Appendix: Objective Type Questions

1. Let $S_1 = \{(x_1, x_2) : 2x_1 + 3x_2 = 5\}$, $S_2 = \{(1, 1)\}$ be two subsets of $\mathbb{R}^2$. Then $S_1 \cap S_2$ is

(a) a convex set (b) not a convex set

2. Let S_1 and S_2 be two convex subsets of $\mathbb{R}^n$. If S_1' and S_2' represent the compliments of S_1 and S_2, respectively. Then

(a) $S_1 + S_2$ (b) $S_1 \cup S_2$ (c) $S_1' \cap S_2'$ (d) $S_1' \cup S_2$

is always a convex set

3. Let S_1 and S_2 be two convex subsets of $\mathbb{R}^n$. If S_1' and S_2' represent the compliments of S_1 and S_2, respectively. Then

(a) $S_1 - S_2$ (b) $S_1' \cup S_2'$ (c) $S_1' \cap S_2$ (d) $S_1 \cap S_2'$

is always a convex set

4. Consider the set $S = \{(x_1, x_2) : x_2^2 \leq x_1\}$. Then S has

(a) no vertex (b) finite number of vertices (c) infinite number of vertices

5. The number of extreme point(s) that a hyper-plane has

(a) infinite (b) finite (c) none of these

6. The set $S = \{(x_1, x_2) : x_1 + x_2 = 1\}$ has no vertex because it is

(a) not convex (b) not bounded
(c) not closed (d) none of these

7. Consider the unit simplex $S = \{(x_1, x_2, x_3) : x_1 + x_2 + x_3 = 1,\ x_1, x_2, x_3 \geq 0\}$. Then number of vertices S has

(a) 2 (b) 4 (c) 5 (d) none of these

8. Let $S = \{X \in \mathbb{R}^2 : |X| < 1\}$. Then S has no vertex because it is

(a) not closed (b) empty
(c) unbounded (d) not convex.

9. Let $S = \{(x_1, x_2) : x_1^2 + x_2^2 \leq 1\}$. Then S has

(a) no vertex (b) finite number of vertices (c) infinite number of vertices

10. Consider the set $S = \{(x_1, x_2) : x_1 + x_2 \geq -1,\ x_1 \leq 0,\ x_2 \leq 1\}$. Then S has

(a) no vertex (b) infinite number of vertices

(c) only two vertices (d) none of these

11 The vertex of the set $S = \{X : X = (1-\alpha)X_1 + \alpha X_2,\ \alpha \geq 0,\ X_1, X_2 \in \mathbb{R}^2\}$ is

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12. The set $P_F \setminus A$, where A is the set of all vertices of P_F is

(a) convex set (b) not a convex set

(c) may or may not be convex (d) none of these

13. The system of equations

$$x_1 - x_2 + x_3 = 4$$

$$2x_1 + x_2 - 5x_3 = 3$$

is equivalent to the following system with inequalities
(a) $x_1 - x_2 + x_3 \leq 4,\ 2x_1 + x_2 - 5x_3 \leq 3,\ -x_1 + 2x_2 + 6x_3 \geq 7$
(b) $x_1 - x_2 + x_3 \leq 4,\ 2x_1 + x_2 - 5x_3 \leq 3,\ -x_1 + 2x_2 + 6x_3 \leq 1$
(c) $x_1 - x_2 + x_3 \leq 4,\ 2x_1 + x_2 - 5x_3 \leq 3,\ 2x_1 - 4x_3 \leq 1$
(d) $x_1 - x_2 + x_3 \leq 4,\ 2x_1 + x_2 - 5x_3 \leq 3,\ 3x_1 - 4x_3 \geq 7$

14. The vertex of the set $S = \{X : X = (1-\lambda)X_1 + \lambda X_2,\ 0 < \lambda \leq 1,\ X_1, X_2 \in \mathbb{R}^2\}$ is

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15. For $x_1, x_2 \geq 0$, consider the system

$$\begin{aligned} x_1 + x_2 - x_3 - 2x_4 + 5x_5 &= 2 \\ x_2 + x_3 + 5x_4 + 5x_5 &= 2 \end{aligned}$$

