Exploratory Data Analysis of Tooth Growth in Gerbils

Hypothesis Testing and Confidence Intervals

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Overview

In this report, we use `ToothGrowth`, a dataset in R, which contains data from an experiment measuring the length of guinea pig teeth, or *odontoblasts*, at three dose levels of Vitamin C, either through Orange Juice or with just ascorbic acid. We will perform exploratory data analysis, followed by hypothesis testing and calculate confidence intervals. Finally, we will show that the tooth growth may increase with increasing dose of Vitamin C, although the type of supplement does not seem to matter.

The Data and Exploratory Analyses

First, we load the ToothGrowth data. The data contains 60 observations in 3 dose levels of 2 supplement types. First, we examine how the response variable, tooth length, varies by dose level and supplement type. We can do this with a simple boxplot.

```
data1 <- ToothGrowth
data1$dose <- as.factor(data1$dose)
```

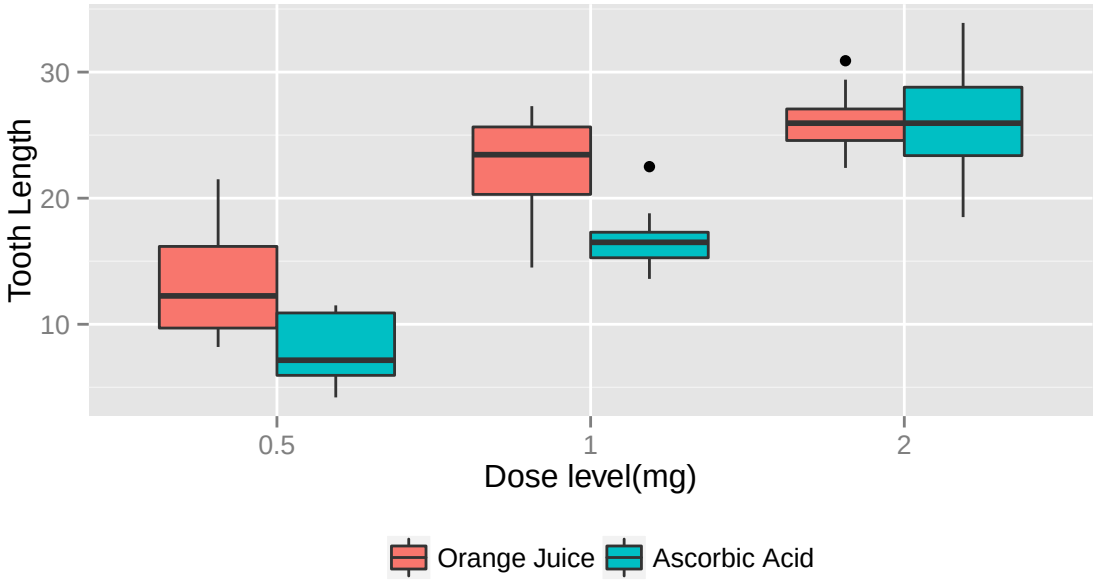

Figure 1: Data Summary

Visually, it looks like Tooth Length is longer at higher dose levels, regardless of the supplement type. However, there may be some effect of supplement type as well. The experimental design of this study essentially has two levels. So, we can group the data in few ways, only by supp, or only by dose. We can take a look at the subgroup means and variances before we get to hypothesis testing.

Group	Means	Variances	Counts
OJ	20.663	43.633	30
VC	16.963	68.327	30
0.5	10.605	20.248	20
1	19.735	19.496	20
2	26.100	14.244	20

Confidence Intervals and Hypothesis Testing

Let's look at the supplement type first. First we compare the mean and variance at each supplement type. Referring to our tables of means and variances above, we see that the variances are `43.6` and `68.32`. So to be safe, we set `var.equal = FALSE`, in our t-test. We know from the description of the dataset that the groups are not paired.

```
test1 <- t.test(len ~ supp,data=data1, var.equal = FALSE, paired = FALSE)
test1$conf.int
test1$p.value
test1$estimate
```

```
## [1] -0.1710156  7.5710156
## attr(,"conf.level")
## [1] 0.95
## [1] 0.06063451
## mean in group OJ mean in group VC
##         20.66333         16.96333
```

