THE UNIVERSITY OF TEXAS AT ARLINGON

DEPARTMENT OF BIOENGINEERING

BE 5346 – MEDICAL IMAGING– SPRING 2016

PROJECT REPORT

FAN BEAM OR CONE BEAM GEOMETRY FOR X-RAY COMPUTED TOMOGRAPHY (CT)

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FAN-BEAM OR CONE-BEAM CT

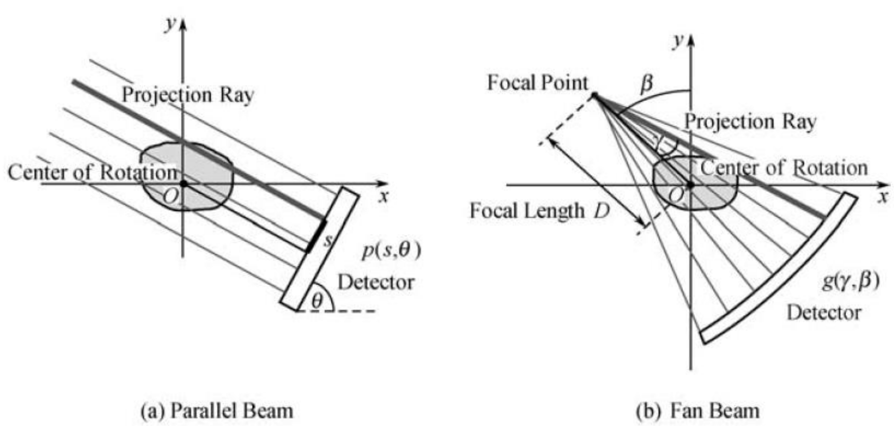
(a) What is Fan-beam CT? Why is it better than the parallel-beam CT?

Fan Beam Geometry

Fan beam geometry is the most widely used geometry for field applications. It employs a single x-ray source with a fan beam structure for each projection. The fan shaped beam of x-rays is directed to an array of detectors that are fixed in position relative to the x-ray source or stationary 360° ring of detectors. The single x-ray source rotates around the object being investigated and a projection is taken at each angle. The fan beam projection data is collected at each angle and is then used for image reconstruction which is done by rebinning technique i.e. by modifying the parallel beam reconstruction algorithm for the fan beam use.

Parallel Beam vs Fan Beam Geometry

Parallel beam geometry is the most basic geometry directly obtained from CT theory. Although parallel beam geometry is relatively simple and straight forward it employs a large amount of radiation source in order to comprise the projections, which themselves are composed of many parallel rays. This is not generally preferred and also becomes expensive and sometimes not feasible. Parallel beam geometry thus faces difficulty where multiple sources are not applicable and also involve ring artifacts, significant noise and vibrations. Fan beam geometry for CT is advantageous to parallel beam CT by employing a single source because handling multiple x – ray or gamma ray sources can be dangerous. Fan CT also significantly reduces the scanning time.



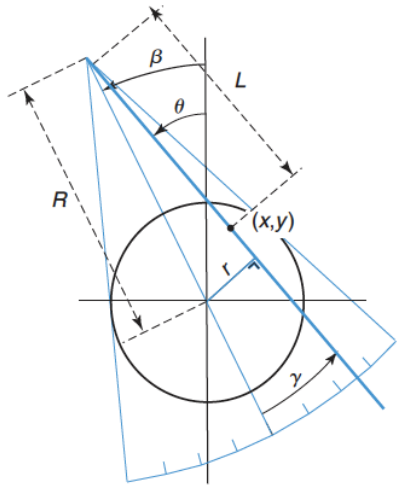
(a) Parallel Beam (b) Fan Beam

Cone beam Geometry

The above two geometries confined their projection to different angles within the same plane (x-y) (2D). Cone beam on the other hand is composed of rays that forms a 3D cone. Similar to the parallel and fan beam geometry the source rotates around the target and multiple projections at different angles are obtained which are then used for three-dimensional image reconstruction. Cone and fan beam geometries are practical and application oriented unlike parallel beam geometry.

(b) Explain the principle of fan-beam CT and show the difference between the two methods, in detection setup and mathematical algorithm.

