3.1 Algebra

Subject content	Suggested teaching activities
• understand the meaning of $\|x\|$, sketch the graph of $y = \|ax + b\|$ and use relations such as $\|a\| = \|b\| \Leftrightarrow a^2 = b^2$ and $\|x - a\| < b \Leftrightarrow a - b < x < a + b$ when solving equations and inequalities, e.g. $\|3x - 2\| = \|2x + 7\|$, $2x + 5 < \|x + 1\|$; graphs of $y = \|f(x)\|$ and $y = f(\|x\|)$ for non-linear functions f are not included;	To introduce the notation, start with a numerical value, e.g. –5, and discuss the meaning of $\|-5\|$. Help learners to deduce the results $\|a\| = \|b\| \Leftrightarrow a^2 = b^2$ and $\|x - a\| < b \Leftrightarrow a - b < x < a + b$ as part of a class discussion. Some really useful resources are at www.tes.co.uk/teaching-resource/a-level-maths-c2-modulus-function-worksheets-6146818: • 'Modulus Function Introduction' provides a worksheet for learners to complete. **(I)** • 'Solving Modulus Equations and Inequalities' could be used for consolidation/practice. **(I)** • 'Modulus Transformations' provides practice at sketching graphs involving a modulus. Demonstrate some initially to learners using a graph plotter. **(I)** • 'Alternative Methods for Solving Modulus Equations' is a worksheet which helps learners to explore the different ways of solving this type of equation. **(I)** Learners investigate the connection between the shape of the graph of $y = ax + b$ and the shape of the graph of $y = \|ax + b\|$ by plotting a range of these using graphing software. **(I)** The graphs of various modulus functions are at: www.mathsmutt.co.uk/files/mod.htm Suitable past/specimen papers for practice and/or formative assessment include **(I)(F)**: 2020 Specimen Paper 2, Q3; Nov 2014 Paper 33, Q1; Jun 2013 Paper 31, Q4 (involves logarithms); Jun 2014 Paper 32, Q1; Paper 32, Q1; Jun 2013 Paper 32, Q1
• divide a polynomial, of degree not exceeding 4, by a linear or quadratic polynomial, and identify the quotient and remainder (which may be zero)	There are several different methods of polynomial division including inspection, the table method, and long division. This PowerPoint presentation introduces all three methods for factorising cubics. You can use the methods for any polynomial and also for division that results in a remainder: www.furthermaths.org.uk/files/sample/files/edx/Factorising_cubics.ppt When teaching any of the methods, start with a numerical example to remind learners of the thought process they need, and use this to introduce the terms 'quotient' and 'remainder'. For example, $54763 \div 8$ leads to a quotient of 6845 and a remainder of 3. Continue with a simple algebraic example $(x^2 + 4x + 1) \div (x + 2)$ which leads to a quotient of $x + 2$ and a remainder of –3. You will probably need to show learners further examples involving more complex

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	polynomials before they practise on their own. Ideas on possible approaches you can take for long division are at: www.khanacademy.org/math/algebra2/polynomial_and_rational/dividing_polynomials/v/dividing-polynomials-with-remainders and www.mathsisfun.com/algebra/polynomials-division-long.html A worksheet of examples for practising any of the methods for division is at: www.mathworksheetsgo.com/sheets/algebra-2/polynomials/dividing-polynomials-worksheet.php **(I)** There is another approach known as synthetic division but learners have to be careful when using it, especially when factorising. Textbooks will have many useful questions for learners to practise.
• use the factor theorem and the remainder theorem, e.g. to find factors and remainders, solve polynomial equations or evaluate unknown coefficients, including factors of the form $(ax + b)$ in which the coefficient of x is not unity, and including calculation of remainders	Summarise the work already done on polynomial division to show that $p(x)$ = (divisor $\times$ quotient) + remainder. Show that algebraic division can often be avoided in questions by substituting into $p(x)$ the value of x that makes the divisor zero (e.g. substituting 3 if the divisor is $x - 3$ and calculating p(3) to find the remainder). Show that the factor theorem is a special case of the remainder theorem when the remainder is zero. A good approach of this type which you could use with a whole class is at: www.mathsisfun.com/algebra/polynomials-remainder-factor.html Show examples involving finding factors, solving polynomial equations and evaluating unknown coefficients to the whole class, questioning learners individually throughout. Remind learners that they should show all their working as the use of a calculator for finding solutions to polynomial equations will not be accepted in an exam. A useful worksheet which covers basic use of the remainder theorem and evaluating unknown coefficients (log in for free download) is at: www.tes.co.uk/teaching-resource/worksheet-on-the-remainder-theorem-6140286 **(I)** More examples on the remainder theorem and on solving polynomial equations are at: www.mash.dept.shef.ac.uk/Resources/A26remainder.pdf Suitable past/specimen papers for practice and/or formative assessment include **(I)(F)**: Nov 2014 Paper 31, Q3; Paper 33, Q3; Jun 2014 Paper 32, Q5; Jun 2013 Paper 31, Q1; Paper 32, Q4