Its solution $x_1 = x_3 = x_4 = 0$, $x_2 = 7, x_5 = -1$ is

(a) a basic solution (b) a basic feasible solution (c) not a basic solution (d) feasible solution

16. Let the optimal of a LP occur at vertices X_1 and X_2. Then we know that it also occurs at each

$$X = (1 - \alpha)X_1 + \alpha X_2, \quad 0 < \alpha < 1$$

(a) X is a basic solution (b) X is not a BFS (c) X is not a basic solution (d) none of these

17. For $x_1, x_2 \geq 0$, consider the system

$$\begin{aligned} x_1 + 2x_2 - x_3 - 2x_4 - 3x_5 &= -1 \\ 2x_2 + x_3 + 5x_4 - 3x_5 &= -1 \end{aligned}$$

Its solution $x_1 = 0$, $x_2 = 1$, $x_3 = 0$, $x_4 = 0$, $x_5 = 1$ is

(a) a basic solution (b) a basic feasible solution

(c) feasible solution (d) none of these

18. In a simplex table, there is a tie for the leaving variable, then the next BFS

(a) will be nondegenerate (b) will be degenerate

(c) may be degenerate or nondegenerate (d) does not exist

19. Two vertices of P_F are $(x_1, x_2, x_3, x_4) = (0, 0, 1, 2)$ and $(3, 0, 0, 1)$. Then a point of P_F which can not be the vertices

(a) $(1, 2, 0, 0)$ (b) $(0, 1, 3, 0)$ (c) $(0, 1, 2, 0)$ (d) $(1, 2, 3, 0)$

20. A LPP in standard form has m constraints and n variables. The number of basic feasible solutions will be

(a) $\binom{n}{m}$ (b) $\leq \binom{n}{m}$ (c) $\geq \binom{n}{m}$ (d) none of these

21. A LPP in standard form has m constraints and n variables. Then number of adjacent vertices corresponding to a vertex are

(a) $n-m$ (b) $\leq n-m$ (c) $n!/m!(n-m)!$ (d) none of these

22. In Problem 19, if $m = 5$ and $n = 8$, and X is basic feasible solution with 3 components at positive level. Then, the number of bases which correspond to X due to degeneracy are

(a) 5 (b) 10 (c) 15 (d) 20

23. In an LPP, let p = number of vertices and q = number of BFS. Then

(a) $p \leq q$ (b) $p = q$ (c) $p \geq q$ (d) none of these

24. In a simplex iteration, if the leaving variable rule is violated, then the next table will

(a) not give basic solution (b) give a basic solution which is not feasible

(c) give a nonbasic solution (d) nothing can be said

25. For max $p_1 = -3x_1 + x_2$, subject to $3x_1 - x_2 \leq 6$; $x_2 \leq 3$; $x_1, x_2 \geq 0$, the optimal table is

B V	x_1	x_2	s_1	s_2	Soln
p_1	3	0	1	0	3
s_1	0	1	1	0	9
x_2	3	0	1	1	3

If max $p_2 = x_1 - x_2$, then optimal solution $(x_1, x_2) = (0, 3)$ remains optimal for the weighted LP: $p = \max\ \alpha_1 p_1 + \alpha_2 p_2$, $0 \leq \alpha_1 \leq \alpha_2$, then α_2 is

(a) 1/2 (b) 3/4 (c) 1 (d) none of these

26. If in any simplex iteration the minimum ratio rule fails, then the LPP has

(a) nondegenerate BFS (b) degenerate BFS

(c) unbounded solution (d) infeasible solution

27. In a max LPP with bounded solution space, a variable having positive relative cost is permitted to enter and the minimum ratio rule is properly followed, then

(a) the next solution will not be BFS
(b) the objective function value decreases
(c) the objective function increases
(d) none of these

28. If x_j is a basic variable in some simplex table, then relative cost of x_j is

(a) positive (b) negative (c) infinite (d) 0

29. In some simplex table of a maximization LPP, the column of x_j is $(3; -2, -1, -3)^T$. Then this shows that

(a) P_F is bounded
(b) solution is unbounded
(c) P_F is unbounded in x_j direction
(d) solution is infeasible

