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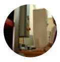

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Review

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From the double pendulum model to full-body simulation: evolution of golf swing modeling

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Golf is one of the most popular sports worldwide. Scientific research into golf has grown substantially over the past four decades. This article reviews the biomechanical models of the golf swing, focusing on how these models can aid understanding of golf biomechanics and the fitting of golf clubs to individual players. It is shown that models range in complexity from the conventional double pendulum model to full-body simulations that include sub-models. The usefulness of any model or simulation is ultimately determined by the assumptions included and the model's complexity. The article summarizes the established areas of golf swing modeling and simulation, discusses the assumptions made by those models, and identifies areas where more research and further development are needed.

Keywords:
- golf
- modeling
- simulation
- review

1. INTRODUCTION

Few if any athletic disciplines have formed the basis for as many models and simulations as the golf swing. Some of the reasons for this could be the sport's general popularity, the large turnovers in the golf industry, or most likely, the underlying complexity of the seemingly simple task of bringing a golf club in contact with a ball. The list of factors influencing the success of a golf swing is long. Very often it is not easy to understand the relationships of the various parameters and variables involved. In fact, a recent review of the research efforts in golf pointed out that 'understanding of the golfer's interaction with the club is still too crude to fit clubs to people properly' [1]. Given that approximately 55 million people play golf worldwide [1], it is important that the interaction between the golfer and equipment is better understood.

When the behavior of a system is too complex to be explained by an observer, it is useful to reduce the system to its essential elements in order to simplify and solve a research problem associated with it. The process of defining a representative alternative system is commonly referred to as 'modeling' [2], whereas the application of this model (for example to run virtual experiments) is associated with the term 'simulation' [2]. In the case of the golf swing, numerous researchers have presented models and simulations using various degrees of sophistication to represent the system. Obviously, the amount of simplifications and assumptions depends on the aims and objectives of each individual project, but in all cases, care needs to be taken to create appropriate models that adequately describe the underlying physics without being too cumbersome. Over-simplifications, as well as too high a degree of fidelity, can make it impossible to use a model to solve a research problem. The true value of a model is being able to extrapolate rather than merely interpolate between observations. This is particularly poignant in multidimensional problems with many degrees of freedom, such as the open system intrinsic to the golf swing. In this context, it appears to be helpful to clearly distinguish between assumptions and simplifications. Throughout this article, the term 'assumption' will refer to model characteristics that are believed to resemble reality very closely. For example, it is known that gravity

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varies depending on geographical position, but is very likely that it is safe to assume that these variations will have no effect on the simulation results of the golf swing; therefore, a constant gravity value is assumed. In contrast to this, simplifications are derivations from reality that could be necessary to make it feasible to create a particular model or compute simulation results in a realistic time. An example of a typical simplification would be that, rather than trying to include all relevant soft tissues, most models of the golf swing deal with the human body as a rigid-body system.

Previous golf reviews have covered areas such as the general physics of the golf swing [3], muscle activity during the golf swing [4], the biomechanics of the swing [5], and golf injuries [6]. None of these reviews have specifically focused on the use of modeling and simulation to understand the dynamic aspects of the golf swing, and to our best knowledge, no review of golf models has been published previously. Given the number of previous models, such a review could potentially be useful in giving suggestions and guidelines for future research. Therefore, the objective of this article is to review and classify previous biomechanical models and simulations of the golf swing in order to aid the design of future modeling/simulation studies.

2. SCOPE AND STRUCTURE OF REVIEW

The focus of this paper is biomechanical models that permit one to draw conclusions on the internal, kinetic reasons for the observed results. There are a plethora of further studies that are not included, for example, studies that only create graphical representations of golf swings based on 3-D motion capture, studies concentrating on club behavior without any information on the body motion are not included (for example impact models or models of the ball), and putting models and simulations were excluded because of the marked differences in the dynamics of full golf swings and the putting stroke. Throughout this article, models and simulations will be classified as follows: (i) by the spatial dimension (2-D and 3-D); (ii) the number of segments involved; and (iii) by whether an inverse dynamics or forward dynamics approach was chosen. The inverse dynamics solution typically involves the measurement of body movement and some forces (for example ground reaction forces), whereas input of the internal forces is required for the forward dynamics approach so that the output (body movement) can be calculated [2] (Figure 1).

3. 2-D MODELS OF THE GOLF SWING

3.1 Double Pendulum Model

In order to gain insight into the basic mechanics involved in the golf swing and to explain the optimal coordination of the swing, Cochran and Stobbs [7] suggested a simple model of the downswing consisting of a double pendulum (Figure 2). They assumed that the two most relevant pivot points of the moving body segments were the wrist and a point 'roughly corresponding to the middle of the golfer's upper chest' [7]. This imaginary point is taken to be fixed in space and connected to an upper lever that is representative of the arms of the golfer. Another segment, representing the club, was connected to the upper lever via a hinge joint. This 'wrist' hinge was assumed to behave passively, restricted only by a stop that prevented the club segment from moving too far back at the top of the backswing. Cochran and Stobbs used this model to explain the basic mechanics of the swing and showed that the combined effect of inertia and centripetal force acting on the

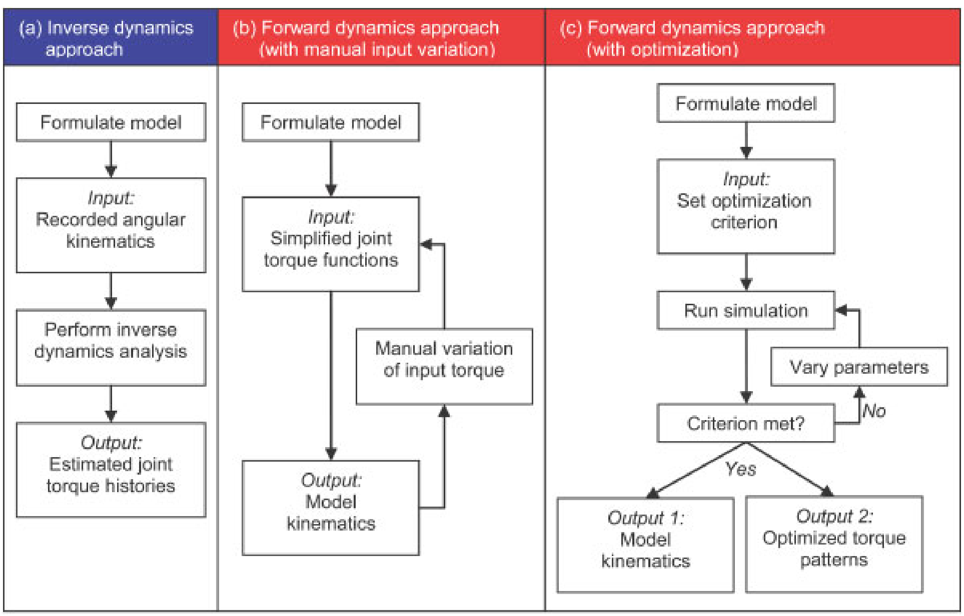

Figure 1. Typical steps involved in (a) inverse dynamics analysis, (b) forward dynamics analysis (with manual or systematic variation of input parameters), and (c) forward dynamics analysis (using the optimization algorithm).

