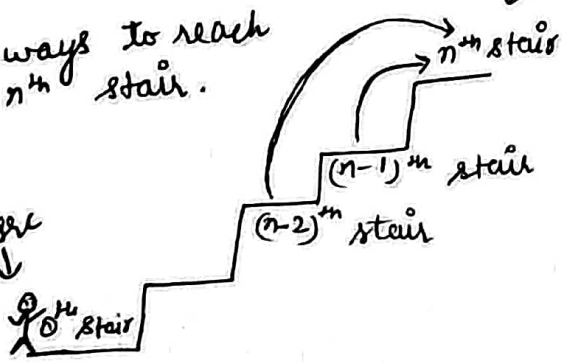
Recursion-2

⊙ **Climb Stairs →** find no. of ways to reach n^{th} stair.



Two steps allowed ⇒

① One stair at a time

② Two stairs at a time.

Easy way is that to find out how we reached n^{th} stair; two ways → from $(n-1)^{th}$ stair → from $(n-2)^{th}$ stair

$$f(n) = f(n-1) + f(n-2)$$

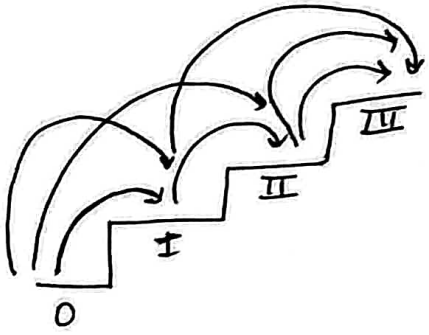
$f(n)$: no. of ways to reach n^{th} stair; $f(n-1)$: no. of ways to reach $(n-1)^{th}$ stair; $f(n-2)$: no. of ways to reach $(n-2)^{th}$ stair

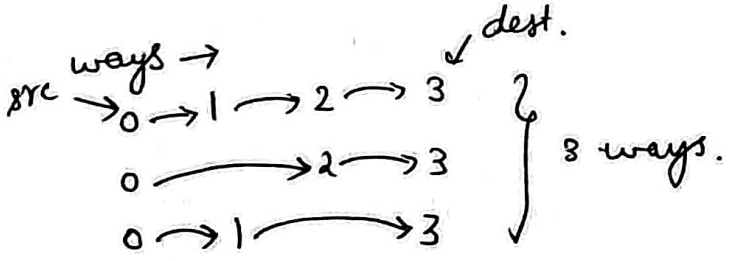
→ **Base case** → termination condition → rukna kaha hai?

0^{th} stair पर पहुँचने के कितने तरीके हैं? → 1 → not 0.

1^{st} stair पर . . . . . → 1 (0 to 1)

⇒ `if(n==0 || n==1) return 1;` ← Base case.

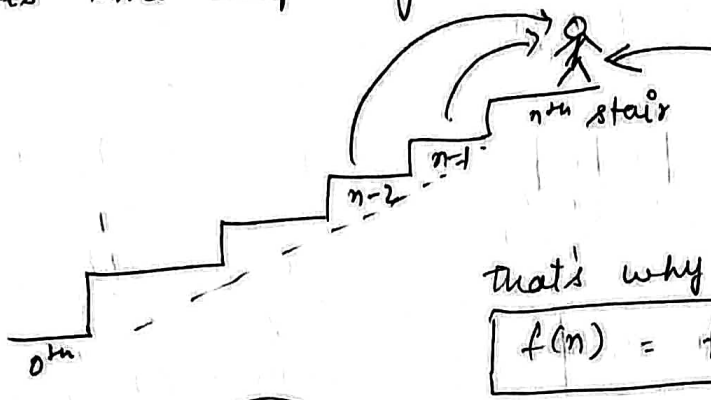




0^{th} stair → 1 way.
We are already standing at 0^{th} stair, but if we say that we have 0 ways to reach 0^{th} stair that means we can't reach 0^{th} stair ever, but we are already there, so that's why we have 1 way to reach 0^{th} stair.

```
int climbStairs(int n){
    if(n==0 || n==1) return 1;   ← Base case
    return climbStairs(n-1) + climbStairs(n-2);
}                                  ← R.R.
```

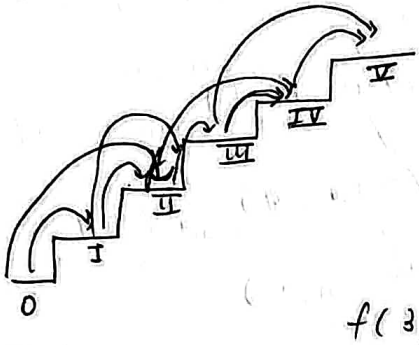
We will end up getting TLE.
It's time complexity is exponential.



