Indian Institute of Technology Mandi

Mathematical Modeling Of Balanduino

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CONTROL SYSTEMS (EE-301P)

Dr. Subashish Datta

Group Members:

Mukesh Kumawat-- B15222

Amit Ghanghas-- B15208

Anand Kumar -- B16046

Deepak Jorwal -- B16054

1 Introduction

The mathematical modeling of Balanduino has been divided into three parts the pendulum, wheel and motor. Translational and rotational equations has been solved to obtain the mechanical model and motor equations comprise of electrical model. The list of parameters used in the modeling are

Symbol	Meaning	Units
For Pendulum		
g	Gravity	m/s^2
J_p	Inertia Pendulum	kgm^2
m_p	mass pendulum	kg
L	Distance bw centre of wheel and pendulum	m
ψ	Angle of pendulum	rad
$\dot{\psi}$	angular velocity of pendulum	rad/s
x_p	x direction for pendulum	m
y_p	y direction for pendulum	m
$N-x$	Force b/w pendulum and wheel in x direction	N
N_y	Force b/w pendulum and wheel in y direction	N
For Wheel		
J_w	Inertia Wheel	kgm^2
m_w	Mass Wheel	kg
r	radius of wheels	m
θ	angle of Wheel	rad
$\dot{\theta}$	anglular velocity of Wheel	rad/s
x_w	x direction for wheel	m
y_w	y direction for wheel	m
N	Normal force from table to wheels	N
F	Frictional force between table and wheels	N
For Electrical		
R_a	Terminal resistance	ohm
k_t	Dc motor torque constant	Nm/A
K_e	Dc motor back emf constant	V sec/rad
T_m	Torque from the motors	Nm
U	Input voltage to motors	V
n	Gear ratio	Unitless

1.1 Pendulum

The parameters and various forces acting on the pendulum has been shown in the given figure.

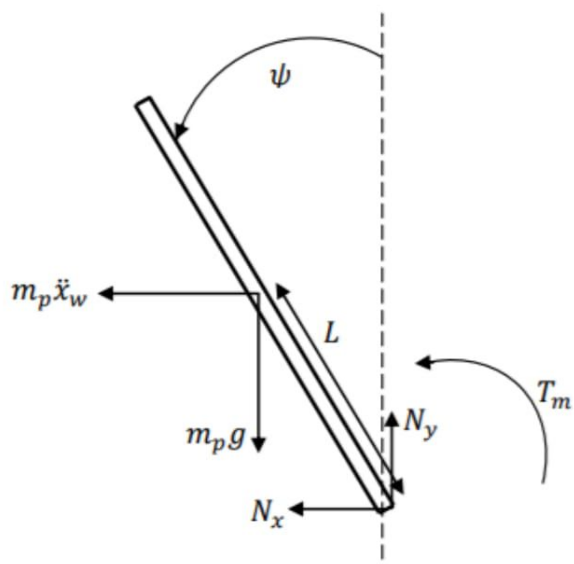

Figure 1: Forces acting on pendulum.

$$T_m + m_pgLsin\psi + m_p\ddot{x_w}Lcos\psi = J_p\ddot{\psi}$$
$$-N_x - m_p\ddot{x_w} = m_p\ddot{x}_p$$
$$N_y - m_pg = m_p\ddot{y}_p$$

From the figure shown above we get

$$x_p = -Lsin\psi$$

$$y_p = Lcos\psi$$

Taking their derivative to obtain velocity

$$\dot{x_p} = -L\psi cos\psi$$

$$\dot{y_p} = -L\psi sin\psi$$

Taking derivative of velocity to obtain accelerations

$$\ddot{x_p} = -L\ddot{\psi}cos\psi + L\dot{\psi}^2 sin\psi$$

$$\ddot{y_p} = -L\ddot{\psi}sin\psi - L\dot{\psi}^2 cos\psi$$

The inertia of pendulum can be calculated by considering the balanduino as cuboid so the inertia is given as

$$J_p = \frac{1}{12}m_p W^2 + \frac{1}{3}m_p H^2$$

Here W and H is the width and height respectively

1.2 Wheel

Various forces acting on the wheel has been shown in the given figure.

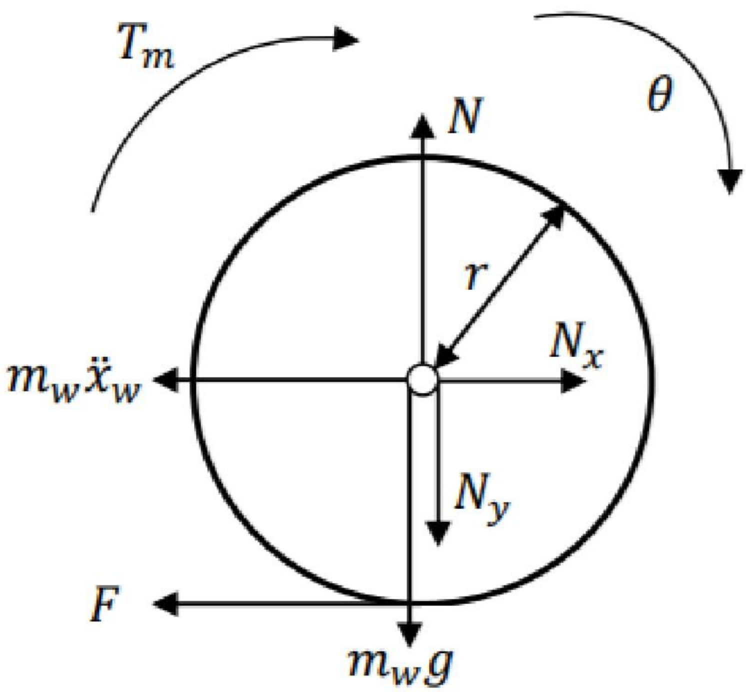

Figure 2: Forces acting on Wheel.

