Landing Probabilities in Monopoly

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Abstract

We consider both a computational method and an analytical method to find the probabilities of landing on each tile in monopoly. To access the code used to make this report see either Joseph's or Anand's Github.

1 Introduction

Monopoly is a divisive game – divisive not because some of us don't like it but because some of us, myself included, have never won a game in our lives. However, no matter what, we can all admit that it is a highly enjoyable and well-designed board game. The dice introduce an element of luck, but this can for the most part made up for by a clever strategy. What does a good strategy in Monopoly entail? Well, the most important question to answer is: **what tiles should one buy and build houses on?**

The answer can be counter-intuitive. Anyone who has played the game more than thrice can (but won't) tell you that building houses on the blue tiles is not that smart of a move in early game. Beginners, however, tend to fall victims to the "blue-tile trap" – they are easily lured in by the promises of sky-high house and hotel rents. But the extravagant costs bankrupt them before they can develop enough real estate. Thinking about the balance between costs and potential revenue is a good place to start from when thinking about which tiles to develop.

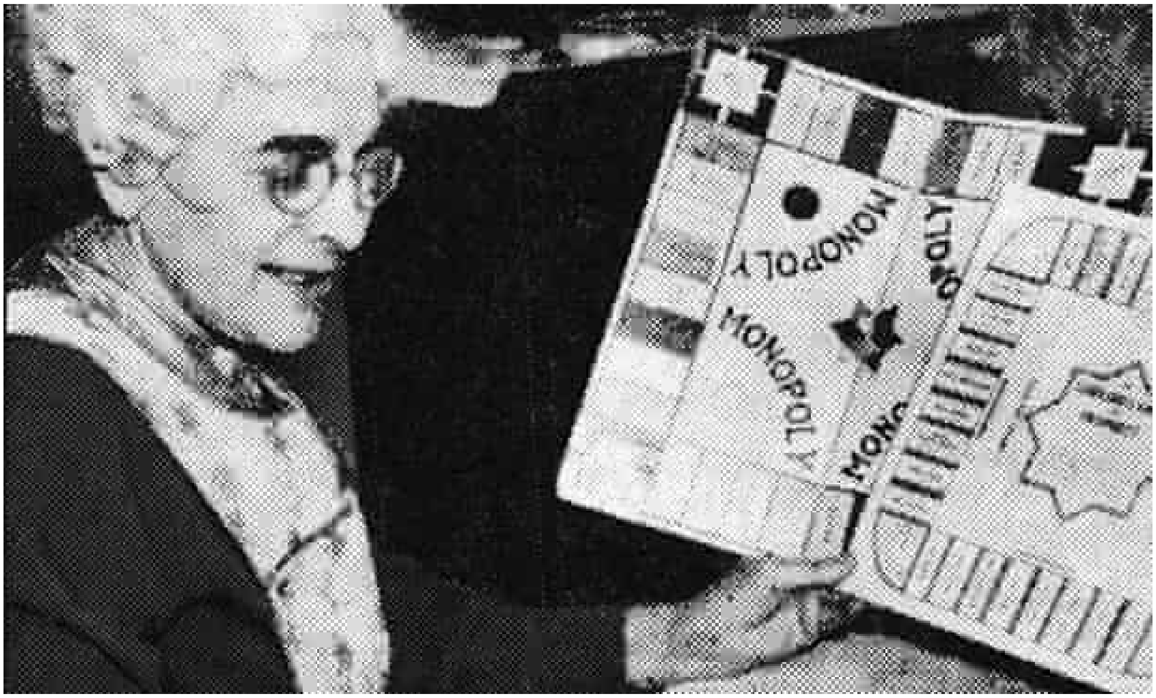

Figure 1: Lizzie Magie, inventor of the precursor to Monopoly game, in 1936. Photograph: Anspach Archives

In this report, we aim to provide players with highly useful and workable information regarding landing probabilites and to answer the question: **what tiles receive the highest amount of footfall during a Monopoly game?** Intuitively, we can make a few guesses. A few tiles such as Jail and GO have chance and community cards instructing players to move to them. This would probably mean these tiles are visited more often than others. In addition, tiles that are easily reachable from GO and Jail might have higher probabilities too.

The way we answer our question scientifically is by modelling Monopoly as a Markov Chain.

2 Monopoly as a Markov Chain

As we play monopoly, the tile on which we land after our next roll only depends on our current position on the board.

Figure 2: The Monopoly game board (UK)

For example, if you are at Euston Road (green), where you move to in the next round depends *only* on your next dice roll and does not depend on anything that has happened previously.