Yha tak Mai kaise aayi thungi?
⇒ Ya to (n-1)th stair se ya (n-2)th stair se,

That's why

$$f(n) = f(n-1) + f(n-2)$$



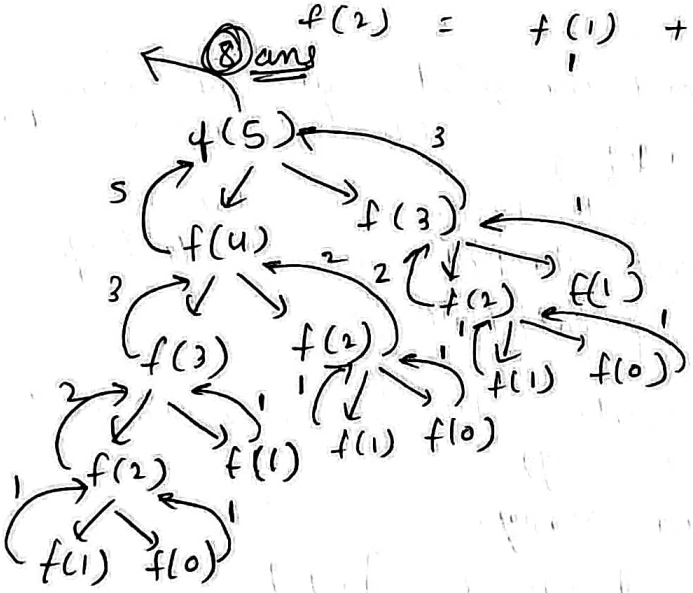
$f(5) = f(4) + f(3) = 8$

$f(4) = f(3) + f(2) = 5$

$f(3) = f(2) + f(1) = 3$

$f(2) = f(1) + f(0) = 2$

$f(1) = 1$
$f(0) = 1$



This is same as fibonacci series.

Ques Array

arr →	10	20	30	40	50

print the array.

Iterative method we know:→

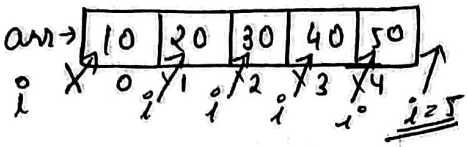
```
for(int i=0; i<n; i++){
    cout << arr[i];
}
```

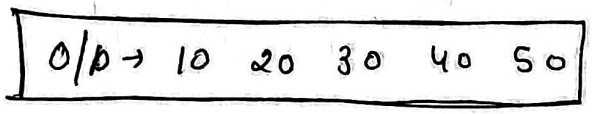
stopping condition.
Can we use it as a base case in recursion?

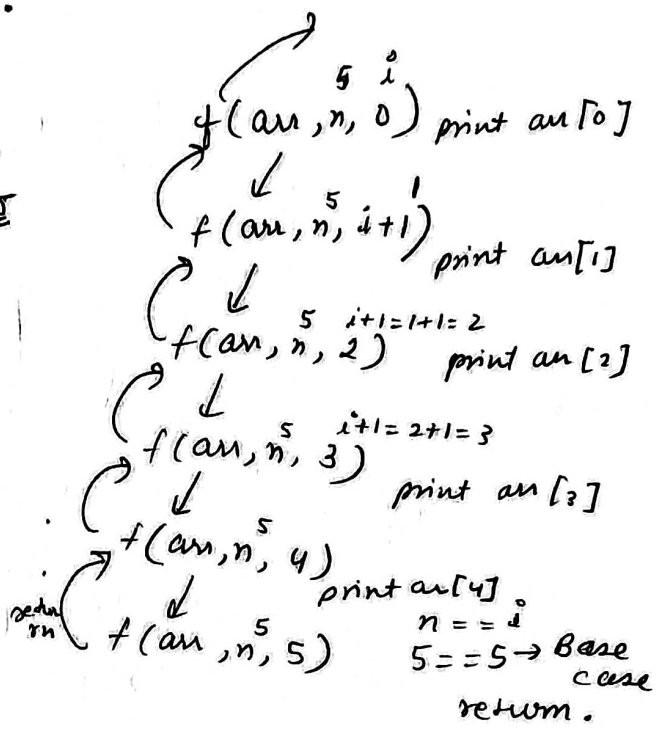
Recursive →

```
void print(int arr[], int n, int i){
    if (i == n) return;          ← base case
    cout << arr[i];              ← print the element in ith index.
    print(arr, n, i+1);          ← call for (i+1)th index.
}
```

1 case solve Kardiya → Baki recursion sambhal lega.