30. In phase-I of the two phase method an artificial variable turns out to be at positive level in the optimal table of Phase-I, then the LPP has

(a) no feasible solution
(b) unbounded solution
(c) optimal solution
(d) none of these

31. In a maximization problem, a basic variable corresponding to minimum ratio leaves the basis, this ensures

(a) largest increase in objective function
(b) the next solution will be a BFS
(c) decrease in objective function
(d) none of these

32. In a maximization problem, a nonbasic variable with most negative relative cost enters the basis ensures

(a) largest increase in objective function (b) the next solution will be a BFS

(c) decrease in objective function (d) none of these

33. Let $B = (A_1, A_3, A_5)$ be a basis for a LPP such that $A_4 = \alpha A_1 + \beta A_2 + \gamma A_3$. Suppose any one column of B is replaced by A_4 to have a new basis. Then

(a) $\alpha, \beta, \gamma > 0$ (b) $\alpha, \beta, \gamma \leq 0$ (c) $\alpha, \beta, \gamma \neq 0$ (d) no such relationship required

34. The optimum of a LPP occurs at $X = (1, 0, 0, 2)$ and $Y = (0, 1, 0, 3)$. Then optimum also occurs at

(a) $(2, 0, 3, 0)$ (b) $(1/2, 1/2, 0, 5/2)$
(c) $(0, 1, 5, 0)$ (d) none of these

35. If in a simplex table the relative cost $z_j - c_j$ is zero for a non-basic variable, then there exists an alternate optimal solution, provided

(a) it is starting simplex table (b) it is optimal simplex table

(c) it can be any simplex table (d) none of these

36. A LPP amenable to solution by simplex method has third and fourth constraint as $x_1 + x_2 + x_3 \leq 3$ and $2x_1 + x_2 + 3x_8 \leq 8$. These constraints can be represented by a single constraint

(a) $3x_1 + 2x_2 + 4x_3 \leq 11$ (b) $x_1 + 2x_3 \leq 5$ (c) $3x_1 + x_2 + 3x_3 \leq 11$ (d) none of these

37. In canonical form of a LPP, the availability vector b

(a) is restricted to ≥ 0 (b) is restricted to ≤ 0 (c) any component may be ≤ 0 or ≥ 0

38. Suppose, in some simplex iteration x_j enters the basis. Then, at later stage in some simplex iteration

(a) x_j can leave the basis (b) x_j can not leave the basis (c) both (a) and (b) are possible

Suggestion. If the rule $\theta_j(z_j - c_j)$ is not followed then answer is (c), otherwise it is (b).

39. Suppose, in some simplex iteration x_j leaves the basis. Then, in just next iteration

(a) x_j can enter the basis (b) x_j can not enter the basis (c) both (a) and (b) are possible

40. By inspecting the dual of the following LPP find the optimal value of its objective function

$$\begin{aligned} \max \quad & z = 2x_1 + x_2 + 3x_3 \\ \text{s.t.} \quad & x_1 - x_2 + x_3 \geq 5 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

41. The following LPP has

$$\begin{aligned} \min \quad & x_0 = -2x_1 + 10x_2 \\ \text{s.t.} \quad & x_1 - x_2 \geq 0 \\ & -x_1 + 5x_2 \geq 5 \\ & x_1, x_2 \geq 0 \end{aligned}$$

(a) alternative solution (b) unique solution (c) unbounded solution (d) none of these

Suggestion. This is an interesting problem in which the LPP has alternate optimal solution. Every point on the line $-x_1+5x_2 = 5$ gives optimal solution with optimal value 10. From the optimal table, it is not possible to find alternate optimal solution, since the solution space is unbounded. Except $x_1 = 5/4$, $x_2 = 5/4$, all other optimal solutions are nonbasic.

