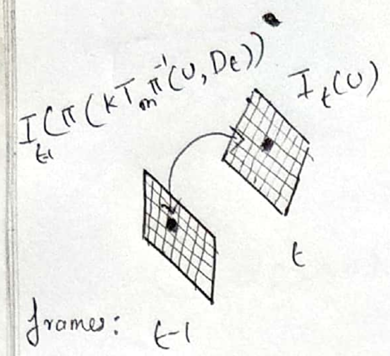
SO3 step

$T_m \approx \xi$

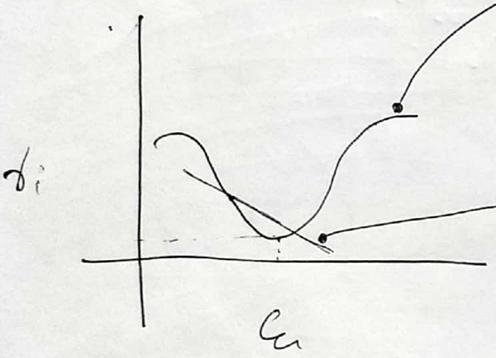


computing image residual

$$\text{residual} = I_t(u) - I_{t-1}(\pi(K T_m \pi^{-1}(u, D_t)))$$

To choose ξ_t (motion params) that minimize the residual (assuming photo consistency)

$$f(\xi_t) = \frac{1}{2} \| r_i(\xi_t) \|_2^2$$



Since residual function is non-linear in nature, we use a second order taylor series expansion to linearize the function.

$$f(x) = f(a) + \frac{f'(a)(x-a)}{1!} + \frac{f''(a)(x-a)^2}{2!}$$

We now use gauss newton method (consider the problem as least squares problem) to find the optimal motion params (ξ_t) by assuming the gradient of $f(\xi_t)$ to be zero.

$$= \min \frac{1}{2} \| r_i(\xi) \|_2^2$$

$$f(\xi + \Delta\xi) = \gamma(\xi) + J_f(\xi)\, d \quad \text{(Linear expansion)}$$

$$= \min \tfrac{1}{2} \| \gamma^k + J_f^k d \|_2^2$$

$$\Delta f = J^{kT} (\gamma^k + J d^k) = 0$$

$$d^k = -(J^{kT} J^k)^{-1} J^T \gamma^k = \Delta\xi = (J^T J)^{-1} J^T \gamma$$

$$J^k = \begin{bmatrix} \frac{\delta \gamma_1}{\delta \xi_1} & \frac{\delta \gamma_1}{\delta \xi_2} & \cdots & \frac{\delta \gamma_1}{\delta \xi_n} \\ \vdots & & \ddots & \\ \frac{\delta \gamma_m}{\delta \xi_1} & & & \frac{\delta \gamma_m}{\delta \xi_n} \end{bmatrix}$$

m = observation space (no: pixels)

n = set of motion param

$$\frac{\delta \gamma}{\delta \xi} = \frac{\delta \gamma}{\delta x_i} \frac{\delta x_i}{\delta X_i} \frac{\delta X_i}{\delta \xi}$$

(Jacobian row)

(left product)

→ image residual vector

→ derivative of projection function

$$\therefore \quad \pi(X_i) = \begin{bmatrix} f_u \frac{x_i}{z_i} + c_u \\ f_v \frac{y_i}{z_i} + c_v \end{bmatrix}$$

↳ differentiating, we get

$$\begin{bmatrix} f_u/z_i & 0 & -f_u\, x_i/z^2 \\ 0 & f_v/z_i & -f_v\, y_i/z^2 \end{bmatrix}$$

→ derivating exponential map (Lie algebra to Lie group)

$n = 3$

To solve for $\Delta\xi$, a hermissian matrix is found using a jacobian now for all pixels.

Cholesky decomposition is used on the hermitian matrix to solve for $\Delta\xi$.

$$\begin{bmatrix} a & 0 & 0 \\ b & c & 0 \\ d & e & f \end{bmatrix} \times \begin{bmatrix} a & b & d \\ 0 & c & e \\ 0 & 0 & f \end{bmatrix} = \begin{bmatrix} a^2 & ab & ad \\ ab & b^2+c^2 & bd+ce \\ ad & bd+ce & d^2+e^2+f^2 \end{bmatrix}$$

($c_\phi = e = f = 0$ since we just have $x, T, 3$ $(n = 3)$)

$$= \begin{bmatrix} a^2 & ab & ad \\ ab & b^2 & bd \\ ad & bd & d^2 \end{bmatrix}$$