lower lever can create a well-coordinated downswing if the upper lever is accelerated using the correct force [7]. In this case, no wrist torque is required to coordinate the rotation

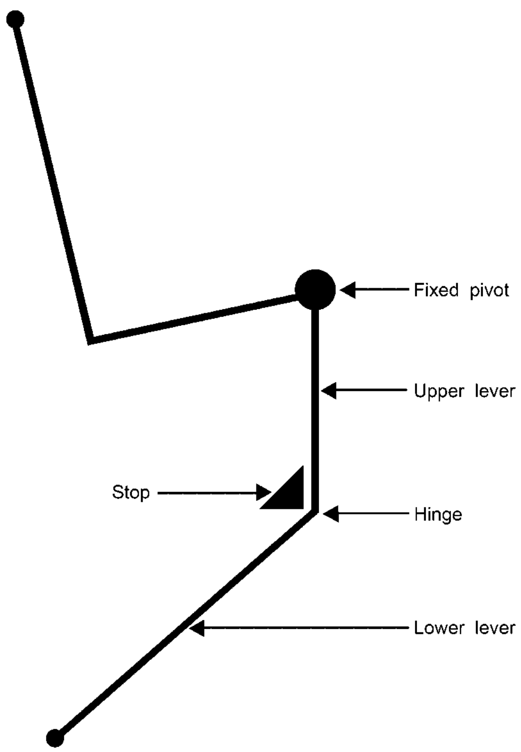

Figure 2. Double pendulum model of the golf swing [7]. All movements occur in one plane and 'stop' prevents angle between upper and lower lever decreasing below 90°. Reproduced by kind permission of Triumph Books.

about the lower hinge ('wrists') other than the passive torque provided by the stopper in the wrist joint. Based on this observation, the concept of natural wrist release emerged. This concept describes a swing pattern in which no active muscular wrist torque is applied to accelerate the club during the downswing. Instead, the motion of the arm is coordinated in a way that allows the club to accelerate 'naturally', driven by the centrifugal force acting on its centre of gravity. Cochran and Stobbs [7] present their model as a simplified mechanical representation of the swing to explain the principles that underlie the golf swing, whereas others successfully used their approach for inverse dynamics analyses [10] and forward dynamics swing simulations. Of these forward dynamics simulations, some applied simplified torque profiles at the fixed pivot and the 'wrist' hinge [8,11,15,18], thereby keeping the number of input parameters manageable for manual manipulation, while others used optimization algorithms to define more complex input torque profiles [13,14]. An overview of studies that applied the double pendulum model or variations of it is shown in Table 1.

3.1.1 Inverse dynamics models

Budney and Bellow [10] used a double pendulum model to predict the joint torques that would be required to replicate angular kinematic data recorded from real swings. They then compared two 'club matching' approaches, both aiming to allow the player to use the same swing kinematics for all clubs from the driver to the shortest iron. The first approach was the conventional static swing weight method. The second approach was based on the dynamic swing model and scaled the club masses so that the 'maximum circumferential force' the player has to overcome when accelerating the club would be

Table 1. Summary of 2-D golf models

Reference	Focus	# of players	Fixed hub	Shaft	Plane	Gravity
	Two-segment models					
[7]	Illustrative model	NA	Yes	Rigid	Fixed	NA
[8]	Wrist release, backswing length	NA	Yes	Rigid	Fixed	No†
[9]	Parametric study	1	No‡	Rigid	Fixed	Yes
[10]	Club matching	4	Yes	Rigid	Fixed	No
[11]	Role of shaft	3	Yes	Flexible	Fixed	Yes
[12]	Club length, backswing range, club head mass	1	Yes	Rigid	Fixed	No
[13]	Ball position (iron)	NA	Yes	Rigid	Fixed	No
[14]	Ball position (driver), energy flow	NA	Yes	Rigid	Fixed	No
[15]	Effect of inward pull	NA	No§	Rigid	Fixed	Yes
[16]	Swing efficiency	NA	Yes	Rigid	Fixed	No
[17]	Robot simulation	NA	Yes	Flexible	Fixed	Yes
[18]	Wrist release, ball position	NA	Yes	Rigid	Fixed	Yes
	Three-segment models					
[19]	Optimum control	NA	Yes	Rigid	Fixed	No
[20]	Parametric study	NA	Yes	Rigid	Fixed	Yes
[21]	Optimum control	1	Yes	Rigid	Fixed	Yes
[22–25]	Wrist release, power sequencing, equipment testing	NA	Yes	Rigid	Fixed	Yes
[26]	Club fitting, Shaft stiffness	3	Yes	Flexible¶	Fixed	No
[27]	Parametric study	NA	Yes	Rigid	Vertical	Yes
	Others					
[28]	Comparison 2- and 4-link model	4	Yes	Flexible	Fixed	Yes

†Gravity replaced by a constant torque; ‡horizontal shift; §vertical shift; ¶used correction factor to account for shaft deformation.

identical for all clubs within the set. They found that the static method as well as the dynamic club matching would result in a similar, monotonic progression of club masses throughout a typical set of clubs.

3.1.2 Forward dynamics: simplified input torque functions

Jørgensen [8] presented a mathematical simulation model based on the double pendulum approach. One of his aims was to analyze how wrist torque could be used to enhance performance. Using the Lagrangian approach, he described the system with a pair of coupled differential equations. After simplifying these equations, he found that the impact velocity of the club head could be increased if torque was applied at the 'wrist' hinge to prevent the lower lever from changing its position too early ('late release').

The double pendulum model has also been applied to study the bending of the shaft throughout the swing by Milne and Davis [11], who included a flexible shaft. They derived simplified ramp torque functions for the wrist and shoulder joints, which were, in the case of the wrist joint, matched to bending moments at the top end of the shaft as recorded with strain gauges. They concluded that the statically measured shaft flexibility does not affect swing kinematics significantly and that common static shaft tests should be replaced by dynamic tests to increase the validity of the tests. Milne and Davis have been criticized for not correctly incorporating the offset of the centre of gravity of the club head in their model [29]. Instead of including the 3-D position of the centre of gravity, they projected it onto the swing plane of their 2-D model using a correction factor obtained from their player testing rather than appropriate torques and forces to change its position relative to the longitudinal axis of the shaft.

Miura [15] used the double pendulum model of the swing to analyze whether a translational displacement of the position of the central rotation axis of the system just before impact could increase impact velocity. Miura noted that skilled players appeared to pull the club towards the rotation axis just before impact (vertical/upwards relative to the global reference frame). Therefore, he created a modified double pendulum model by including an additional translational actuator and found that the club head velocity at impact could in fact be increased by this inward pull.