Hence, we can model the way we travel through the board as a Markov Chain. A Markov Process is a random process through time with the property that, stated roughly, the future depends only on the present and not on the past. We can then use our knowledge of Markov chains to compute the limiting probabilities for being at each tile on the board. See this online book to learn more Markov chain theory.

2.1 Convergence of Markov Chains

A question that is frequently pondered upon in the study of Markov chains, and the question we consider with respect to the game of Monopoly, is how our positions will be distributed as time goes on. The form of this distribution is mainly dictated by the transition probabilities. The Convergence Theorem can be seen stated fully as Theorem 4.9 in Freedman.

3 Computational Approach

Complex systems are often intractable to study using analytical methods. This makes it difficult to study real-life phenomena (which are often stochastic in nature) on paper. Fortunately, over the past few decades, dramatic improvements in computing power has allowed the use of simulation-based methods to study stochastic systems.

One such powerful class of methods is called **M**onte **C**arlo methods. Developed during the second World War, MC methods have grown over the past 80 years to be an indispensable tool in the scientist's toolbox. They involve using simulations to obtain approximate solutions to quantitative problems.

3.1 Method Used

A computer can simulate thousands of dice rolls in just a few seconds, whereas it would take a human many hours to do so. This gives computers an intrinsic advantage that can be exploited to study long-term behaviour of many systems including the game of Monopoly. We simulate a million dice rolls (and the corresponding movements) around the board. By keeping track of the number of times each tile is visited, we can easily calculate the long-term probability of each tile being visited.

By virtue of the Convergence Theorem (see section 2.1), we can make a claim about the probabilities thus calculated: the probabilities are unique, i.e. you would arrive at the same probabilities how many ever times you carry out the simulation. This gives us a great amount of confidence, and we can further study the probabilities resulting from our simulation without any validity concerns.

Staying true to the good practice of not reinventing the wheel, we base our simulation code on the well-designed Monopoly simulation created by [Bell, 2020]. We make a few modifications (such as being stuck in jail for three turns unless one rolls a double) to make the simulation more complete.