Principle of a fan beam CT

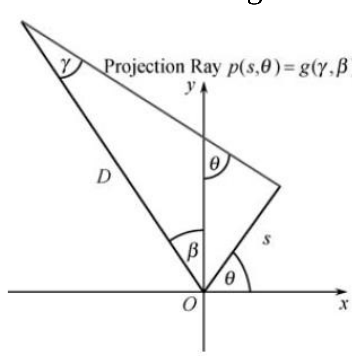


The above figure can be used to explain the principle of a fan beam geometry in which the coordinates (r, θ) used in parallel-beam geometry is combined together with the coordinates (γ,β) used in fan-beam geometry. The detector is place along a circular arc; The x-ray beam makes an angle β with the y-axis, and is at an angle γ from the center line of the fan (or origin). The fan-angle γ is the angle formed by the fan or the angle needed to cover the entire object. An intensity profile $I_\beta(\gamma)$ is measured for every view (defined by an angle β). I_0 is the attenuated intensity. The attenuation profiles g_β (γ) is obtained by log converting the intensity profiles $I_\beta(\gamma)$ and is the projection of the function μ(x,y) at an angle β.

$$I_\beta(\gamma) = I_{0\,e}{}^{-} \int \mu(x,y)dl$$

$$g_\beta(\gamma) = g(\gamma, \beta) = -ln[I_\beta(\gamma)/ I_0] = \int \mu(x,y)dl$$

Parallel-beam geometry utilizes central slice or projection theorem to derive reconstruction algorithms. There is no equivalent theorem for the fan beam geometry. Therefore, image reconstruction is done by converting the fan-beam imaging situation into a parallel-beam imaging situation and modifying the parallel-beam algorithms for fan-beam use i.e. each fan beam projection g(γ, β) has to rebinned into a parallel beam projection p(r, θ). For this two primary conditions have to be made from the below figure.



$$\theta = \gamma + \beta$$

and

$$r = D \sin \gamma$$

We then assign,

$$p\ (r,\ \theta) = g\ (\gamma,\ \beta)$$

In parallel beam CT, a new co-ordinate system (r,s) is obtained by rotating the (x,y) over an angle θ. Such that,

$$r = x \cos\theta + y \sin\theta;\ s = -x \sin\theta + y \cos\theta$$
$$x = r \cos\theta - s \sin\theta;\ y = r \sin\theta + s \cos\theta$$

In the radon space.

$$x = R \cos\varphi;\ y = R \sin\varphi;$$
$$p(r,\theta) = \iint \delta(R,\varphi)\ \delta(x \cos\theta + y \sin\theta - r)\ dx\ dy$$

Therefore, $r = R \cos(\theta - \varphi)$

Applying the above rebinning conditions for fan beam CT,

$$g(D \sin\gamma,\ \gamma + \beta) = \iint \delta(R,\varphi)\ \delta(x \cos(\gamma+\beta) + y \sin(\gamma+\beta) - D \sin\gamma)\ dx\ dy$$

Therefore, the sonogram is $r = R \cos(\gamma + \beta - \varphi)$

We known that for a parallel beam CT, the projection angle θ can be measured from 0 to 2π but since x-ray beams coming from the opposite direction result in identical measurements, θ is limited to 0 to π. The redundant data are related as p(r, θ) = p(-r, θ+π). In case of fan beam CT, β ranging from 0 to π do not include all possible line measurements as shown in the below figure. Therefore, a range from 0 to [π+2 fan angle] is required in order to include all the line measurements and is generally referred as short scan. Fan angle is the angle made by the fan or the angle that is required to cover the entire object. The redundant data are related as g(γ, β)=g(-γ, β+2γ+π). A fan-beam scan that covers an angular range of 2π is referred to as a full scan.

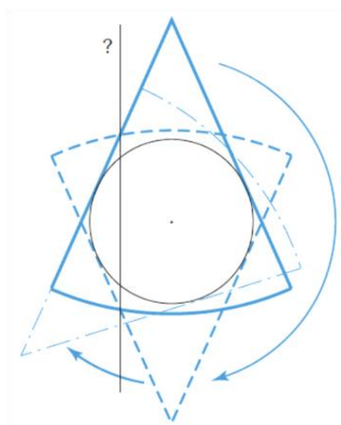


Figure 3: Fan beam scan from 0 to π do not include all possible line measurements