Recently, White [16] used the double pendulum model to revisit the effects of fundamental physical principles on the efficiency of the golf swing. Using an undriven double pendulum model, it was confirmed that arm and club would rearrange themselves without any active wrist contribution. The author then split the overall stroke efficiency into swing and collision efficiency. Swing efficiency was defined as 'the fraction of the total kinetic energy of the system that is present in the club head immediately before the collision', and the collision efficiency was determined as the ratio between ball mass and club head mass ($m_{ball}/m_{club\ head}$). Total strike efficiency was then calculated by multiplying swing efficiency and collision efficiency so that an optimum between the two conflicting factors, swing and collision efficiency, could be found. White [16] then used a modified version of the equations from Pickering and Vickers [14] with a constant shoulder torque to explore the effects of changes of some of the input parameters on stroke efficiency. These parameters were club head mass, shaft length, shaft mass, wrist angle at initiation of the downswing, and time of wrist release. When analyzing the effects of an active wrist torque after wrist release, White [16] noted that this caused the club head velocity to peak before impact. This demonstrates the limitations of the proposed model as it does not adjust its torque patterns to maintain an efficient swing when the model configuration changes.

Chen *et al*. [18] examined the effect of different wrist torque profiles and ball positions on club head speed at impact. Using the Lagrangian approach, they obtained the equations of motion for a double pendulum swing model. They then applied four different wrist torque profiles: (i) natural release; (ii) passive (wrist is held rigid until preset negative torque is reached [PW]); (iii) active wrist (positive torque is used to accelerate wrist joint after release [AW]); and (iv) a combination of PW and AW (PAW). Keeping the maximum torque values constant, the highest impact speed could be achieved using the PAW wrist torque pattern. Because no impact model was included that would cause the club head to slow down after impact, the peak club head speed could always be determined, even if it occurred after the club head passed the intended impact position. It was found that peak club head speed always occurred after the club head passed a neutral tee position. Therefore, (right-handed) golfers would benefit from placing the tee closer towards their left, which is in agreement with Pickering and Vickers's results [14]. It should be noted that the position of the ball within the stance is also associated with launch conditions rather than just optimal club head speed.

3.1.3 Forward dynamics: torque functions

The double pendulum model was used to analyze the effects of a variation of impact position between club head and ball (tee position for driver swings). Pickering [13] observed that for swings performed with an iron, there is an advantage in positioning the ball aligned with the pivot axis of the swing because this results in the club face having a velocity component directed towards the ground. This gives the ball more backspin, which they believe is desirable for iron hits to prevent the ball from rolling too far on the ground after landing. It could be postulated that even more backspin would be achieved if the ball was placed further backward, but this option was not simulated by Pickering. For drivers, however, another study by Pickering and Vickers [14] provided evidence that higher impact velocities could be achieved if the ball was moved to the left (for a right-handed player), towards the target.

3.2 Three-Segment Models of the Swing

Obviously, the two-segment, double pendulum model of the golf swing does not account for rotations of the arms about the shoulder joint and rotations of the torso as there is only a single body segment representing the arms. Therefore, some researchers have introduced another hinge in the model representing a simplified shoulder joint. Cochran and Stobbs

[7] state that they considered this option when suggesting their model and mention that they found that adding another joint to their model helped to achieve increased club head velocity at impact. However, they concluded that utilizing another joint would make it more difficult to coordinate the segments in a way that would result in an efficient swing.

3.2.1 Inverse dynamics models

It appears that Tsujiuchi *et al*. [26] used an inverse dynamics approach to study the shoulder, elbow, and wrist torques applied during downswings performed by three players. The methods that were used are not described in detail in the paper. They state that marker trajectories were recorded in three dimensions and then projected onto a single plane. The Newton–Euler method was used to derive equations of motion for a three-segment model, including a flexible shaft. However, it is not clear how the slope and position of the projection plane were determined or how the rigid body model was modified to account for the flexibility of the shaft. Tsujiuchi *et al*. [26] found that all three golfers analyzed in their study had a negative torque acting in their wrist joints that would slow down the 'uncocking' of their wrists just before impact. There was a weak correlation between the magnitude of this negative torque and shaft deflection, indicating that the forward bending of the shaft at impact could be caused by the torque pattern applied by the player. However, the swing model presented by Tsujiuchi *et al*. was based on a 2-D approach. Neither the off-centre position of the club head's centre of gravity nor the wrist rotation that golfers use to square the face before impact was considered. The centre of gravity position of the club head would also tend to cause forward deflection of the shaft at impact. Therefore, in a real swing, the centre of gravity position may cause the forward bending that the authors explained by an active wrist torque.

3.2.2 Forward dynamics: simplified torque functions

Turner and Hills [20] proposed a three-segment model consisting of a shoulder, arm, and a club segment, all of which move in one plane. After formulating the equations of motion for the system, forward dynamics simulations were run using constant torque settings. The timing of these constant torques was then varied, and the wrist torque manipulated by using a cubic polynomial. The authors then present a number of effects that could be achieved by modifying the torque function.

3.2.3 Forward dynamics: optimized torque functions

A study presented by Campbell and Reid [19] was the earliest application of optimum control theory that was found as part of this article. Using Pontryagin's maximum/minimum principle, they predicted torso, shoulder, and wrist torque profiles under different optimization criteria. The first optimization criterion was to achieve maximum impact speed while not exceeding predefined torque limits. The second criterion was to reach a predefined impact speed with minimum mechanical work. The results obtained by Campbell and Reid showed that, with the minimum work optimization criterion, the mechanical work decreased by 67 per cent, while impact speed was only reduced by 25 per cent at the same time. The presented torque profiles for the more efficient swing showed a notable sequencing of peak torques. In a similar but more comprehensive study design, Kaneko and Sato [21] obtained initial torque profiles by using inverse dynamics. Based on this, they found that a 'minimum power' optimization criterion led to the most accurate torque predictions when applying optimum control theory. They calculated how torque profiles were likely to change when the mechanical characteristics of the club were altered (length, mass distribution, and total mass). Their simulations showed how heavier clubs or clubs with a centre of gravity closer to the club head caused increases of the joint torques. When the club length was increased, less power was required to reach the same club head velocity.

Neal and Sprigings [22] used a three-segment model to determine whether using active wrist torque to accelerate the club just before impact rather than just releasing the wrist joint passively ('natural release') has advantages for the golfer. Besides including a third segment (torso) in their model, they also installed torque generators in the joints that do not neglect the physiological properties of human muscle. Namely, these characteristics include the velocity–force relationship ('Hill's relation') and the maximum activation rates of human muscles. After setting up Newtonian equations of motion for the system, they used an optimization algorithm to find the torque patterns that produced maximum club head velocities under three different boundary conditions. By comparing the results of these simulations, they found that club head speed could be increased if the golfers actively applied a wrist torque accelerating the club just before impact rather than letting the club head accelerate only passively as suggested in previous studies. Unsurprisingly, they were able to demonstrate that if the model configuration was set to ignore physiological muscle limits, higher club head velocities could be achieved. However, the unrealistic, steep changes in joint torque showed that the resulting motion patterns could never be reproduced by real golfers. This shows that the physiological properties of muscle have to be kept in mind when modeling the swing. Using the same model, Sprigings and Mackenzie [23] were able to analyze the mechanical power flow among the segments. They found that there is a proximal to distal power flow with maximum power values reached for the wrist joint. The majority of energy transferred to the club originates from passive joint forces rather than active muscle forces. Additionally, Sprigings and Mackenzie [23] showed that their three-segment model applied more realistic shoulder joint torques (peak value: 87 Nm) than previous two-segment models.