42. Let $\min f(X) = C^T X$, $AX \geq b$, $X \geq 0$ be a primal LPP. Suppose X_0 and Y_0 are the primal and dual feasible. Then

(a) $C^T X_0 \leq b^T Y_0$ (b) $C^T X_0 \geq b^T Y_0$ (c) $C^T X_0 = b^T Y_0$ (d) none of these

43. Let $\max f(X) = C^T X$, $AX \leq b$, $X \geq 0$ be a primal LPP. Suppose X_0 and Y_0 are the primal and dual feasible. Then

(a) $C^T X_0 \leq b^T Y_0$ (b) $C^T X_0 \geq b^T Y_0$ (c) $C^T X_0 = b^T Y_0$ (d) none of these

44. The dual simplex method is applicable provided

(a) optimality remains satisfied (b) feasibility remains satisfied

(c) both remain unsatisfied (d) optimality is satisfied but feasibility is disturbed

45. Application of dual simplex method requires that availability vector b must satisfy

(a) $b \geq 0$ (b) $b \leq 0$ (c) no restriction of (a) and (b) type

46. In dual simplex table x_j is the only variable with negative value in solution column, but all other entries in x_j-row are ≥ 0. Then LPP has

(a) unbounded solution (b) infeasible solution

(c) alternate optimal solution (d) none of these

47. If the primal has degenerate optimal solution, the dual has

(a) alternate optimal solution (b) degenerate optimal solution
(c) no feasible solution

48. If a variable x_j is unrestricted in sign in a primal LPP, then the corresponding dual jth constraint in the dual will be

(a) $\leq$ (b) $\geq$ (c) equality constraint (d) none of these

49. If the jth constraint in the primal is an equality, then the corresponding dual variable is

(a) unrestricted in sign (b) restricted to ≥ 0 (c) restricted to ≤ 0

50. The primal LPP is

$$\begin{aligned} \max \quad & x_0 = x_1 - 2x_2 \\ \text{s.t.} \quad & x_1 - 3x_2 \leq -3 \\ & -x_1 + 2x_2 = -2 \\ & x_1, x_2 \geq 0 \end{aligned}$$

The $C_B^T B^{-1}$ in optimal table of above LPP is $(0,1)^T$. Then the the optimal solution of the dual is

(a) $(0,1)^T$ (b) $(0,-1)^T$ (c) $(0,2)^T$ (d) none of these

51. Consider the LPP

$$\begin{aligned} \max \quad & z = x_1 + 5x_2 + 3x_3 \\ \text{s.t.} \quad & x_1 + 2x_2 + x_3 = 3 \\ & 2x_1 - x_2 = 4 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

Given that in optimal table of this problem x_1 and x_3 are basic variables, then the optimal solution of the dual problem is

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Suggestion. First and third dual constraints are satisfied as equality constraints.

52. Consider the LPP

$$\begin{aligned} \max \quad & z = x_1 + 5x_2 + 2x_3 \\ \text{s.t.} \quad & x_1 + 2x_2 + x_3 = 15 \\ & 2x_1 - x_2 = 10 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

Given that in optimal table of this problem x_1 is a basic variable and x_3 is a nonbasic variable with relative cost $1/5$, then the optimal solution of the dual problem is

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53. Let primal be a min LP and let a feasible solution which is not optimal of primal causes objective function value to 25. Then which of the following can be the value of dual objective function

(a) 25 (b) 24.5 (c) 26 (d) none of these

54. If a slack or surplus variable s_i is positive in optimal BFS of primal, then in optimal dual solution

(a) the dual variable y_i is 0 (b) the dual variable $y_i > 0$

(c) the slack or surplus of ith dual constraint is 0 (d) none of these

55. If the primal LPP has an unbounded solution, then the dual problem has

(a) an optimal solution (b) infeasible solution (c) an unbounded solution (d) none of these

56. If the dual LPP has an unbounded solution, then the primal problem has

(a) optimal solution (b) infeasible solution (c) unbounded solution (d) none of these

57. A primal LPP has nondegenerate optimal solution, then the optimal solution of the dual

(a) is nondegenerate (b) is degenerate

(c) may be nondegenerate or degenerate (d) none of these

58. If the primal LPP has infeasible solution, then the solution of the dual problem is

(a) unbounded (b) infeasible (c) either unbounded or infeasible (d) none of these