Image Reconstruction for Fan beam CT vs Parallel beam CT

As mentioned above, the detector rotates around at a constant speed and has a uniform angular interval when data are taken. For both parallel and fan beam imaging geometry, this assumption results in a PSF of the projection/ backprojection i.e. a point in the x-y plane will always result in a blurred version of the original object regardless of whether it's a parallel or fan beam geometry. In the parallel-beam case, when you find the backprojection at the point (x, y), you draw a line through this point and perpendicular to each detector. This line meets the detector at a point, say, r*. Then add the value p (r*,θ) to the location (x, y). In the fan-beam case, when you find the backprojection at the point (x, y), you draw a line through this point and each focal-point location. This line has an angle, say, γ^*, with respect to the central ray of the detector. Then add the value g ($\gamma*$, β) to the location (x, y).

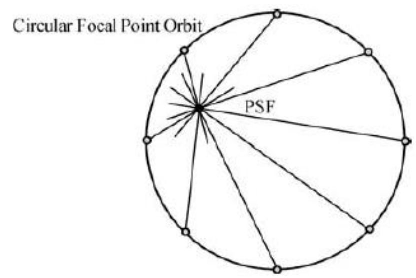


Figure 4: The fan-beam 360◦ full-scan PSF is the same as that for the parallel-beam scan

If f(x,y) if the original image and if the backprojection of the projection data is b(x,y), then PSF can be shown to be 1/r where $r = 1/\sqrt{x^2 + y^2}$.

$$b(x,y) = f(x,y) \otimes 1/\sqrt{x2 + y2}$$

In the Fourier domain,

$$B(\omega x, \omega y) = F(\omega x, \omega y) \otimes 1/\sqrt{\omega x^2 + \omega y^2}$$

The 2D ramp filter $|\omega| = \sqrt{\omega x^2 + \omega y^2}$ can be applied to the backprojected image $B(\omega_x, \omega_y)$ to readily obtain the image $F(\omega_x, \omega_y)$ in the 2D Fourier space. The original image f(x,y) can therefore be obtained by taking 2D inverse Fourier space. This is called as filtered backprojection algorithm.

$$F(\omega_x, \omega_y) = B(\omega_x, \omega_y) \otimes |\omega|$$

Therefore, $$f(x,y) = IF[F(\omega_x, \omega_y)] = IF[B(\omega_x, \omega_y) \otimes |\omega|]$$

There are two types of fan beam detectors: flat detector fan-beam and curved detector fan-beam In a flat detector, the data points are sampled with equal distance Δs or intervals, while in a curved detector, the data points are sampled with equal angle $\Delta\gamma$ intervals.

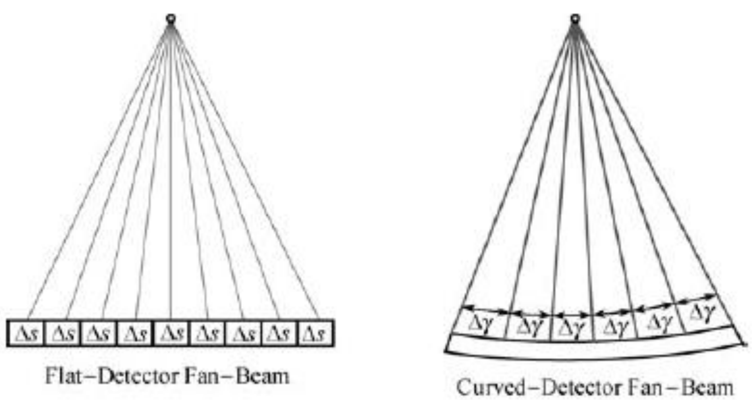
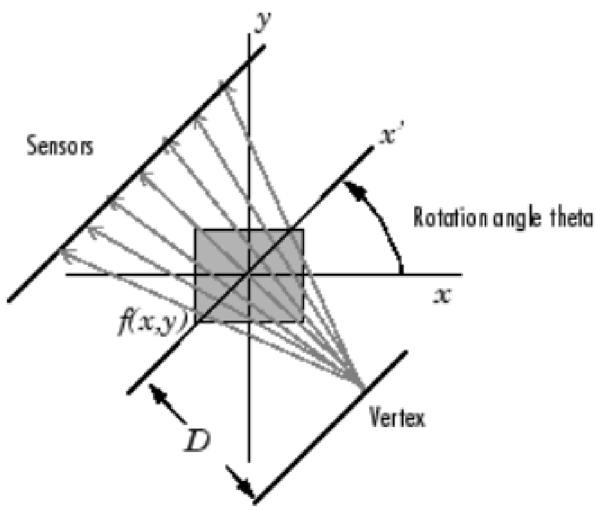


Figure 5: Fan beam detectors of equiangular and equispaced

c) Please the Run an example using MATLAB. You can u head model given in MATLAB, or you can use any image you like. Note: You are allowed to use any existing code you can find through google. But you need to be able to explain the functions you use when you reconstruct the images.