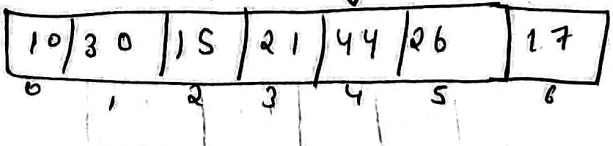
→ Code without index parameter:→

```
void print(int arr[], int n){
    if (n==0) return;
    cout << arr[0] << " ";
    print(arr, n);
}
```

In the func writing post increment will give error. (prints 10 again and again)
(means i++ in place of i+1.)
It is because again and again i's value as 0 will be passed, and we are stucked in infinite loop.
pre increment (++i) works fine.

HOW→ What if we wrote cout after Recursive relation? (processing)

Ques→ Maximum element in an array.



Iterative →

```
int maxi = INT_MIN;
for(int i=0; i<n; i++){
    if(arr[i] > maxi)
        maxi = arr[i];
}
return maxi;
```

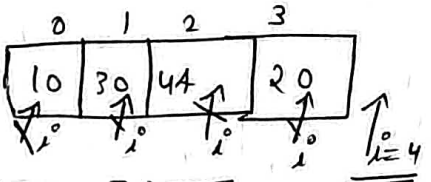
or

```
for(int i=0; i<n; i++){
    maxi = max(maxi, arr[i]);
}
return maxi;
```

```
void findMax (int arr[], int n, int i, int &maxi){
    if (i >= n) → whole array is traversed
        return;
    // EK case solve krna hai
    if (arr[i] > maxi)
        maxi = arr[i];
    // baki recursion sambhal lega.
    findMax(arr, n, i+1, maxi);
}
int main(){
    int arr[] = {10, 20, 30, 40, 50, 5, 15};
    int n = 8;
    int i = 0;
    int maxi = INT_MIN;
    findMax(arr, n, i, maxi);
    cout << "Max Element: " << maxi;
}
```

↖ pass by reference.

without passing reference variable



f(arr, 4, 0, INT_MIN) n==i → F

↓ maxi = 10 ∵ 10 > INT_MIN → T

f(arr, 4, 1, 10) n==i → F

↓ maxi = 30 ∵ 30 > 10 → T

f(arr, 4, 2, 30) n==i → F

↓ maxi = 44 ∵ 44 > 30 → T

f(arr, 4, 3, 44) n==i → F

↓ maxi = 44 ∵ 20 > 44 → F

f(arr, 4, 4, 44) n==i → T Base case reached.
4==4 return

Here each func call has it's own maxi and when function returns, the scope of that maxi ends. At last we have INT_MIN (in main() func.) as maxi, so that's why we are getting wrong ans.

⊙ **By reference →**

main(){

mani [INT_MIN / 10 / 44]

}

(main() ← INT-MIN)

arr | 10 | 44 | 30 | (indices 0 1 2, i → 0)

f(arr, n, i, mani) — n = 3, i = 0, mani = INT-MIN

↓ mani = 10

f(arr, n, i, mani) — n = 3, i = 1, mani = 10

↓ mani = 44

f(arr, 3, 2, 44)

↓ mani = 44

f(arr, 3, 3, 44) ⟶ i == n Base case reached
return.

Ques- Find Minimum in the array.

```
void findMin ( int arr[], int n, int i, int& mini){
    if(i == n)
        return;
    mini = min(mini, arr[i]);
    findMin (arr, n, i+1, mini);
}
```

Ques- i/p → string str = "love babbar",
key = 'r'

o/p → 'r' is present or not? (using recursion).

| l | o | v | e | b | a | b | b | a | r |

> We will check for first character, baaki recursion sambhal lega.

```
bool checkKey (string& str, int& i, int& n, char& key)
{
    //base case.
    if (i >= n)
        return false;
    // Ek case solve karo.
    if ( str[i] == key)
        return ~~key~~ true;
    // baaki recursion sambhal lega.
    return checkKey( str, i+1, n, key);
}
```

> we can send the variable by reference. (Sometimes it's removes TLE).