Sprigings and Neal [24] describe another application of their model in order to analyze the feasibility of a simple device for club performance testing. This test device consisted of a pendulum where clubs could be mounted and only driven by gravitational force acting on the club after its release. Sprigings and Neal modeled both this simple device, only driven by gravitational torque, and the same set-up, but driven by a more realistic torque function. Their results showed that a simple, one-link pendulum model driven by gravitational torque is not suitable to test the efficiency of new club designs because the results were inconsistent with the more realistic model that included a wrist torque function. By this, Sprigings and Neal present a good example of a model that used too extreme a simplification to create valid results (in this case, the unrealistic torque driving the club in the one-link pendulum).

More recently, a new approach to manipulate the shape of torque curves when optimizing forward dynamics swing simulations has been suggested [27]. This method reduces the number of input parameters to three per joint without restricting the shape of the input torque curve. This was achieved by assuming: constant segment masses and dimensions, that joint velocities and torques at the top of the backswing were zero, that joint torques at impact were zero, that impact would occur at a predefined impact position, and that the joint angles would change monotonically throughout the downswing. After making these assumptions, an upper torque limit was set for each joint, and maximum impact velocity was used as an optimization criterion. While the previous assumptions could probably be justified based on previous studies, it was also necessary to set the downswing duration to a fixed value so that the optimization could be performed. This does not seem realistic, as it is likely that for an optimized swing with an increased club head velocity the downswing duration would shorten.

3.3 Other 2-D Models

Iwatsubo *et al*. [28] presented a comparison of a two- and four-link model of the golf swing. Using the Newton–Euler method, they found the equations of motion for two different swing models and used motion capture data to obtain the joint angle histories of the swings of four golfers. Using inverse dynamics, they calculated the joint torques applied by the golfers. Results for the wrist joint were almost the same for both models. For the shoulder joint, however, some characteristics of the individual swings were no longer distinguishable when using the two-link model of the swing. Iwatsubo *et al*. [28] concluded that the more complex, four-link model of the swing is better suited to analyze the skill level of golf players.

4. 3-D MODELS OF THE GOLF SWING

Golfers perform their swing in 3-D space, and it is unclear how much the simplified view of regarding the swing as a planar motion influences the results of the studies presented in the previous section. Therefore, potentially more realistic, yet also more complex models have been developed that consider the movement of up to 15 body segments in 3-D space (Table 2).

4.1 Models Including Club and Upper Body Only

4.1.1 Inverse dynamics models

An early study of the 3-D kinetics of the golf downswing was carried out by Vaughan [30]. Using inverse dynamics and a rigid model of a golf club, he analyzed the torques and forces applied by four golfers to their clubs. The motion of the club was recorded using two cameras ($f = 300$ Hz). He found that the players he studied did not swing the club in one static plane. Nevertheless, he confirmed some observations made with the earlier 2-D models of the swing by finding that golfers applied negative torque to the club just before impact causing the hands to slow down and the club head to accelerate [30]. Neal and Wilson [31] used an approach similar to Vaughan to determine the 3-D forces and torques applied to the club. They confirmed most of Vaughan's findings, added estimates for shoulder joint forces, and described the angular kinematics of the club segment in more detail. Interpretation of some of their results is difficult because all kinematic and kinetic data are presented in a global coordinate system rather than in the local club or arm reference system.

Table 2. Summary of 3-D golf models

Reference	Focus	♯ of players	Fixed hub	Shaft	Plane	Gravity[†]
	Partial body models					
[30]	Inverse dynamics	4	No	Rigid	Free	Yes
[31]	Inverse dynamics	6	No	Rigid	Free	–
[32]	Parametric study	NA	Yes	Rigid	Restricted	No
[33]	Inverse dynamics	1	No	Flexible	Free	–
[34]	Robot simulation	NA	Yes	Flexible	One plane	–
[29]	Shaft stiffness effects	4	Yes	Flexible	Two planes [1]	Yes
	Full body					
[35]	Modeling framework	NA	No	Flexible	Free	Yes
[36]	Shaft stiffness effects	1	No	Flexible	Free	Yes
[37–39]	Full kinematic and kinetic analysis	84	No	Flexible	Free	Yes
[40]	Club length effects	1	No	Flexible	Free	Yes
[41]	Club shaft and length effects	1	No	Flexible	Free	Yes

[1]Torso and arm/club segments moved in different planes.
[†]Where dashes are present, article does not mention whether gravity is included or not.

Tsunoda *et al*. [33] presented a more complex swing model that was based on recorded golf swings. Their model was made up of two rigid elements that represented the upper arm and the forearm and a flexible shaft. Full golf swings, including backswing, were recorded using a motion capture system (10 cameras, f = 250 Hz, 19 body markers, four shaft markers). All relevant joint angles of the left arm were then calculated and imposed upon the model. The disadvantage of the increased complexity was that parameters could not be modified as easily, and that the mathematical model behind the simulation could only be solved by specific multibody kinetic software. Therefore, Tsunodo *et al*. utilized the software MADYMO (Mathematical Dynamic Models) to run their simulations. The model was validated by comparing shaft strain measurements and model outputs. This validation determined that the model outputs correlated with the measurements from the real swing, but the model over-predicted shaft strain in the critical phase just before impact, and some of the model outputs included distinct oscillations that did not occur in reality. One possible explanation for these differences could be that the rigid connection of grip and arm segment in the model neglected any dampening that in reality, may be provided by the hands of the player.

4.1.2 Forward dynamics: simplified torque functions

While the initial inverse dynamics analyses of the 3-D kinetics of the golf swing mainly focused on just one rigid segment (the club), more recent studies also involved the consideration of two or three segments and of flexible club models. Jones [32] presented a mathematical, 3-D model of the golf swing. The model consisted of two segments, one of which represented the arms and the shoulder and the other one the club. The two segments were connected to each other by using a spherical joint. This joint allowed movement in all three rotational degrees of freedom and represented the wrist joint of the player. The arm/shoulder segment was connected to a joint whose coordinates were fixed in space and that allowed two kinds of rotation: about a ‘neck’ axis (similar to the 2-D double pendulum model), and rotation about an axis, connecting both shoulders (comparable with the shoulder’s flexion/extension axis in the anatomical position and thus controlling the elevation of the swing plane). For his swing simulation, Jones split the downswing into two phases. In the first phase, the wrists were locked while the arm segment started to rotate. In the second and final phase of the downswing, the wrists were released, allowing a ‘natural’ acceleration of the club towards the ball caused by centrifugal forces. To demonstrate the capabilities of the model, Jones presented examples for the effects of changing the club orientation at the beginning of the downswing or changing the position of the centre of gravity of the club. Even small variations of these parameters caused major changes in the position of the model segments at impact, jeopardizing the accuracy of the swing. This demonstrates how small changes in the mechanical characteristics of golf equipment can have significant effects on the outcome of the swing unless the coordination of the motion is adapted to these changes by the golfer. Unfortunately, Jones provided no details on the joint torques driving the model and acting on the wrist joint, and gave no information regarding the validation of his model.

