1 Functions

Syllabus content	Suggested teaching activities
Whole unit	We recommend that learners cover **2 Quadratic functions** before this unit so that the manipulative skills required are established and well-practised. It is possible that trigonometric functions will be used together with the material detailed in this topic, so we also recommend that learners have covered **10 Trigonometry**. The graphing elements of **7 Logarithmic and Exponential Functions** could be done with **1 Functions**, so that the concept of an inverse function being the reflection of the function in the line $y = x$ can be more easily established. The section on understanding the relationship between $y = \mathrm{f}(x)$ and $y = \lvert\mathrm{f}(x)\rvert$, could be done before the rest of the syllabus content for **1 Functions** as part of **3 Equations, inequalities and graphs**. This unit takes the basic skills that have been acquired for Cambridge IGCSE or Cambridge O Level Mathematics and builds on them so that learners are able to handle a variety of functions in more challenging settings. The key skills, notation and concepts relating to functions that learners will have met in IGCSE or Cambridge O Level Mathematics will need to be revisited and formalised with more rigour here.
• understand the terms: function, domain, range (image set), one-one function, inverse function and composition of functions	All of these terms might have been met in Cambridge O Level or Cambridge IGCSE Mathematics. Even if this is the case, they will need formalising in the more challenging settings that learners will encounter in this course. Visual presentations will work better and be more memorable and meaningful for learners – as will relating the function to its graph. Bubble diagrams such as that shown below should be good visual introductions to the key difference between a basic mapping and a function, and the idea can be extended to consider domains and ranges more fully and also one-one functions and subsequently inverses.

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	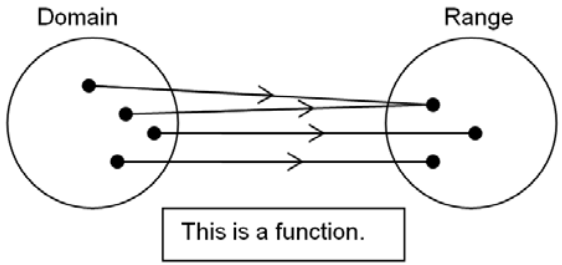 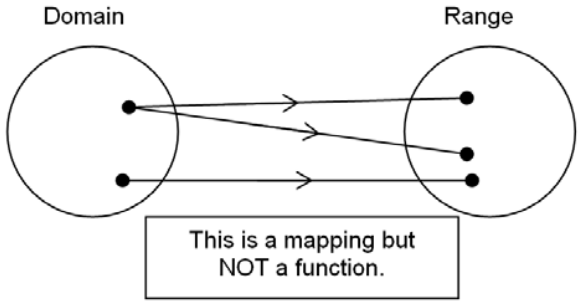 'What is a function?' at www.mathsisfun.com/sets/function.html gives a good solid overview of the basics. Emphasise the difference between a mapping and a function. 'Is it a function' by user8 at: www.geogebratube.org/m/1871 will help with this. The first five pages of the 'Functions and graphs' resource at: www.stem.org.uk/resources/elibrary/resource/30372/functions-and-graphs is a useful reminder to learners about the structure of functions and the idea of domain and range.
• use the notation $f(x) = \sin x$, $f: x \mapsto \lg x \ (x > 0)$, $f^{-1}(x)$ and $f^2(x)$ [= f(f(x))]	Recap notation ready for the more challenging settings that learners will encounter in this course. Composition notation might be new for some learners.

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• understand the relationship between $y = f(x)$ and $y = \|f(x)\|$, where $f(x)$ may be linear, quadratic, or trigonometric	It is expected that this will be undertaken with a graphical approach. Learners should both be able to draw the graph of $y = \|f(x)\|$ and find the equation of such a function given the graph. Lots of work can be done using GeoGebra software (free to download from www.geogebra.org) to consider graphs such as $y = x^2$ and $y = \|x^2\|$. Discuss any issues that arise. It is easier to start with straight lines here and consider what is useful information to establish the relationship between the distinct sections of $y = f(x)$ and $y = \|f(x)\|$. 'Understanding absolute value' at: www.geogebratube.org/material/show/id/5538 is a simple way to introduce this concept – starting with $y = 2x - 1$ and adjusting the gradient and intercept to consider various linear functions; you can adjust the gradient and intercept yourself, or ask a learner to do this. Encourage learners to carry out the manual process of drawing graphs of such functions using the ' Graphing Absolute-value functions' resource at: www.purplemath.com/modules/graphabs.htm. This resource also moves on to give an example of what the absolute values of a quadratic function would look like, which is the natural next step. This work could be combined with topic **3 Equations, inequalities and graphs**.
• explain in words why a given function is a function or why it does not have an inverse	Learners should, for example, understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. Again, the use of the bubble diagram can help learners understand this better by visualising, as will the consideration of graphs (for example trigonometric graphs). Both situations where learners are considering whether a mapping is a function or a function has an inverse, can and should be considered using bubble diagrams. Learners should use graphs. Give learners plenty of practice. Initially this should be teacher led but learners should then attempt some work independently**.** (**I**) Graphical calculators or graphing software such as GeoGebra will be really useful tools here, for example 'Is it a function' and 'Relation or Function?'.