> This i+1 can give error (Because i is passed as by reference)
> So we will make an var, and pass that.
> ⇒ int newi = i+1;
> return checkKey (str, newi, n, key);

→ If we have to print the index.
Then (We have to make minor changes)

```
int checkkey( - - - - - - ){
    if( i>=n)
        return -1;
    if (str[i] == key)
        return i;
    - - -
    - - -
}
```

→ And if we have print all the occurence of a character.
Then,

```
void checkkey( - - - - ){
    if( i>=n)
        return;
    if ( str[i] == key)
        cout << "Found at: " << i << endl;
    - - -
    - - -
}
```

→ Now what if we don't have to print the indexes but we have store them into a datastructure, then,

```
void checkkey(string& str, int n, int i, char& key, vector<int>* ans){
    if( i>=n) return;   // base case.
    if( str[i] == key){
        ans->push_back(i);
    }
    int newi = i+1;
    checkkey(str, n, newi, key, ans);
}
```

Note → Whenever we want to store something in a datastructure we have to pass that datastructure as pass by reference.

EK case hum solve karenge.

↖ Baaki recursion sambal lega.

→ If we want to count the occurence of a character then we will just pass a counter variable which will be increased by 1 if the target (key) is found.

```
void checkKey( string& str, int& n, int& i, int& count, char key){
    if (i>=n) return;
    if (str[i]== key)
        count++;
    int newi= i+1;
    checkKey (str, n, newi, count, key);
}
```

Ques → i/p → 647
o/p → print all digit of this no.

loop →

647 %10 → 7
/10
64 %10 → 4
/10
6 %10 → 6
/10
0 → stop.

↳ Base case in recursion

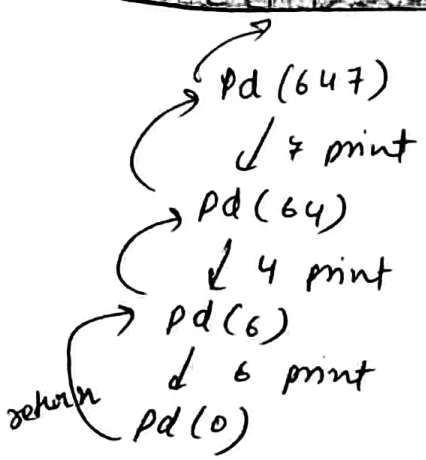
```
void printdigit( int n){
    if (n==0)
        return;
    int digit = n%10;
    cout<<digit<<" ";
    int newValueofn = n/10;
    printdigit (newValueofn);
}
```

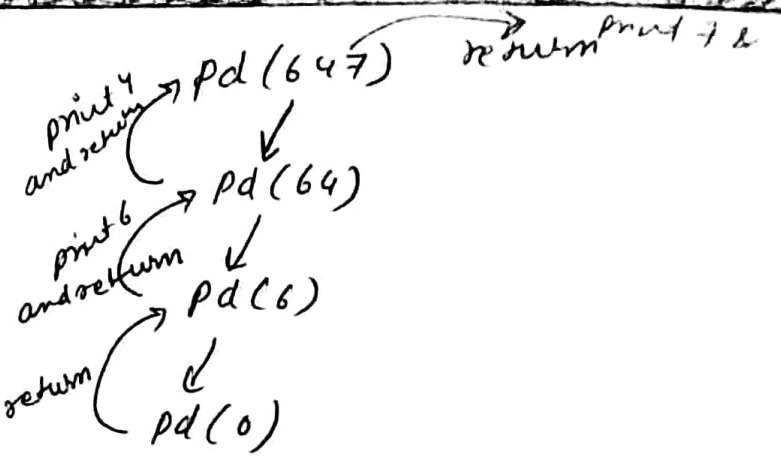
← 1 case main solve karungi
← baki recursion sambhal lega.

i/p → 647
o/p → 7 4 6 (reverse order)

To print in original order.

```
void printdigit ( int n){ - -
- -
- -
    int newValueofn = n/10;
    printdigit (n/10);
    int digit = n%10;
    cout<<digit<<" ";
}
```





This code doesnt work of $n = 0$
so we will handle this case explicitely.

```
if (n == 0)
    cout << 0 << endl;
```

(1) i/p n = 0647

o/p → 4 2 3

It is converting 0647 into 423. why? Find out.