The 'fanbeam' function computes projections of an image matrix along specified directions. A projection of a two-dimensional function f(x,y) is a set of line integrals. The fanbeam function computes the line integrals along paths that radiate from a single source, forming a fan shape. To represent an image, the fanbeam function takes multiple projections of the image from different angles by rotating the source around the center of the image. The following figure shows a single fan-beam projection at a specified rotation angle.

Fan-Beam Projection at Rotation Angle Theta

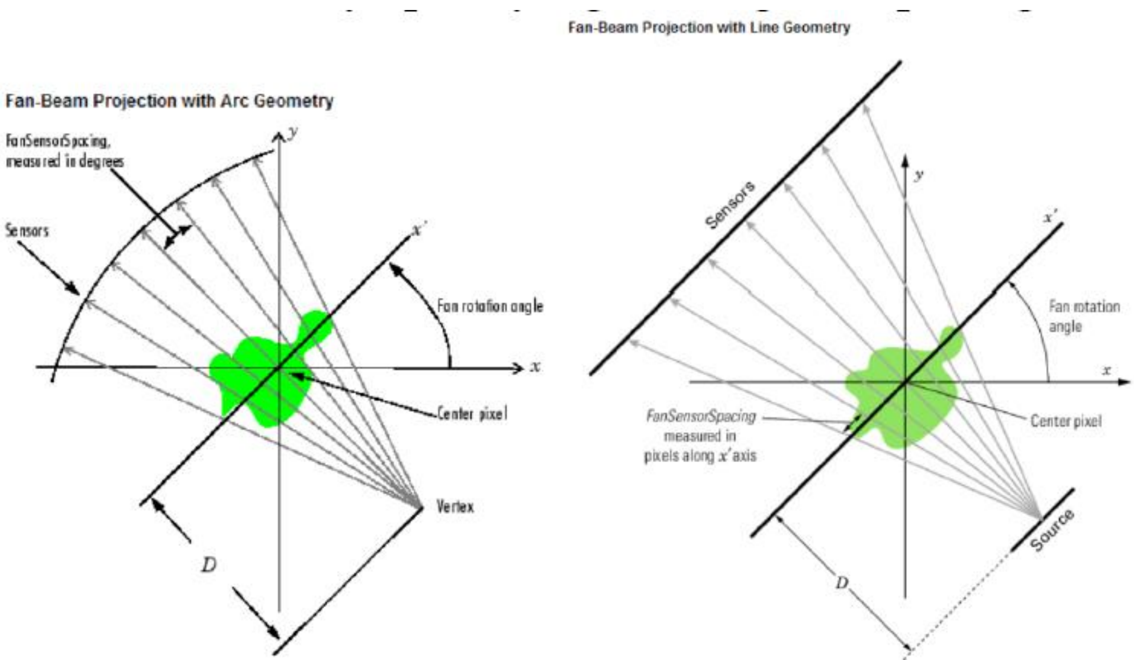


To use the fanbeam function specify as arguments an image and the distance between the vertex of the fan-beam projections and the center of rotation (the center pixel in the image). The fanbeam function determines the number of beams, based on the size of the image and the settings of fanbeam parameters.

The FanSensorGeometry parameter specifies how sensors are aligned. If you specify the value 'arc' for FanSensorGeometry (the default), fanbeam positions the sensors along an arc, spacing the sensors at 1 degree intervals. Using the FanSensorSpacing parameter, you can control the distance between sensors by specifying the angle between each beam. If you specify the value 'line' for FanSensorGeometry parameter, fanbeam position sensors along a straight line, ratherthan an arc. With 'line' geometry, the FanSensorSpacing parameter specifies the distance between the sensors, in pixels, along the x´ axis.

