{\text{ascending order}}$    1   2   3   8

1   13   9   6   12

# factors → 1   2   3   4   6

Why →  
1) organizing  
2) analyzing  
3) searching

**Question ( Elements Removal )**

Given N elements, at every step remove an array element.

Cost to remove an element = Sum of array of elements present in an array

Find minimum cost to remove all elements.

NOTE : First add the cost of removal and then remove it.

<sup>0 1 2</sup>  
arr - [ 2 1 4 ]

<i>index to remove</i>	<i>cost</i>	<i>array</i>
0	$2 + 1 + 4 = 7$	[1 4]
1	$1 + 4 = 5$	[4]
2	$4 = 4$	x

16 x

<sup>0 1 2</sup>  
arr - [ 2 1 4 ]

<i>index to remove</i>	<i>cost</i>	<i>array</i>
2	$2 + 1 + 4 = 7$	[2 1]
0	$2 + 1 = 3$	[1]
1	$1 = 1$	x

11 ✓

<sup>0 1 2</sup>  
A = [ 4 6 1 ]

<i>index</i>	<i>cost</i>	<i>rem array</i>
1	$4 + 6 + 1 = 11$	[4 1]
0	$4 + 1 = 5$	[1]
2	$1 = 1$	x

17



$A = [3 \quad 5 \quad 1 \quad -3]$

index	cost	rem array
1	$3 + 5 + 1 - 3 = 6$	$[3 \quad 1 \quad -3]$
0	$3 + 1 - 3 = 1$	$[1 \quad -3]$
2	$1 - 3 = -2$	$[-3]$
3	<u><math>-3</math></u>	$\times$

2

Observation  $\rightarrow$  Remove large values.

	$[a \quad b \quad c \quad d]$	Remove	cost
	$\begin{matrix} 0 & 1 & 2 & 3 \end{matrix}$		
		a	$a + b + c + d$
		b	$b + c + d$
		c	$c + d$
		d	<u><math>d</math></u>

minimise  
cost

$\rightarrow \underline{a + 2b + 3c + 4d}$   
 $a > b > c > d$

// sort  $A[]$  in descending  $\rightarrow TC = \underline{O(N \log(N))}$

cost = 0

$SC = \underline{O(N) / O(1)}$

for  $i \rightarrow 0$  to  $(N-1)$  {

    cost +=  $A[i] * (i+1)$

}

return cost

$TC = O(N \log(N) + N) = \underline{O(N \log(N))}$

$SC = \underline{O(N) / O(1)}$

**Question** ( Noble Integers ) { Distinct data }

Given N array elements, calculate number of noble integers.

An element  $ele$  in  $arr[]$  is said to be noble if { count of smaller elements =  $ele$  itself }

arr = [ 1, -5, 3, 5, -10, 4 ]

# ele < A[i] → 2 1 3 5 0 4      Ans = 3

arr = [ -3, 0, 2, 5 ]

# elements < A[i] → 0 1 2 3      Ans = 1

Brute force →  $\forall A[i]$ , find the # elements < A[i]  
 $O(N)$  ↙ & check for noble integer. ↘  $O(N)$

TC =  $O(N^2)$       SC =  $O(1)$

```
ans = 0
for i → 0 to (N-1) {
    cnt = 0
    for j → 0 to (N-1) {
        if (A[j] < A[i]) cnt++
    }
    if (cnt == A[i]) ans++
}
return ans
```


$$A = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 3 & 8 & 2 & -1 & 5 & -3 \end{bmatrix}$$

$\#e < A[i] \rightarrow$  3 5 2 1 4 0      Ans = 2

↑                    ↑

move all elements  $< A[i]$  on 1 side

⇒ counting may be faster

Arranging data s.t smaller values are on

1 side  $\Rightarrow$  sorting

(left)  $\rightarrow$  (ascending order)

Sorted  $\rightarrow A = [-3 \quad -1 \quad 2 \quad 3 \quad 5 \quad 8]$

#e < A[i]  $\rightarrow$  0 1 2 3 4 5 = index

$\#e \in A[i] \rightarrow$       0      1      2      3      4      5       $\swarrow$  = index

// sort A[] in ascending order

$$\text{cnt} = 0$$

```
for i → 0 to (N-1) {  
    if (A[i] == i) crt ++  
}
```

return crt

$$T_C = O(N \log(N) + N) = \underline{O(N \log(N))}$$

$$SC = O(N) / O(1)$$



**Question ( Noble Integers ) :** { Data can repeat }

0 1 2 3 4  
arr = [ -10, 1, 1, 3, 100 ]

#e < A[i] → 0 1 1 3 4      Ans = 3

0 1 2 3 4 5 6 7 8  
arr = [ -10, 1, 1, 2, 4, 4, 4, 8, 10 ]

#e < A[i] → 0 1 1 3 4 4 4 7 8      Ans = 5

0 1 2 3 4 5 6 7 8 9 10 11 12 13  
arr = [ -3, 0, 2, 2, 5, 5, 5, 5, 8, 8, 10, 10, 10, 14 ]

