3.2 Logarithmic and exponential functions

Subject content	Suggested teaching activities
• understand the relationship between logarithms and indices, and use the laws of logarithms (excluding change of base)	Start by defining the terms 'logarithm' and 'exponential', linking to the concept of indices. To help learners understand a statement such as $\log_a x = b$, describe it to them in words such as "What power of *a* is *x*? Answer: *b*" An introduction with animation showing the relationship between logarithms and exponentials is at: www.purplemath.com/modules/logs.htm . Learners should practise converting expressions from logarithmic to exponential form and from exponential form to logarithmic. Most textbooks will have plenty of examples of this type. A useful worksheet (includes the laws of logarithms) is at: maths.mq.edu.au/numeracy/web_mums/module2/Worksheet27/module2.pdf **(I)** To introduce the laws of logarithms, start with statements $\log_a x = b$ and $\log_a y = c$. Use targeted questioning to encourage learners to write the exponential forms of these statements and reach the conclusion that $a^{b+c} = xy$, rewriting this in logarithmic form to obtain $\log_a xy = \log_a x + \log_a y$. Ask learners to obtain the other two laws in a similar way. Learners will then need to practise applying these laws. Eight files of notes, worksheets and revision (log in for free download) are at: www.tes.co.uk/teaching-resource/a-level-maths-logarithms-worksheets-and-revision-6146791 **(I)** An additional resource which demonstrates the above approach is at: www.mathsisfun.com/algebra/exponents-logarithms.html Suitable past/specimen papers for practice and/or formative assessment include **(I)(F)**: Nov 2014 Paper 31, Q1; Jun 2014 Paper 31, Q6
• understand the definition and properties of e^x and $\ln x$, including their relationship as inverse functions and their graphs; including knowledge of the graph of	You can introduce the exponential function e^x in various ways. One approach is to use a graph plotter to show learners the graphs of various exponential functions, e.g. $y = 2^x$, $y = 3^x$, $y = 5^x$. Develop the idea of a particular exponential function that lies between $y = 2^x$ and $y = 3^x$, such that its gradient function

Subject content	Suggested teaching activities
$y = e^{kx}$ for both positive and negative values of k	is the same as itself. With a suitable graph plotter you can demonstrate that the gradient function of $y = e^x$ is e^x. **Extension activity:** There are other, formal, approaches that you could use with more capable learners. For example you could consider compound interest and the limit of the series $\left(1+\frac{1}{n}\right)^n$ as shown at: www.mathsisfun.com/numbers/e-eulers-number.html Encourage learners to obtain the logarithmic form of the statement $e^x = a$ and so introduce them to natural logarithms. Building on the work done in Pure Mathematics 1.2 'Functions', develop this into the inverse relationship between e^x and $\ln x$ and demonstrate the inverses on a graph plotter. An interactive exercise covering this relationship is at: http://hotmath.com/help/gt/genericalg2/section_8_5.html **(I)**
• use logarithms to solve equations and inequalities in which the unknown appears in indices, e.g. $2^x < 5$, $3 \times 2^{3x-1} < 5$, $3^{x+1} = 4^{2x-1}$	As a whole class exercise, work through some examples of increasing difficulty, using carefully directed questioning to work through the solutions. Textbooks will include many examples of this type of question and the interactive exercise at the link above includes some too. Demonstrate examples using inequalities, with learners finding critical values first and then deducing the set of solutions. It is helpful to highlight to learners the sign of ln *x* for $0 < x \le 1$, perhaps through an example where the inequality reverses. Past/specimen papers and mark schemes are available to download at www.cambridgeinternational.org/support **(I)(F)** Jun 2014 Paper 32, Q2; Paper 33, Q1
• use logarithms to transform a given relationship to linear form, and hence determine unknown constants by considering the gradient and/or intercept, e.g. $y = kx^n$ gives $\ln y = \ln k + n \ln x$ which is linear in ln *x*and ln y; $y = k(a^x)$ gives $\ln y = \ln k + x \ln a$ which is linear in *x* and ln *y*	If you relate this technique to practical situations, this will help learners when they need to use it in their scientific subjects. Common forms of equation are $y = Ab^x$ and $y = Ax^b$. Learners will need to be able to write these equations in logarithmic form and hence relate them to the equation of a straight line. Sometimes the variables will be letters other than *x* and *y* so learners need to spot the form of the equation in order to distinguish the variables from the constants. A useful summary for dealing with situations involving $y = Ax^b$ is at: http://mathbench.umd.edu/modules/misc_scaling/page11.htm Either work through this with learners in class or they could study it independently. **(I)** Use a similar approach for equations of the type $y = Ab^x$. Work through such an

Subject content	Suggested teaching activities
	example in class, making use of a graph plotter to demonstrate the straight line obtained. Textbooks will provide learners with many useful practice questions. For variety, try to choose examples which involve variables other than x and y. Often, learners are asked to work from a given graph in straight line form. Common errors involve learners considering y values rather than ln y values, so practising questions will help to avoid such errors. The Paper 2 past exam papers have examples of this type. To help reinforce this point, split learners into groups or pairs and ask each of them to prepare a question. A simple way to do this is for learners to 'work backwards' from a logarithmic relationship e.g. $P = At^b$. Each group chooses values for A and b, works out the coordinates of two pairs of coordinates and draws an appropriate straight line graph. Learners circulate their graphs around the other groups who then identify the logarithmic equations used to draw the graphs. Suitable past/specimen papers for practice and/or formative assessment include **(I)(F)**: Jun 2013 Paper 32, Q3

Past/specimen papers and mark schemes are available to download at www.cambridgeinternational.org/support
The resource list for this syllabus, including textbooks endorsed by Cambridge International, is available at www.cambridgeinternational.org