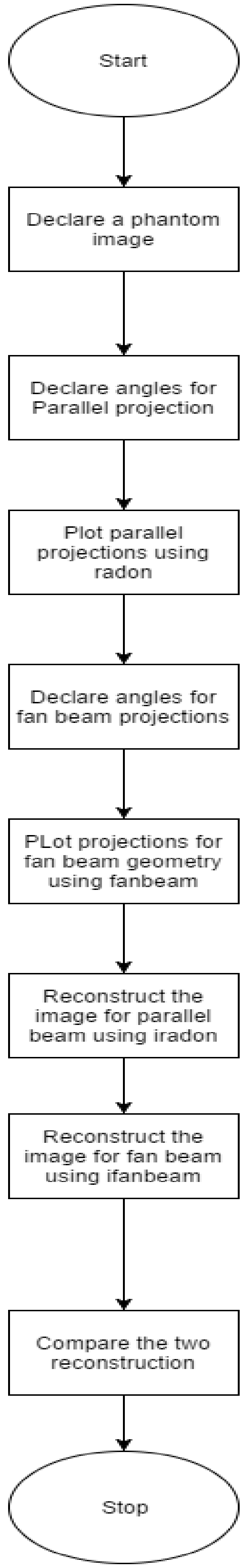
Fanbeam takes projections at different angles by rotating the source around the center pixel at 1 degree intervals. Using the FanRotationIncrement parameter you can specify a different rotation angle increment.

The following figures illustrate both these geometries. The first figure illustrates geometry used by the fanbeam function when FanSensorGeometry is set to 'arc' (the default). Note how you specify the distance between sensors by specifying the angular spacing of the beams.



To reconstruct an image from fan-beam projection data, use the ifanbeam function. With this function, you specify as arguments the projection data and the distance between the vertex of the fan-beam projections and the center of rotation when the projection data was created. For example, this code recreates the image I from the projection data P and distance D.

Flow chart

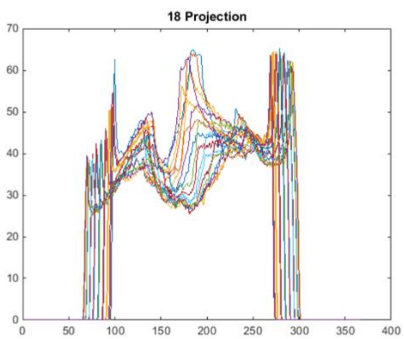

Start
Declare a phantom image
Declare angles for Parallel projection
Plot parallel projections using radon
Declare angles for fan beam projections
PLot projections for fan beam geometry using fanbeam
Reconstruct the image for parallel beam using iradon
Reconstruct the image for fan beam using ifanbeam
Compare the two reconstruction
Stop

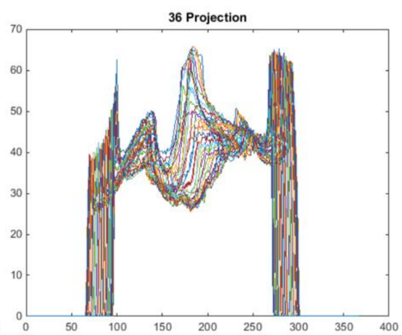
I = ifanbeam(P,D);

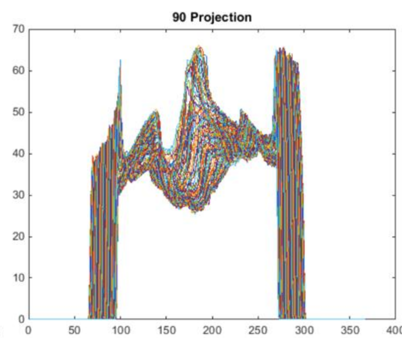
The "fanbeam" command executes the function "radon" and "para2fan" commands. The "ifanbeam" command can inverse the sinogram into the real picture by using "fan2para" command.