A recent study utilized a 3-D model of the golf swing in order to characterize the relationship between the point of wrist release, shaft deformation, and club head velocity. Suzuki *et al*. [34] modeled the swing as the motion of a double pendulum. Furthermore, their model incorporated a flexible club and allowed supination of the forearm, thereby making the model 3-D because of the offset of the club head’s centre of gravity relative to the shaft axis. Using this model, they showed that by keeping the shoulder torque constant, the kinetic energy of the club head at impact could be maximized if the wrist release coincided with the time point at which the deflection of the oscillating shaft became zero for the second time. Once the optimum release point had been identified, they theorized that additional energy could be transmitted to the club if the players applied a ramp-like shoulder torque profile. They proved that these torque profiles could in fact increase club head velocity and suggested that highly-skilled players could use their full-body motion in order to be able to create these torque profiles. However, multiple shaft oscillations, as seen in their simulation, are not typical for downswings [42]. Once again, the reason for these oscillations could be a lack of dampening provided by the model, both in terms of the material characteristics of the simulated shaft and the missing elasticity of the simulated hand-grip connection.

MacKenzie [29] performed a comprehensive simulation study to determine the effect of shaft stiffness on impact velocity. Simulations were based on a model consisting of three body segments (torso, arm, and hand) and the club (Figure 3). The club part of the model was further divided into three segments that were connected with flexible links. These links were designed to account for the bending and dampening characteristics of the shaft. Body segments were driven with torque generators with torque curves derived using evolutionary optimization algorithms. MacKenzie’s conclusion was that ‘no particular level of shaft stiffness had a superior ability to increase clubhead speed’ [29]. However, he still found that up to 9.65 m/s of the total impact velocity was generated by the unloading of the flexible shafts, but because of the dynamic interaction of the model segments and the optimization

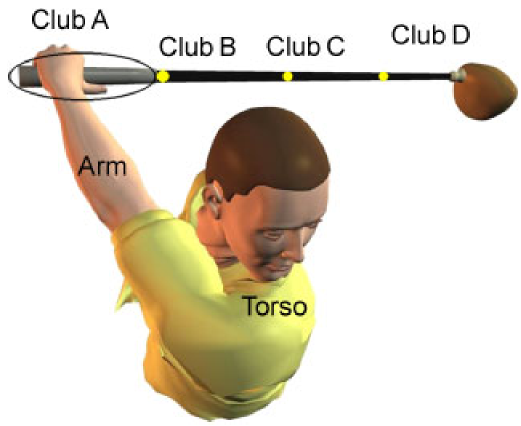

Figure 3. Six-segment model [29]. Reproduced by kind permission of the University of Saskachewan.

approach chosen, the model was able to reach almost the same absolute club head velocities with a rigid shaft with identical maximum joint torque levels in place.

4.2 Full-Body Models

Compared to the numerous models focusing on the club and arms, relatively few researchers have attempted to model the full-body movement of the golfer (Table 2).

An early full-body model of a golfer performing a downswing was presented by McGuan [36], who utilized the multibody dynamics software ADAMS (Automatic Dynamic Analysis of Mechanical Systems). The body segment trajectories of McGuan's simulation were based upon data obtained from a golf swing. McGuan [43] pointed out that there are two intrinsic problems when driving complex rigid body models with motion capture data obtained from real movements. In most cases the equations of motion of the model will be mathematically over-determined when the trajectories of multiple markers are constraining the system, and the motion capture markers will change their position relative to their corresponding body segments because of skin movement or instrumental errors. McGuan [36] overcame these problems by introducing weightless spring elements, which connect each motion capture marker with the corresponding virtual marker on the rigid body model of the golfer, thereby allowing the rigid body movement to fit the recorded marker trajectories. He then used a two-step approach in order to allow a simulation of the movement. During the first step, the body segments were moved by the marker trajectories as described earlier, and angular kinematics were recorded at each joint. During the second step, the marker trajectories from motion capture were ignored and the model was set into motion by joint torques. McGuan [36] demonstrated that this model could be used to show the effects of shaft stiffness variations on club head velocity and dynamic loft angle at impact. It is interesting to note that the simulated swings were relatively inefficient in terms of club head speed and loft angle when the shaft stiffness was changed, unless the torque curves of the model were adapted. Unfortunately, McGuan did not provide any information on further results obtained from this model and how the model was validated.

Nesbit *et al.* [35] simulated the downswing of a golfer by means of a model of the golfer (15 segments) and a finite element model of the club (Figure 4). Again the model was created using ADAMS. However, only five of the 15 body segments were actually driven by motion capture data from real golfers, and Nesbit *et al.* gave no detailed information regarding the validation of their model.

More recently, Nesbit presented the results of another simulation study using a more complex model [37] (Figure 4), consequently applied this model for a work and power analysis of the swing [38], and published a detailed description of the model [39]. His objective was to characterize the complete 3-D kinetics and kinematics of the golf swings of several subjects. After doing so, his aim was to highlight similarities and differences among golfers. He analyzed one swing of each of 85 subjects. All players used the same driver for their swings. The

Figure 4. Fifteen-segment full-body model [37]. Reprinted from *Journal of Sports Science and Medicine* Volume A: Nesbit SM; A three dimensional kinematic and kinetic study of the golf swing, 499–519, 2005, with kind permission from the *Journal of Sports Science and Medicine*.

angular displacement histories of each joint were used to define the movement of a full-body model of the golfer, which included sub-models of a rigid android, a flexible club, an impact model, and a ground surface model. It was assumed that the load between both hands was distributed equally, all joints were either ideal ball and socket or hinge joints, and the model did not include any representation of muscles or tendons, so no strain energy could be stored (for example, at the top of the backswing). Validation was performed by comparing manually-calculated joint torques, results from other studies, and ground reaction force data, and showed reasonable agreement. However, it was not possible to use the derived joint torque profiles to drive all the degrees of freedom of the model's joints in a forward dynamics way because this resulted in unpredictable results and simulation failure [39]. Nesbit's results [37] support the idea that each golfer has a unique kinematic and kinetic swing 'signature'. The overall coordination was found to be a very important factor for maximizing club head velocities: the subjects did not use hindrance torques to block their wrists as proposed by earlier simulation studies; instead they coordinated the full-body motion in such a way as to delay wrist release and to achieve peak club velocity at impact. These findings highlight the value of including the full-body motion of the player in golf simulation studies, as important information regarding body segment coordination might be missed if only 2-D data are considered.

Another study concerning the full-body kinetics of a golfer was presented by Kenny *et al.* [40]. Their objective was to validate a full-body computer simulation of a golfer swinging three clubs with different lengths (46'', 48'', and 50''). One subject performed eight swings under each club condition, which were recorded using a five camera motion capture system (f = 240 Hz). A full-body model was scaled to the anthropometrics of the subject based on 54 body measurements, and both inverse and forward dynamics simulations were performed using the LifeMOD plug-in of the ADAMS software. As the primary goal of the authors was to validate the model, they correlated the marker trajectories and club head velocities of the model and the real player and found good agreement (Pearson coefficient >0.99). No data were presented to show that there was no linear shift between the model and human trajectories, which could occur even if the correlation coefficient between the two data sets is high. The only available data set to validate the kinetic output of the model were grip force measurements from previous studies, which also showed reasonable agreement. Using the validated model, Kenny *et al.* were able to demonstrate that selected muscles need to produce significantly higher amounts of force when swinging longer clubs to maintain the same club head velocity.