<table>
<tr><th>Subject content</th><th>Suggested teaching activities</th></tr>
<tr><td>

- recall an appropriate form for expressing rational functions in partial fractions, and carry out the decomposition, in cases where the denominator is no more complicated than:
- $(ax + b)(cx + d)(ex + f)$
- $(ax + b)(cx + d)^2$
- $(ax + b)(cx^2 + d)$

excluding cases where the degree of the numerator exceeds that of the denominator

</td><td>Examples of the three main types of partial fraction are here (log in for free download):
www.tes.com/teaching-resource/partial-fractions-examples-6140352

Some worked examples and 10 practice questions for learners to try (at the end of the document) are at:
www.mathsisfun.com/algebra/partial-fractions.html

Textbooks will also contain many examples for learners to practise.

In many questions, the first part will involve breaking down rational functions into partial fractions and later parts will use partial fractions with another mathematical technique such as binomial expansion, integration or solving differential equations. Set learners questions involving these topics when they have covered them.

Suitable past/specimen papers for practice and/or formative assessment include (I)(F):
Nov 2014 Paper 31, Q9 (includes a binomial expansion); Paper 32, Q9 (includes a binomial expansion)
Jun 2014 Paper 31, Q9 (includes a binomial expansion); Paper 33, Q8 (includes integration)
Jun 2013 Paper 31, Q3; Paper 32, Q8 (includes differential equations)</td></tr>
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- use the expansion of $(1 + x)^n$, where n is a rational number and $|x|<1$; finding a general term in an expansion is not included; adapting the standard series to expand e.g. $\left(2-\frac{1}{2}x\right)^{-1}$ is included, and determining the set of values of x for which the expansion is valid in such cases is also included

</td><td>Learners have already met the binomial expansion in Pure Mathematics 1.6 ‘Series’ so, to check their understanding, set them some preparatory questions on basic binomial expansions using the formula $(a+b)^n$, where n is a positive integer. (I)

Ask learners to work out the first few terms of the expansion of $(1+x)^n$ from the formula for expanding $(a+b)^n$, to obtain $(1+x)^n = 1+nx+\frac{n(n-1)}{2!}x^2+\frac{n(n-1)(n-2)}{3!}x^3\ldots$ This is now in a useful form for introducing negative and fractional powers.

This tutorial shows that you need the condition $|x|<1$ for negative powers because they generate an infinite series. The first few terms are only a good approximation if the values of x meet this condition and the series converges:
www.examsolutions.net/tutorials/binomial-expansion-validity/?level=International&board=CIE&module=P3&topic=1308

This link uses an example with $n = \frac{1}{2}$ and has an interesting graphical display of the approximation.</td></tr>
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	www.intmath.com/series-binomial-theorem/4-binomial-theorem.php Textbooks will include many examples for learners to practise expanding and finding the range of values for which each expansion is valid.**(I)** Demonstrate to learners how to rewrite examples of the type $\left(2-\frac{1}{2}x\right)^{-1}$ as $\frac{1}{2}\left(1-\frac{x}{4}\right)^{-1}$ so that they can go on to expand them. Suitable past/specimen papers for practice and/or formative assessment include **(I)(F)**: Nov 2014 Paper 31, Q9 (includes partial fractions); Jun 2014 Paper 31, Q9 (includes partial fractions); Paper 33, Q2 Jun 2013 Paper 31, Q2

Past/specimen papers and mark schemes are available to download at www.cambridgeinternational.org/support
The resource list for this syllabus, including textbooks endorsed by Cambridge International, is available at www.cambridgeinternational.org