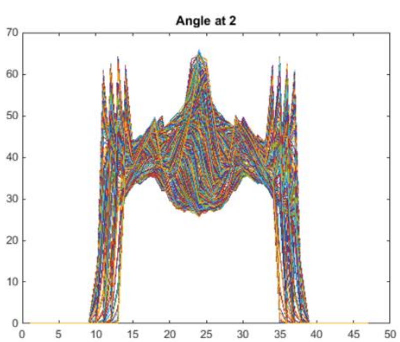
```
%% Image Reconstruction of a Phantom Brain Model using parallel beam and fan beam technique
close all; clear all
%% Defining an phantom brain model in the x,y space
figure;
I=phantom('Modified Shepp-Logan',256);
imshow(I,[],'InitialMagnification','fit'), title('The phantom brain model in the x-y space')
%% Parallel Projection of the brain model in the Radon Space
theta1=0:10:170;
theta2=0:5:175;
theta3=0:2:178;
[p1,r]=radon(I, theta1);
[p2,r]=radon(I, theta2);
[p3,r]=radon(I, theta3);
figure;plot(radon(I,theta1)),title('18 Projection');
figure;plot(radon(I,theta2)),title('36 Projection');
figure;plot(radon(I,theta3)),title('90 Projection');
%% Fan Projection of the brain model in the Radon Space
D=250;
[g1,f_loc,f_angle1]=fanbeam(I,D,'FanSensorSpacing',2);
[g2,f_loc,f_angle2]=fanbeam(I,D,'FanSensorSpacing',1);
[g3,f_loc,f_angle3]=fanbeam(I,D,'FanSensorSpacing',0.25);
figure;plot(fanbeam(I,D,'FanSensorSpacing',2)),title('Angle at 2');
figure;plot(fanbeam(I,D,'FanSensorSpacing',1)),title('Angle at 1');
figure;plot(fanbeam(I,D,'FanSensorSpacing',0.25)),title('Angle at 0.25');
%% Sinogram for 90 parallel projections in Radon Space
figure,imagesc (theta3,r,p3);colormap(hot);colorbar
xlabel('\theta'); ylabel('r');
%% Sinogram for fan rotation angle of 0.25 in Radon Space
figure,imagesc(f_angle3,f_loc,g3);colormap(hot);colorbar
xlabel('Fan Rotation Angles (degree)'); ylabel('Fan sensor location(degree)');
%% Back parallel-projection to the object space with filter
% This returns raw back projection data
b1=iradon(p1,10);
b2=iradon(p2,5);
b3=iradon(p3,2);
figure,imshow(b1),title('Back Projection at 18 angles');
figure,imshow(b2),title('Back Projection at 36 angles');
figure,imshow(b3),title('Back Projection at 90 angles');
%% Back fan-projection to the object space with filter
% This returns raw back projection data
```

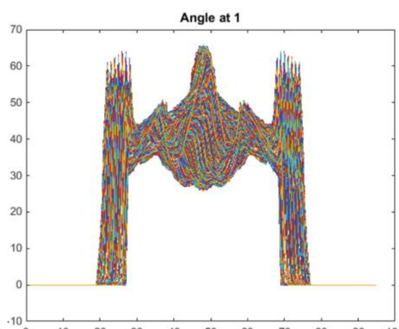
```
output_size=max(size(I));
B1=ifanbeam(g1,D,'FanSensorSpacing',2,'OutputSize',output_size);
figure;imshow(B1), title('Back Projection at fan angle of 2');
B2=ifanbeam(g2,D,'FanSensorSpacing',1,'OutputSize',output_size);
figure;imshow(B2), title('Back Projection at fan angle of 1');
B3=ifanbeam(g3,D,'FanSensorSpacing',0.25,'OutputSize',output_size);
figure;imshow(B3), title('Back Projection at fan angle of 0.25');
```