Using a similar approach, yet without performing a detailed analysis of muscle forces, swings performed by one golfer with four different club models have also been simulated [41] (Figure 5). Once again, ADAMS software and its LifeMOD plug-in were used, but in this case, the shaft model was more detailed and the club head properties were based on a Computer Aided Design (CAD) model derived from stereoscopic images. Similar to the findings of Tsunoda *et al.* [33], shaft deflection patterns were overlaid by unrealistic oscillations that were probably caused by the rigid modeling of the grip-hand interface or an incorrect dampening factor. Nevertheless, after filtering the estimated shoulder joint torque curve, the resulting pattern was similar to the ramp-like pattern that was suggested by Suzuki *et al.* [34].

It is interesting to note that so far, the studies by Kenny *et al.* [40] and Betzler *et al.* [41] appear to be the only studies of the full-body kinetics of the golf swing that include

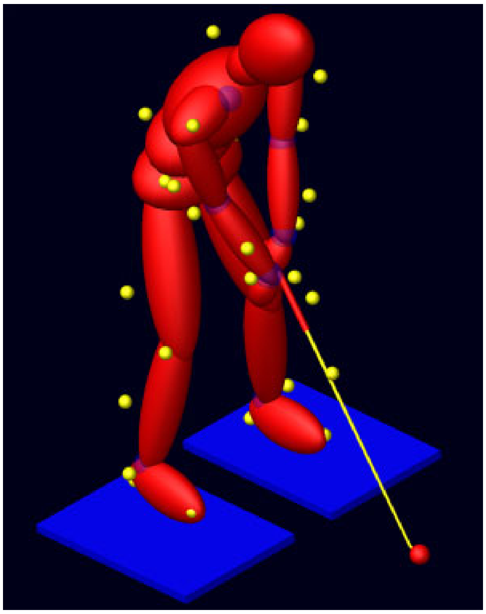

Figure 5. Eighteen-segment model [41]. ©2008 Taylor and Francis. Reproduced by kind permission.

a comparison of the effects of different golf clubs on the kinetics of the swing. There is great potential in this area for researchers to increase the understanding of the effects of equipment changes on golfers.

5. ASSUMPTIONS AND SIMPLIFICATIONS IN CURRENT MODELS OF THE SWING

5.1 Limitations of Current 2-D Models

When looking at the presented 2-D models of the golf swing, it is not always easy to differentiate between cases where authors intend to use simplifications and where they assume that their model represents the actual behavior of the system under investigation. The following section discusses typical features of the 2-D studies and makes reference to findings from human swings to determine whether the 2-D models made justified assumptions or made unrealistic simplifications. These simplifications may be justified in order to create the models, but it is important to know the limitations and trade-offs of a model when applying it.

Typical characteristics of some of the 2-D swing models are:

1. Club head, hands, and a fixed pivot point ('shoulder' or 'neck') all stay in one static plane for the entire downswing.
2. There is no translational movement of the pivot point of the upper segment.
3. The distance between the central fixed pivot and the wrist pivot point remains constant throughout the downswing.
4. Only rotations at a small number (two to four) of simplified joints are assumed to have a meaningful effect on the outcome of the swing; the movement of the rest of the body is neglected.
5. The effect of gravity on the swing dynamics is neglected.
6. The shaft is considered to be rigid.
7. The centre of gravity of the club head is projected to the tip end of the shaft; there is no offset from the swing plane.
8. The torque profiles that drive the model are simplified.
9. The right arm is not included.
10. Analysis starts from the point at which the downswing commences; the backswing is not included.

The validity of these assumptions and their possible effects are considered in more detail in the following subsections.

5.1.1 Planar swing

In a total of 24 studies, it was assumed that club head, shaft, hands, and the left arm all move within one plane and that this plane does not change its elevation throughout the downswing [7–28,32,34]. This facilitates analyzing the swing as a 2-D movement in this plane, thereby simplifying swing models considerably. It is interesting to note that some authors [11,20] do not provide any references to studies to justify this assumption, and other authors [10] refer to the early work of

Cochran and Stobbs [7]. In their book, however, Cochran and Stobbs [7] present four pictures of golfers with traces of club head and hands to show qualitatively, without referring to 3-D quantitative data, that the arms, hands, and the club head remain in one plane. Only one previous study [44] suggested that Cochran and Stobbs' simplification is true. The majority of studies indicate that the segments do not move in a single plane or that the plane is not static for at least some part of the downswing [30,31,37,45–47]. As the majority of the studies, and in particular, the more recent investigations, came to the conclusion that the segments do not remain in one plane for the majority of players, it is more likely that it is a simplification rather than a valid assumption that the downswing is planar. Furthermore, Teu *et al.* [48] pointed out that the external rotation of the humerus and supination of the forearm also contribute to the acceleration of the club. These actions would tend to rotate the club out of the observation plane unless it is in line with the arms or the club's position is adjusted at the same time to stay in plane. Therefore, it is suggested that the effect of these simplifications is looked at in more detail by comparing appropriate planar and non-planar models. If for a specific research question, a planar model can be shown to produce comparable results to a non-planar model, the simpler model may be preferable as it is easier to implement and its results are easier to interpret.

5.1.2 Fixed pivot point

Another feature of the majority of the 2-D studies is the use of a fixed pivot point as the central axis of rotation [7,8,10–14,16–28]. Some authors do not provide a reference as to why they make this assumption [25], while others refer to the work of Cochran and Stobbs [7]. Cochran and Stobbs only present three drawings based on stroboscopic, 2-D pictures supporting the idea that the pivot point remains in a static position. A more recent study using 3-D motion capture methods stated that the pivot point can change during the downswing [37]. In this context, it is interesting to note that Jørgensen presented a revised version of his original swing model [8] by adding a horizontal translation of the central pivot point towards the target during the downswing [9]. He showed the effect of this potentially more realistic design of the model by comparing the club head velocities achieved by the updated version to results from a model with a fixed pivot point. The peak club head velocities were up to 17 per cent faster when the pivot point moved [9]. Jørgensen also suspected that skilled golfers swing in a way that elevates the central rotation axis slightly vertically just before impact [9]. This upward shift was also observed by others [33], and has been shown to increase club head velocity by approximately 3 per cent [15].

5.1.3 Distance between fixed pivot and wrists

It has been noted [49] that all two-segment models assume the distance between the fixed pivot and the moving wrist pivot to be constant throughout the downswing, despite the fact that golfers may adjust this distance either by changing the pivot point of their arms while maintaining the club in the same position, by flexing or extending the elbow, or because of shifts in the position of the wrist joint centre of rotation relative to the forearm as observed by Milburn [50]. It is difficult to determine which of these three factors contributes to the 'constantly changing radius' of the inner hub reported by Nesbit [37] for human swings, but it may play a significant role in delaying the wrist release, which cannot be reproduced by two-segment models with a fixed pivot point and fixed arm length [37].