3.2 Computational Results

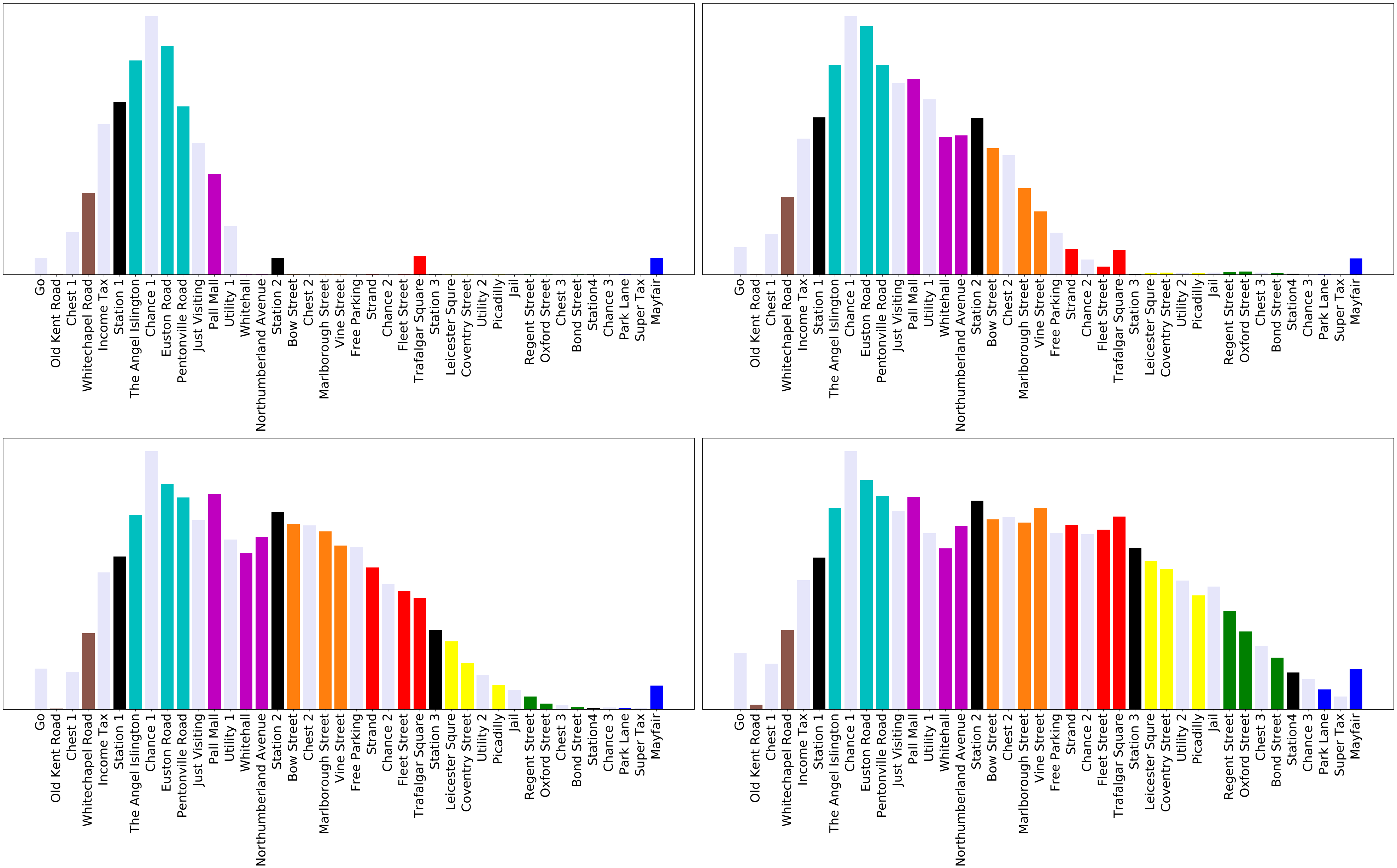

Figure 3: Empirical probabilities of landing on each tile after 'n' moves (top-left: n=1; top-right: n=2; bottom-left: n=3; bottom-right: n=4)

In Figure 3, we see clearly how movement evens out across the board as ones plays more turns. After the first turn, we can see that the majority of tiles are unreachable. The tiles with positive landing probabilities are those that can be reached with the dice roll or by a chance/community card. After two dice rolls, we see there are only a few tiles that cannot be reached. By the end of the third dice roll, we notice that virtually all tiles can be reached. We can infer from these plots that the probabilities even out quite quickly and that there is a high level of variability even in as few as four dice rolls.

Simulating a million dice rolls, we arrive at the long-term probabilities shown in Figure 4. As expected, Jail has the highest landing probability, followed by Just Visiting. Other tiles are mostly similar in probability – though some colours such as orange and red seem to stand above the rest. Another interesting fact: one is almost 1.5x more likely to land on Trafalgar Square than on Park Lane.