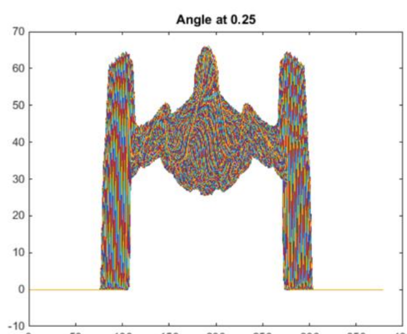


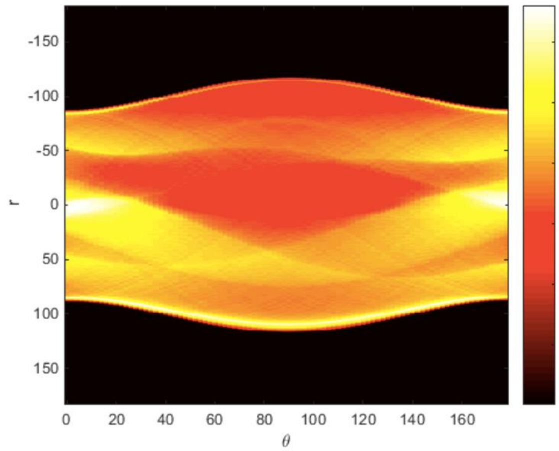


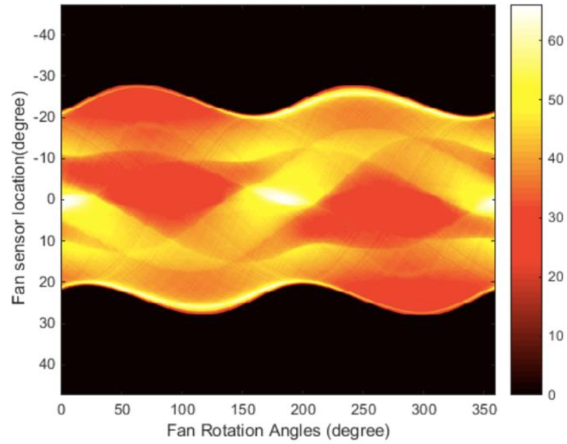




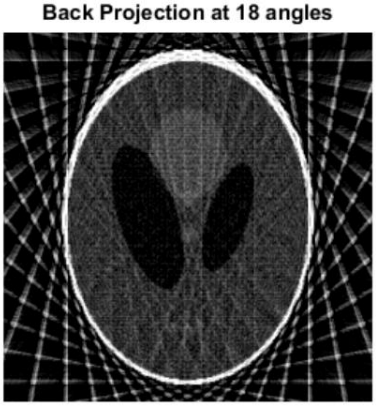






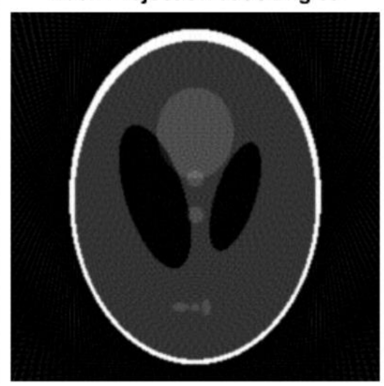


Back Projection at 18 angles

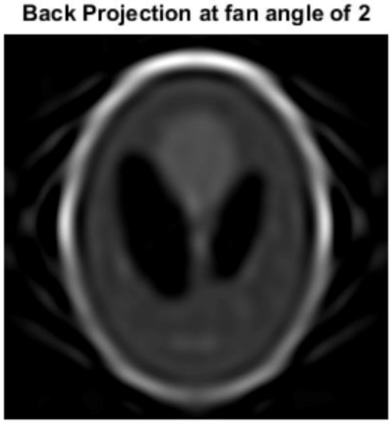


Back Projection at 36 angles

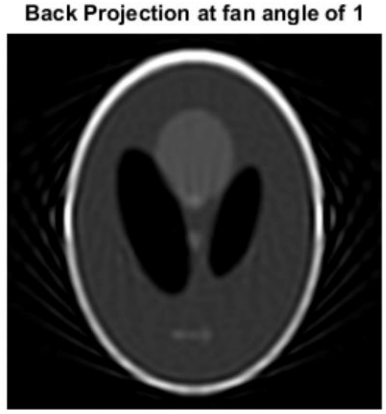
Back Projection at 90 angles



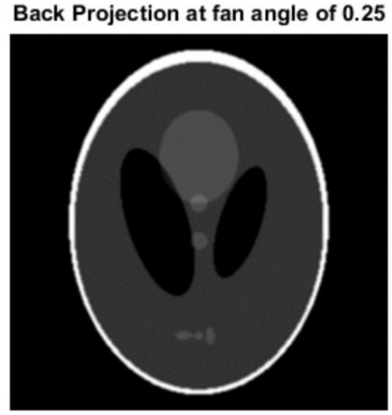
Back Projection at fan angle of 2



Back Projection at fan angle of 1



Back Projection at fan angle of 0.25



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[2] Paul Suetens. Fundamentals of Medical Imaging (2nd edition) (2011). European journal of nuclear medicine and molecular imaging 38(2):409-409.
[3] Gengsheng Zeng. Medical Image Reconstruction: A Conceptual Tutorial (2010). ISBN: 978-3-642-05367-2 (Print) 978-3-642-05368-9.