5.1.4 Role of full-body motion

It is difficult to determine the effect of using one or two 'torso' and/or 'shoulder' torque generators to represent the contribution of the complete body motion of the golfer. The torso and legs may, for example, just compensate the activity of the shoulders and arm in terms of keeping the body in balance, in which case it is justified to neglect their action and to assume that they only play a secondary role. However, it has been found that the full-body motion of the player can increase the club head velocity, for example, by shifting the pivot point of the arms [9]. Therefore, it could prove to be useful to analyze the full-body contribution to club head velocity generation in 3-D, and to attempt a more realistic segmentation, as presented in the more recent 3-D full-body simulations [35–41].

5.1.5 Gravity

The effect of gravity on the dynamics of the downswing is neglected by many simple models of the downswing because it is assumed that it has a low-order effect. For example, Jørgensen [8] decided to disregard the fact that the contribution of gravity towards the total torque acting at the shoulder would change depending on the angular position of the levers. Instead, Jørgensen introduced a torque factor acting at the fixed pivot point rather than at the centre of gravity of the levers. The amount of torque was constant and therefore independent of the position of the levers. Other models do not include gravity at all [10,12–14,16,32], stating, for example, that the centripetal acceleration of the wrist just before impact can be up to 20 times greater then the acceleration due to gravity [32]. In his revised model, however, Jørgensen [9] included gravity as a force acting on the moving levers rather than the simplified torque used previously. He showed that ignoring gravity completely can decrease the impact velocity by approximately 8.5 per cent. This demonstrates that gravity might be a more important factor than initially thought, especially at the beginning of the downswing when centrifugal forces are still relatively low. This was also confirmed by recent findings using a double pendulum model [18]. In their study, Chen *et al.* suggested that 10 per cent of the overall work throughout the downswing can be attributed to gravity [18]. Even if both of these findings could be used to argue that gravity has a second-order effect, the computing capacities that are available today should allow the incorporation of gravity into models.

5.1.6 Shaft properties

The vast majority of previous models do not include a flexible representation of the golf club [7–10,12–16,18–25, 27,28,30–32]. In the studies that do include a flexible shaft, the level of sophistication of the shaft models is very broad. It ranges from simple models that assume uniform stiffness along the shaft [17,34,36], over three- [29], eight- [40], and 15-segment [37,39] models, up to detailed models in which shaft stiffness varies along the shaft [11] or in which the finite element approach is used [33,41]. No study has attempted to include a model of the non-homogenous structure of graphite shafts into simulations or has considered the fact that carbon fiber-reinforced composites may have strain rate-dependent material properties [51].

It is unclear in which cases the ability of the shaft to bend during the swing needs to be included in swing models, and it is unknown what the implications of not including it are. Tsujiuchi *et al.* [26] compared shoulder, elbow, and wrist joint torques with and without considering shaft stiffness, but it was unclear how these joint torques were determined and how the model accounted for shaft deflection. More recently, optimized swings with a model, including shafts with different stiffness properties, were compared in an extensive simulation study [29]. It was found that using a flexible shaft resulted in relatively small increases in impact velocity (up to 1.87 m/s), but it was noted that using a rigid shaft had effects on the kinematics of the body segments included in the model. For instance, the angular velocity of the wrist angle decreased by almost 20 per cent when the model swing was optimized with the flexible shaft. It is difficult to determine whether this is an indicator that studies that have used a rigid representation of the shaft (looking for example at wrist release regimes) should be repeated with a flexible shaft representation. It is worth noting that the characteristics of the shaft may have more of a role in the presentation of the club head to the ball at impact rather than generating additional club head velocity per se.

5.1.7 Club head properties

The majority of studies combine the properties of the shaft and the club head within one rigid club segment or assume that the club head is a mass point located at the end of the shaft, without offset from the shaft axis [8–10,12–28]. Of the 2-D simulations, only one has tried to account for the offset of the club head's centre of gravity by incorporating an offset [11]. From simulations in which the effect of the centre of gravity position on shaft deflection was studied [29], it could be seen that assuming the centre of gravity of the club head to be aligned with the club head will have a marked effect on shaft deflection at impact. Therefore, if the shaft is modeled as a flexible element, it appears necessary to consider the offset position of the centre of gravity of the club head (unlike Suzuki *et al.* [17]). Furthermore, it may be necessary to consider the inertial properties of the club head if the torsional deformation of the shaft is analyzed. It could be hypothesized that modern, high moment of inertia club heads impart an increased torsional load on the shaft when the club is rotating around the shaft's longitudinal axis just before impact to square the clubface.

5.1.8 Choice of torque functions

Unless the inverse dynamics approach is chosen, it is necessary to make an assumption as to which shape of joint torque pattern is chosen so that the model is set into motion when a simulation is performed. The choice of the shoulder torque function reaches from simple constant torque values (see for example [14,16,18]) to complex patterns that are determined using optimization algorithms (see for example [23–25,27,29]). In many cases, the wrist torque is adjusted so that the angle between the arm and the club lever remains constant until wrist release [14,18,23]. After this, the wrist torque is typically set to zero or adjusted to simulate different types of wrist action (for example, to delay wrist release or to give the club extra acceleration). While the assumption of a constant shoulder torque simplifies simulation significantly, it is important to note that it would be very difficult for a golfer to maintain a constant shoulder torque throughout the downswing. First, it is unlikely that golfers would be able to coordinate their body motion and muscle output in a way that would keep the shoulder torque constant even when the load changes during the swing. Second, the capacity of human muscle to produce force changes with its contraction length (force–length relationship). This would induce changes in the resultant joint torque as the arm moves through different orientations.

Another simplification that is made by some studies is that joint torque generators are simply activated and deactivated, resulting in steep increases or decreases in the applied torques when the system changes from one state into another. This has been criticized because muscle contraction that would result in such steep changes in joint torques may well exceed the physiological limits of muscle in terms of the maximum activation rate [25]. Furthermore, the ability of muscle to produce force decreases with contraction velocity, which is also neglected when simplified (constant) torque functions are applied. Sprigings and Neal [25] suggested the use of modified torque generators that would be limited to more realistic changes in joint torque. Consequently, these generators have been used in a number of simulation studies [23,24,29].

As it is not possible to measure the muscle forces applied by the golfer directly to verify any of the approaches described here, care should be taken when generalizing results obtained using simplified torque functions. It appears that more research into inverse dynamics modeling of golf swings performed by golfers of varying ability is needed before it is possible to find guidelines for simple but more realistic joint torque functions. From the shoulder torque profile determined by Betzler *et al.* [41] for one golfer, it would appear that a ramp-like shoulder torque that falls back to zero at impact as suggested in Suzuki *et al.* [17] could be a good estimate.

5.1.9 Right arm not included

Golfers generally grip the golf club with both hands, and consequently a closed-looped configuration is created. For any kind of inverse or forward dynamics modeling, this creates difficulties because an assumption has to be made with regards to the load distribution between the two arms. One option is to assume an equal load distribution between the two arms [39]. The vast majority of swing models, however, ignore the right arm of the player or combine both arms in one segment [7–20,22–34].