Syllabus content	Suggested teaching activities
• find the inverse of a one-one function and form composite functions	**Inverse function** f^{-1} exists if the function is one-one (so one input value gives only one output value). If $y = f(x)$ then $x = f^{-1}(y)$, so to find the inverse function: 1. Rearrange the formula to make x the subject, so that x = …….. (BEWARE, when taking $\sqrt{}$ you must have $\pm\sqrt{}$; look at the domain of the original function to determine if you should take the positive or negative root – the x value must fit it.) $x = f^{-1}(y)$, so now the inverse function has been found but with y as the input letter 2. The input letter is usually x, so change the input letter to x 3. Domain (f) = Range(f^{-1}) 4. Range(f) = Domain(f^{-1}) The resource 'Finding inverse functions: quadratic (Ex.2)' at www.khanacademy.org/math/algebra/algebra-functions/v/function-inverses-example-2 provides excellent examples for demonstrating how to find inverse functions whilst being careful of the original domain and also demonstrating why a function and its inverse are reflections in the line $y = x$. **Composite function** Forming the composite function gf means putting the function f into the function g, wherever x was in g before. Composite functions like gf only exist if the range of values coming out of f is allowed as input for the function g. The domain of f might need to be restricted for gf to exist: $\text{Domain}(gf) \subseteq \text{Domain}(f)$ $\text{Range}(gf) \subseteq \text{Range}(g)$ Learners should verify by example that gf is not usually the same function as fg. **Extension activity:** For an extra challenge, investigate with learners when fg does not equal gf. The 'Risp 18: When does fg equal gf?' resource on the Rich Starting Points for A Level Mathematics website (Risps www.risps.co.uk) is a great way to investigate and reinforce this concept . (If the link breaks, then from the home page, go to 'The List of Risps from 1 to 40' then scroll to find 'Risp 18: When does fg equal gf?')

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• use sketch graphs to show the relationship between a function and its inverse	The resource 'Function inverse (Ex.3) at www.khanacademy.org/math/algebra/algebra-functions/v/function-inverses-example-3 provides excellent examples for demonstrating finding inverse functions whilst being careful of the original domain and also demonstrating why a function and its inverse are reflections in the line $y = x$. It should be stressed that, graphically, f and f^{-1} are reflections of each other in the line $y = x$ So for example, if the point (2, 4) lies on f, then the point (4, 2) will lie on the inverse function. Lots of practice (with answers) at finding inverse functions, restricting domains (including trigonometric functions) as well as graphing functions and their inverses can be found at: www.stem.org.uk/resources/elibrary/resource/30372/functions-and-graphs . **(I)** 'Inverse of a function' at www.geogebra.org/m/DV64WEtT is an excellent resource for relating the graph of a function to its inverse.
Past and specimen papers	
Past/specimen papers and mark schemes are available to download at www.cambridgeinternational.org/support **(I)(F)** 2020 Specimen Paper 2 Q3 (including 14 Differentiation and integration) Nov 2017 Paper 11 Q6; Nov 2017 Paper 12 Q6; Nov 2017 Paper 23 Q6 Jun 2017 Paper 11 Q4 (including 7 Logarithmic and exponential functions); Jun 2017 Paper 22 Q9 (including 2 Quadratic functions), Q12 Jun 2017 Paper 23 Q2, Q9 Mar 2017 Paper 22 Q11 Nov 2016 Paper 13 Q1; Nov 2016 Paper 23 Q10 Jun 2016 Paper 11 Q6; Jun 2016 Paper 22 Q11 Mar 2016 Paper 12 Q6 (including 14 Differentiation and integration) Mar 2015 Paper 12 Q8 (including 14 Differentiation and integration)	