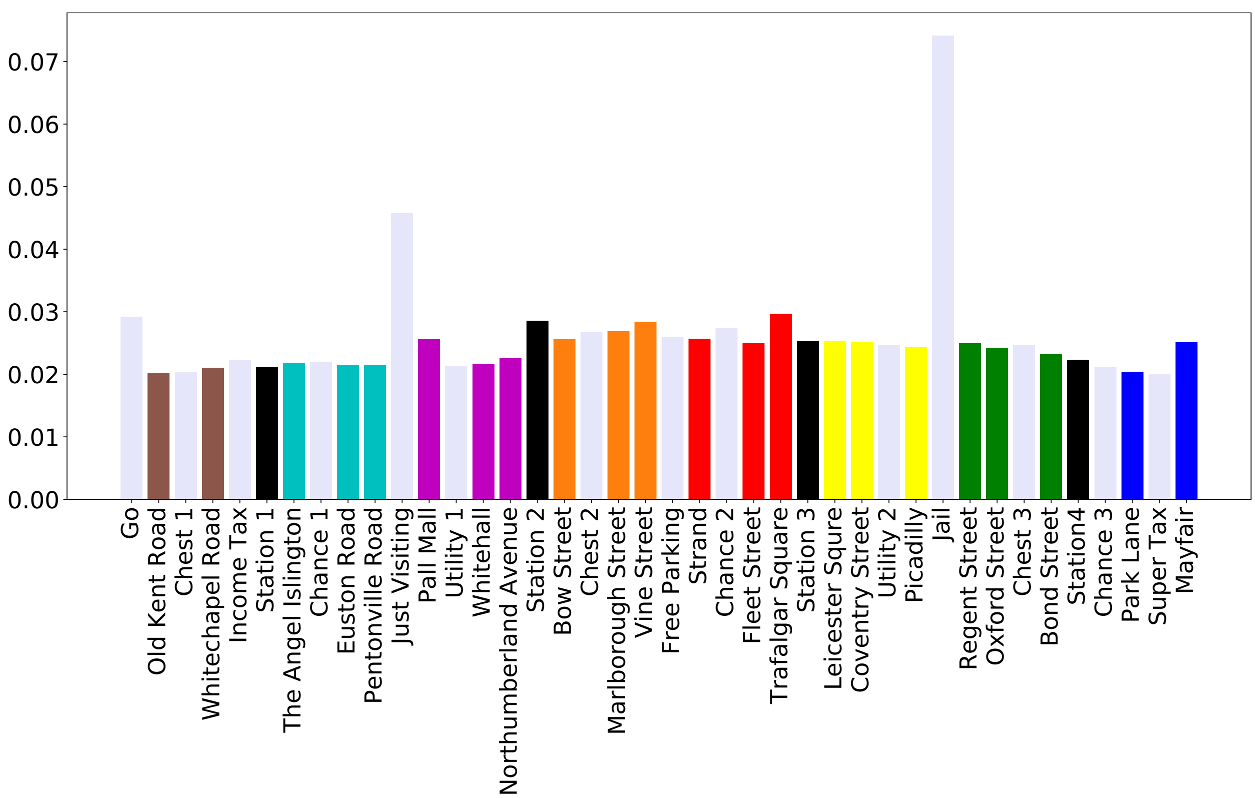

Figure 4: Long-term empirical landing probabilities (at the end of a million dice rolls)

4 Analytical Approach

4.1 Method used to compute probabilities

Using this new Markov Chain perspective on the game, we can find the stationary distribution associated with the board to represent the long run probabilities of landing on any given square. The biggest hurdle in finding this is constructing our transition matrix P. Namely we need to write out the probability of getting from one position to another on any given turn.

As mentioned before, there are two main things that can make this more challenging. Firstly, when we are in jail, the number of times we unsuccessfully roll out will affect out next turn in that we are released from jail after 3 turns. To account for this, we consider jail to actually be 3 states (one for each time we roll). If we don't roll a double, we simply move onto the next state within jail. The second challenge to implement is the cards we can pick up. Some of these cards can take us to other positions, others don't move us. By assuming we put the cards we pick up randomly back into the decks, we can consider our choice of card to be random and independent of previous turns.

Remembering that the stationary distribution is defined as one that satisfies $\pi = \pi P$ (see 2. We can consider π to be a left eigenvector for P with eigenvalue 1. By writing this up in Python, we do indeed find that P has an eigenvector 1 (as expected) and we can normalise the respective eigenvector to find the stationary distribution as follows.

4.2 Analytical Results

By constructing P considering the decks of cards to be the same as in Section 3, we plot a bar chart (Figure 5) of the stationary distribution of the board and order the values in Figure 6, to more easily compare the limiting probabilities.

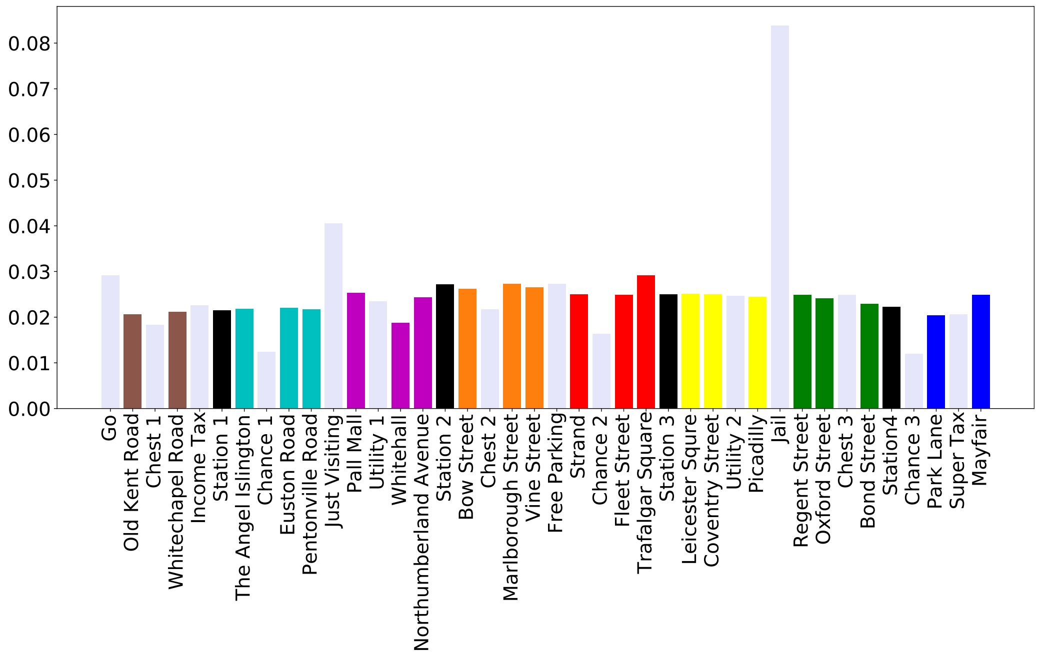

Figure 5: Plot of the standing distribution of the Monopoly board.