Although the p-value is `0.06`, the confidence interval contains `0`. Because the 95% CI includes 0, we cannot reject the null hypothesis that the difference in means is 0. Next, we will test the group differences by dose level, using `t.test` as before. Again, to be safe, based on the calculated group variances, we set `var.equal=FALSE`.

```
dlong <- melt(data1, id.vars = c("dose"), measure.vars="len")
g1 <- dlong[dlong$dose == "0.5",]$value
g2 <- dlong[dlong$dose == "1",]$value
g3 <- dlong[dlong$dose == "2",]$value
t1 <- t.test(g2, g1,var.equal = FALSE, paired = FALSE, alternative = "two.sided")
t2 <- t.test(g3, g1,var.equal = FALSE, paired = FALSE, alternative = "two.sided")
t3 <- t.test(g3, g2,var.equal = FALSE, paired = FALSE, alternative = "two.sided")
conf_int <- rbind(t1$conf, t2$conf, t3$conf)
p_vals <- format.pval(c(t1$p.value, t2$p.value, t3$p.value), eps = .001, digits=3)
t_stat <- rbind(t1$statistic, t2$statistic,t3$statistic)
labels <-c("1mg vs. 0.5mg", "2mg vs. 0.5mg", "2mg vs 1mg")
results1 <- data.frame(labels, conf_int, p_vals, t_stat)
colnames(results1) <- c("Comparison", "95% CI", "95% CI", "p", "t")
print(xtable(results1, digits = 3), comment=FALSE,include.rownames=FALSE)
```

Comparison	95% CI	95% CI	p	t
1mg vs. 0.5mg	6.276	11.984	<0.001	6.477
2mg vs. 0.5mg	12.834	18.156	<0.001	11.799
2mg vs 1mg	3.734	8.996	<0.001	4.900

We have now done 4 successive t-tests on the data, comparing various combinations of groups. But what about power? We have estimates of the 'true' mean difference and its variation from our data.

```r
d1 <- 3.7 #smallest possible group difference, from endpoint of CI of 2mg vs 1mg
sd1 <- sqrt(19) #variance for dose of group1
d2 <- 6.4  #larger group difference, from endpoint of CI of group 1 vs. 0.5mg
sd2 <- sqrt(20)
#power test for a range of difference and assoc. sd estimate
power.t.test(n=20, type = "two.sample", alternative = "two.sided",
             sig.level = 0.01, sd=sd1, delta=d1)$power
```

```
## [1] 0.496575
```

```r
power.t.test(n=20, type = "two.sample", alternative = "two.sided",
             sig.level = 0.01, sd=sd2, delta=d2)$power
```

```
## [1] 0.9598137
```

At an n of 20/group, and $\alpha = 0.01$, we have 95% power to correctly reject a null hypothesis assuming a true mean difference of ~6.4. However, for the smaller differences observed in our tests, we may be underpowered to correctly reject the null hypothesis. Thus, our analysis may not be able to detect the smaller group differences that are present in this data set. From this, we may infer that dose level does have an effect on tooth growth and the previous t-test showed that the type of supplement does not have a net effect. However, we cannot fully test for the entire range of possible group differences at an adequate power.

Conclusions

1. When comparing the means at each dose level (0.5, 1 and 2 mg), all 3 combinations of comaprisons produce confidence intervals that do not contain 0, $t > 1$, and $p < 0.001$.
2. We infer from this data that while the *type* of Vitamin C supplement *does not* significantly affect tooth growth, higher *doses* of Vitamin C *has an effect on tooth growth.*
3. We have 95% power to correctly reject the null hypothesis at a ($\Delta = 6.4$)

Assumptions

1. We assumed that the variances across the group (either by dose or supplement) was not equal.
2. We assumed that the 60 gerbils used for this data were randomy assigned to each of the dose/supplement groups.
3. We assumed that the 60 gerbils are representative of the entire population of gerbils.
4. We assumed that the factors dose and supplement type were independent of each other. However, if they are related, we would need methods outside of hypothesis testing and confidence interval (perhaps regression?)
5. Finally, we assumed that the response variable is not itself affected by other variables that we have not included in our statistical inference.

Note: Code for graph was not show here because of space, but markdown files available here.