#e < A[i] → 0 1 2 2 4 4 4 4 8 8 10 10 10 13      Ans = 7

// sort A[] in ascending order

ans = 0      cnt = 0

for i → 0 to (N-1) {

if (i == 0 || A[i] != A[i-1])

cnt = i

if (cnt == A[i])      ans++

}

return ans

$$TC = O(N \log N) + N = \underline{O(N \log N)}$$

$$SC = \underline{O(N)} / O(1)$$



# Selection Sort

**idea:** Select the minimum element and send that elements to correct position by swapping.

$A = [ \overset{0}{3} \quad \overset{1}{8} \quad \overset{2}{2} \quad \overset{3}{-1} \quad \overset{4}{5} \quad \overset{5}{-3} ]$

Find max element in  $A[] \rightarrow TC = O(N) \quad SC = O(1)$

Find second max element  $\rightarrow TC = O(2N) = O(N)$

$SC = O(2) = O(1)$

Find third max element  $\rightarrow TC = O(3N) = O(N)$

$\vdots$   
 $SC = O(3) = O(1)$

Find  $K^{th}$  largest element  $\rightarrow TC = O(K * N)$  ✓

$SC = O(K)$

$\rightarrow O(1)$  ✓

$A = [ \overset{0}{\cancel{3}} \quad \overset{1}{\cancel{8}} \quad \overset{2}{2} \quad \overset{3}{\cancel{-1}} \quad \overset{4}{\cancel{5}} \quad \overset{5}{\cancel{-3}} ]$   
 $\quad \quad \quad -1 \quad -3 \quad \quad \quad 3 \quad 5 \quad 8$

K elements is sorted position

$\therefore$  if  $K = N-1 \Rightarrow$  sorted array

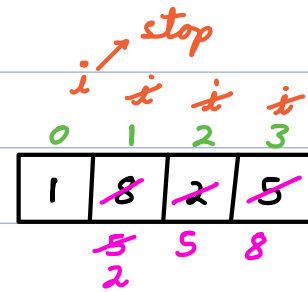
$A = [ \overset{0}{\cancel{3}} \quad \overset{1}{\cancel{8}} \quad \overset{2}{\cancel{2}} \quad \overset{3}{\cancel{-1}} \quad \overset{4}{\cancel{5}} \quad \overset{5}{\cancel{-3}} ]$   
 $\quad \quad \quad -1 \quad -3 \quad 2 \quad 3 \quad 5 \quad 8$

selection sort



&lt;/&gt; Code

```
for i → (N-1) to 1 { // 0 — i
    m = 0 // index of max element
    for j → 1 to i {
        if (A[j] > A[m]) m = j
    }
    swap (A, m, i)
}
```



$$TC = O(N^2)$$

$$SC = O(1)$$

Max → last

Max → first

Min → last

Min → first

Descending

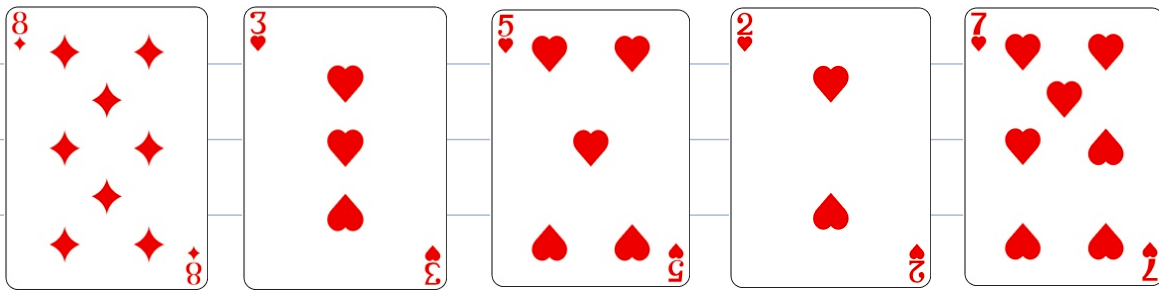
Ascending

H.W → Study Bubble Sort.





# Insertion Sort (Arrangement of playing cards)



Why used  $\rightarrow$  It can sort running stream of data.

$i/p \rightarrow$  7 9 12 10 8  $x$

$i$	$(i+1)$					
0	1	2	3	4	...	$(N-1)$
7	8	9	10	12		

$n = 4$

for any input  $\rightarrow$  min swaps = 0

max swaps = # of elements in array

$n = 0$

for  $\forall$  inputs,  $x$  {

$i = n - 1$  // 0 —  $i$  (current array)

while ( $i \geq 0$ ) {

if ( $A[i] > x$ )

$A[i+1] = A[i]$  // shift right

else

break //  $i \rightarrow$  index of small / equal

$i--$

}  $A[i+1] = x$

} return A

TC =  $O(N^2)$

SC =  $O(1)$

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