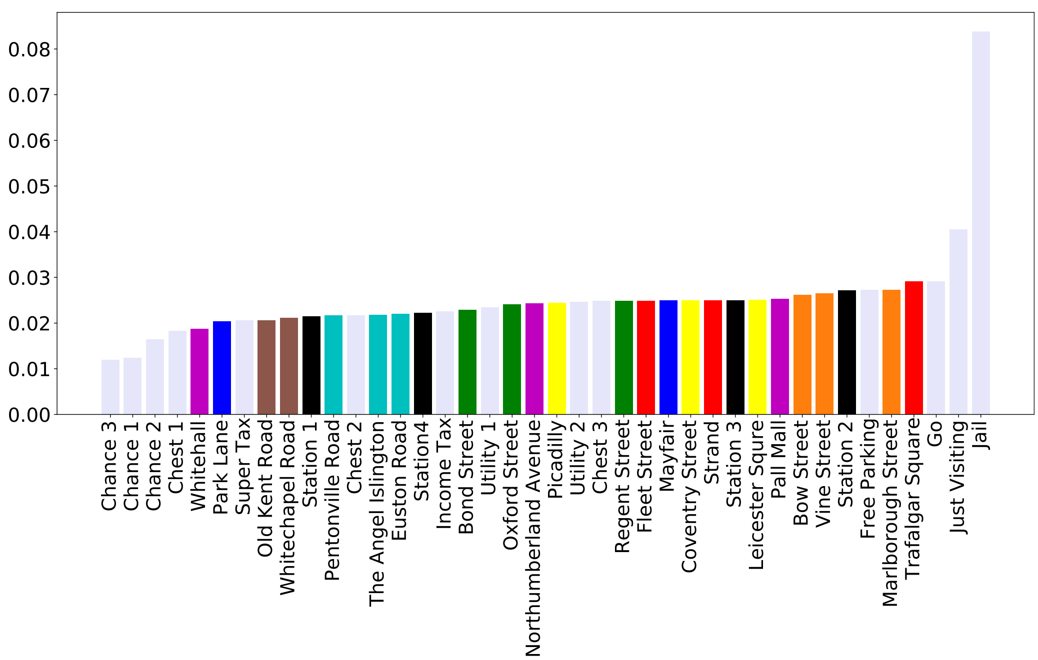

Figure 6: Plot of the ordered standing distribution of the Monopoly board.

We see from figures 5 and 6 that by far the most landed on positions are Jail and Just Visiting, which might be expected given that once you are in jail, you are likely to stay there for several turns, eventually being released into just visiting. The least probable positions in the limit are the Chance tiles, which also makes sense that there is a decent probability of receiving a card that moves you to another tile from Chance (much more so than Community Chest). The rest of the positions are fairly similar and differences between them are much smaller.

5 Comparison of methods

Comparing Figures 4 and 5/6 in the computational and analytical approaches, we see that the structure of the computed long run landing probabilities are very similar. Namely, we have Jail and Just Visiting having the highest probabilities and the rest are very similar. There are some slight differences with the more extreme probabilities across the board, This is likely due to the fact that the stationary distribution in the more theoretical approach assumes that infinite time has passed and so the probabilities have converged completely. Hence, we would expect there to be some slight differences across the probabilities, particularly because these small differences are distributed across all of the 40 positions on the board.

5.1 Conclusion

In order to calculate landing probabilities for each tile on the Monopoly board, we have done a computational as well as analytical study. We found that the results match our intuitive expectations quite well and also that the analytical and computational results are compatible in the long-run. We noted similarities and differences between the two approaches in 5.

This study can be taken further along multiple paths. We mention two of them here:

1. Model Monopoly as a Markov Decision Process (MDP) rather than a simple Markov chain. Doing so would enable one to also include decision-making in the simulation, which is relevant for various cards & money-related activities.

2. Study various buying strategies and the long-term effects of each of those in conjunction with the random process of rolling each turn.

References

[Bell, 2020] Bell, W. H. (2020). Monopolysimulation. https://github.com/williamhbell/MonopolySimulation.