

→ gamma function defined everything except non-positive integers defined for positive real numbers  
Gamma function:  $\int_0^{\infty} e^{-x} x^n dx$

→ Converts ~~the~~ function real variable to a function complex variable  $s$  (in some cases imaginary part is 0)

### e) Advantage of Laplace.

→ Turns Differential equation into algebraic equation and Convolution into multiplication in Laplace domain

→  $e^{t^2}$ ,  $\tan t$  → not exponential order

→ Piecewise cont. on  $[0, \infty)$

→ S-Shifting Theorem - Shifting happens in  $s$  domain

→ Transform of derivative → same thing as above.

→ This can be extended for  $n^{\text{th}}$  order derivative.

→ initial value problems

is a differential equation along with an appropriate initial condition

② Unit Step function

→ Also called as Heaviside function

→ Discont function

→ 0 for neg arg.

Ex. Voltage switched on / off.  
For calculations it is periodic. (specify interval)

→ Unit step function is ~~finite~~ bounded. but it's not stable ~~bec~~ because the absolute integral is infinite.

Dirac Delta.

auxiliary function 
$$g(t-a) = \begin{cases} t/a & a < t < a+h \\ 0 & \text{otherwise.} \end{cases}$$

$$\delta(t-a) = \lim_{h \rightarrow 0} g(t-a).$$



- Dirac Delta is not a function, as it has nothing to do with real line and everything to do with what occurs if we integrate ~~on~~ it against ~~an~~ another function.

### Convolution Theorem

- Geometrical  $\rightarrow$  area
- Physical  $\rightarrow$  Amount.
- Convolution is a mathematical operation on two functions that produces a third function expressing how the shape of one is modified by the other.
- imp digital Signal Processing.
- commutative, distributive, associative

Periodic function.

→  $f(x+T) = f(x)$   $T \rightarrow$  non zero positive or negative

Fourier Transform

→ To find difficult Integral.

→ If the period used in  $c$  to  $c+2\pi$  is infinity Fourier Series changes to Fourier Transform.

tion

Series.

Fourier Series — Expansion of Unfamiliar function in terms of ~~periodic~~ basic periodic functions.

→ The Fourier Transform is an important image processing tool which is used to decompose an image into its Sine and Cosine components.

→ Fourier Transform decomposes any function into sum of basic simple sinusoids.

→ Self reciprocal or Hankel transform

Transform of derivative

If  $f(t)$  is cont.  $f'(t)$  is ~~piece~~ piecewise continuously differentiable,  $f(t)$  and  $f'(t)$  are absolutely integrable in  $(-\infty, \infty)$  and  $\lim_{t \rightarrow \pm\infty} f(t) = 0$  then

$$F\{f'(t)\} = (is) F(s)$$