The torque balance and force balance equations are

$$T_m + Fr = J_w\ddot{\theta}$$

$$N_x - m_w\ddot{x_w} - F = 0$$

$$N - N_y - m_w g = m_w\ddot{y_w}$$

Changing to rotational system

$$\ddot{x_w} = \ddot{\theta}r$$

$$\ddot{y_w} = 0$$

The inertia is can be founf out by using formula of circle

$$J_w = \frac{2rsin2\alpha}{3\alpha}$$

where r is the radius and α is the angle.

1.3 Electrical motor model

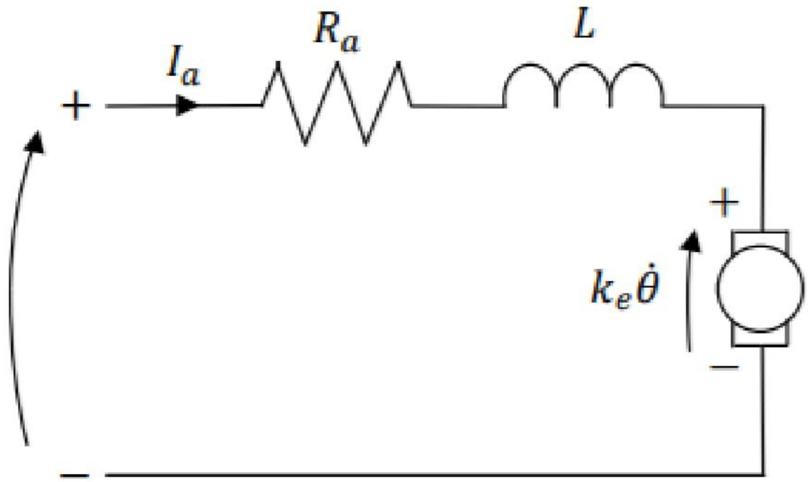

Figure 3: Equivalent model for DC Motor.

In the balanduino two DC- motors have been used to rotate the wheels. Equations governing the dynamics of motor are given as

$$U = R_aI_a + k_e\dot{\theta} + L\dot{i}$$

Here the value of inductance is very small and hence it can be neglected and we get

$$U = R_aI_a + k_e\dot{\theta}$$

The torque acting on motor is given as

$$T_m = nk_tI_a$$

Factors like viscous damping, inertia and motor friction have been neglected in this case.
k_t can be estimated by using rated values of motor and making simplifications. We get k_t as

$$k_t = \frac{T_r}{I_r}$$

where k_t and I_r are rated values of torque and armature current. The back emf is given as

$$V_e = \omega k_e$$

In most of data sheets ratings are given in rpm hence we change rpm radians per sec.

$$V_e = \frac{2\pi N k_e}{60}$$

From these equations we get K_e as

$$k_e = \frac{(V_r - I_r R)60}{2\pi N_r}$$

The torque equation of each motor is given as

$$T_m = n k_t I_a$$

$$I_a = \frac{U - k_e\dot{\theta}}{R_a}$$

Hence,

$$T_m = \frac{n k_t U}{R_a} - \frac{n k_e k_t \dot{\theta}}{R_a}$$

2 Non Linear model

From the equations mentioned above we get non linear equations for pendulum's angular acceleration $\ddot{\psi}$ and wheels angular acceleration $\ddot{\theta}$. The equations are mentioned above

$$\ddot{\psi} = (J_w(gLR_a m_p sin\psi + 2nk_t(U - k_e\dot{\theta})) + r(LrR_a m_p sin\psi(m_p(g - L\dot{\psi}^2 cos\psi) + gm_w)$$
$$2nk_t(U - k_e\dot{\theta})(m_p(Lcos\psi + r) + rm_w)))/(R_a(J_p(J_w + r^2(m_p + m_w)) - L^2r^2m_p^2 cos\psi^2))$$

$$\ddot{\theta} = \frac{Lrm_p cos\psi(gLR_a m_p sin\psi + 2nk_t(U - k_e\dot{\theta})) + J_p(2nk_t)(U - k_e\dot{\theta}) - LrR_a m_p\dot{\psi}^2 sin\psi}{(R_a(J_p(J_w + r^2(m_p + m_w)) - L^2r^2m_p^2 cos\psi^2))}$$

3 Linearized Model

The four states to be used in state space model are

$$x = \begin{bmatrix} \theta \\ \psi \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}$$

The linearized state space model can be written as

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

We get A,B,C and D matrices as

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{gL^2rm_p^2}{-L^2r^2m_p^2+J_p(J_w+r^2(m_p+m_w))} & \frac{-2nJ_pk_ek_t-2Lnrk_ek_tm_p}{(-L^2r^2m_p^2+J_p(J_w+r^2(m_p+m_w)))R_a} & 0 \\ 0 & \frac{gLJ_wm_pR_a+Lr^2m_p(gm_p+gm_w)R_a}{(-L^2r^2m_p^2+J_p(J_w+r^2(m_p+m_w)))R_a} & \frac{-2nJ_wk_ek_t+2nrk_ek_t((L+r)m_p)+rm_w}{(-L^2r^2m_p^2+J_p(J_w+r^2(m_p+m_w)))R_a} & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ \frac{2nJ_pk_t+2Lnrk_tm_p}{-L^2r^2m_p^2+J_p(J_w+r^2(m_p+m_w)))R_a} \\ \frac{2nJ_wk_ek_t+2nrk_t((L+r)m_p)+rm_w}{(-L^2r^2m_p^2+J_p(J_w+r^2(m_p+m_w)))R_a} \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad , \quad D = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$