5.1.10 Backswing not included

The simulation models referenced in sections 3 and 4 of this review do not attempt to include the backswing [7–32,34,36], with the exception of Tsunoda *et al.* [33]. For the full-body swing models [37–41], it appears that the backswing is included, but the results are generally only reported for the downswing portion of the swing. While it is a reasonable simplification to start simulations from the top of the backswing when the shaft is modeled as a rigid link, it is unclear what effect the lack of prebending at the top of the backswing will have in cases where flexible shafts are included, but the backswing is omitted [11,17,29,34]. It has also been highlighted that current swing models lack the ability to include any strain energy that might be stored in the human soft tissue at the top of the backswing, which is released during later parts of the swing [39].

5.2 Limitations of Current 3-D Models

Two groups of 3-D models of the golf swing could be identified: those focusing on the club and the arms, and those that simulate the full-body motion of the player. The first group has many similarities to the 2-D models previously discussed. For example, most of them still assume that there is a fixed pivot point [29,32,34], so the limitations outlined earlier still apply.

The second group of 3-D studies overcomes this problem by including the full-body movement of the golfers. In some studies [35,36], the authors give very little information regarding the validation of their models. While the validation results of Kenny *et al.* [40] were very promising in terms of the correlation between recorded and simulated marker trajectories and club head velocity, their validation lacked a comparison of the ground reaction forces produced by the subject and the model as a means of validating the kinetic output of the simulations. A comparison of recorded and simulated ground reaction forces has been attempted by other authors [52], but results were only reported for one subject, and while the general weight shift pattern of the simulation model was realistic, there were still distinct differences between simulated and measured ground reaction forces (Figure 6).

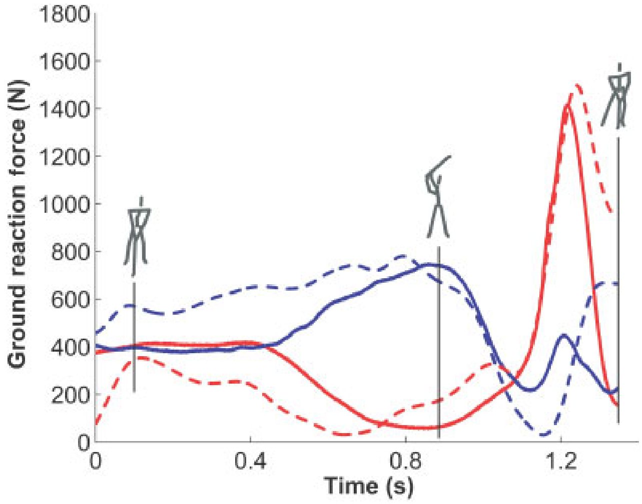

Figure 6. Comparison of ground reaction forces produced by a human player (solid lines) and a forward dynamics simulation model (dotted lines) [53]. Reproduced by kind permission of Savvy Knowledge Systems Corporation.

6. CONCLUSION

This article shows that swing modeling and simulation has significantly improved our understanding of a number of factors influencing golf performance. The wrist action has been of particular interest to researchers, and it has been shown that impact velocity can be increased by delaying wrist release [8,14,23]. There is no consensus whether a delay in wrist release should be achieved by a well-coordinated full-body motion [17,34] or by an active, negative wrist torque [18] that keeps the wrist angle constant despite the centrifugal forces pulling on the club segment when a certain angular shoulder velocity is reached. It is unclear whether an additional positive wrist torque after wrist release will enhance impact speed [18,23,25] or whether this is unnecessary [8]. In inverse dynamics analyses, this positive torque has been registered [30,37], but it is unclear whether it can be actively produced by the golfer despite the high angular velocity of the club relative to the arm at that stage of the swing. A number of studies have reported a drop in wrist torque below zero just before impact, which would slow down the angular wrist velocity [26,30,37].

Modified double pendulum models have shown that a shift of the central pivot point of the swing during the last stage of the downswing (just before impact) could enhance club head velocity [9,15]. Another factor that has been found to have the potential to increase impact velocity is to move the tee position towards the left (for a right-handed player), along the target line towards the target [14,18].

It is encouraging that these findings appear to be in general agreement with perceived golf wisdom. Nevertheless, a number of these findings still lack experimental verification with golfers. Verifying these findings, however, is difficult because of the fact that it is likely that individual golfers will employ different swing styles, depending on their ability, anthropometrics, or 'feel'.

If swing models are not based on experimental data obtained from human swings, it is suggested that at least a basic validation of the simulated swing characteristics should be included (for example, based on published stroboscopic picture sequences). Sometimes this step is omitted (for example, [18, 27]). In the case of one study [27], a comparison of the simulation results with photos or videos of actual swings would have shown that the elbow joint flexion and extension proposed by one of the simulations is not usually used by skilled golfers.

It is noted that the majority of studies regard an increase in impact velocity as the main means to optimize performance in increasing impact velocity [8,12,14–19,21–23,25–27,29,34,36]. In golfing, many other factors can have a performance-enhancing effect; for example, a more consistent impact position. No study was found that attempted to simulate the effect of perturbations on the accuracy of the swing, even if one could argue that the majority of golf players would benefit more from improving their accuracy instead of their distance.

It would be desirable if future models integrate mechanical, anthropometrical, and motion analysis aspects. As such, model simulations could supply information of body equipment coupling for swing skill improvements and/or realistic suggestions for equipment optimization.

In summary, this article has shown that modeling and simulation have been used in various investigations to analyze the mechanics of the golf swing. The sophistication of the models range from very simple (for example, a gravity-driven double pendulum model in the first part of White's study [16]) to very complex (for example, a full-body model that includes various sub-models, such as that proposed by Nesbit [39]). Each model has its justification and purpose, but it appears at this point in time that more realistic models have the greatest potential to improve our understanding of the dynamics of the golf swing, while not becoming over-complicated. Full-body modeling has its merits but still appears to be in a stage in which the methodology needs to be improved. Potentially, more specialized models with a reduced number of segments, and focusing only on specific questions, could help to overcome the problem of the complexity of full-body models, such as that proposed by MacKenzie [29]. These models may include more realistic muscle representations or could be based on a large number of recorded human swings and then be used for a cluster analysis to group individual swings with similar kinetic characteristics. There is also a need for models that explore the effects of variations in equipment properties while ensuring that players react the same way to the equipment variations as the model. Furthermore, more realistic representations of the shaft, the grip, and the properties of the hands could potentially help to understand the dynamic interaction between player and equipment.

The article identifies various limitations of the double pendulum-based swing model that is broadly used in simulation studies, yet this simple model appears to be the one model that has been used most successfully to understand the underlying physics of the golf swing. The effect of the simplifications applied by the double pendulum model on the conclusions drawn from these models is not known, even if these conclusions generally agree with the perceived wisdom of golf coaching. No study could be found that systematically compares the effect of different degrees of model sophistication on simulation results. Similar studies have been performed in other cases (for example, comparing different foot models [54]), and there is a need for similar studies that justify the degree of simplicity of many models of the golf swing.

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