59. If in dual simplex method, the rule for entering variable is not followed, then

(a) the feasibility will further deteriorate (b) the optimality will be disturbed

(c) there will be no change (d) none of these

60. Consider the LPP:

$$\begin{aligned} \max \quad & z = 5x_1 + 2x_2 \\ \text{s.t.} \quad & x_1 + x_2 \leq 3 \\ & 2x_1 + 3x_2 \geq 5 \\ & x_1, x_2 \geq 0 \end{aligned}$$

Which of the following primal-dual solutions are optimal:

(a) $x_1 = 3, x_2 = 1;$ $y_1 = 4, y_2 = 1$ (b) $x_1 = 4, x_2 = 1;$ $y_1 = 1, y_2 = 0$

(c) $x_1 = 3, x_2 = 0;$ $y_1 = 5, y_2 = 0$ (d) $x_1 = 2, x_2 = 5;$ $y_1 = 1, y_2 = 5$

61 In dual simplex method, let the variable x_i leave the basis and the variable x_j enter. Let $x_j > 0$. Then later on

(a) x_j can become negative (b) x_j will remain positive (c) none of these

62. For every maximization LPP with m equality constraints and n variables $(m < n)$, the number of unrestricted dual variables will always be

(a) $\leq m$ (b) m (c) $\leq n$ (d) n

Suggestion. Consult the LPP max $x_1 + 5x_2 + 3x_3$, subject to $x_1 + 2x_2 + x_3 = 3$, $2x_1 - x_2 = 4$, $x_1, x_2, x_3 \geq 0$.

63. Suppose primal is infeasible and dual is feasible. Then dual will have

(a) unbounded solution (b) finite solution (c) alternate optimal solution (d) none of these

64. The jth constraint in dual of a LPP is satisfied as strict inequality by the optimal solution. Then the jth variable of the primal will assume a value

(a) $\neq 0$ (b) ≤ 0 (c) ≥ 0 (d) 0

65. Let P_F be the feasible set of a LPP which is bounded and nonempty. If a constraint is deleted then the feasible set of the new LPP

(a) may be unbounded (b) may be empty (c) will always be bounded (d) none of these

66. A LPP is given in canonical form with $b \geq 0$, and its optimal table is

B. V.	x_1	x_2	s_1	s_2	Soln
x_0	0	0	2	1	34
x_2	1	0	2	−3	2
x_1	0	1	−3	5	1

Then, for $b' = (1, 1)^T$, $C_B^T B^{-1} b'$ is

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67. Consider the optimal table of Problem 65. The objective function of the LPP is

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68. Consider the optimal table of Problem 65. The right hand vector of the LPP is

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69. Consider the optimal table of Problem 65. The column A_1 associated with x_1 in constraint matrix of the LPP is

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Suggestion. For Problems 67 to 69, compute B from B^{-1}.

70. The optimal table of a LPP in which s_1 is a surplus variable and s_2 is a slack variable in standard form of the LPP is

B V	x_1	x_2	s_1	s_2	Soln
z	0	0	−1	−2	2
x_1	1	0	−3	−2	1
x_2	0	1	1	1	1

The right hand side vector is assigned $b' = (4, 5)^T$. Then the new optimal solution is

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71. If the variable x_2 is deleted from the LPP whose optimal table is in the preceding problem, then the optimal solution of the changed problem is

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72. Following is the optimal table of a LPP (s_1, s_2, $s_3 \geq 0$ are the slack variables when LPP is written in standard form)

B V	x_1	x_2	s_1	s_2	s_3	Soln
x_0	0	0	7/6	13/6	0	218/5
s_3	0	0	3/2	−25/2	1	5
x_1	1	0	1/3	−2/3	0	16/5
x_2	0	1	−1/6	5/6	0	10/3

If the third constraint is deleted, then optimal solution of the revised LPP

(a) $x_1 = 14/5,\ x_2 = 11/3$ (b) $x_1 = 3,\ x_2 = 4$
(c) $x_1 = 16/5,\ x_2 = 10/3$ (d) none of these

73. Change in all the coefficients of a particular variable in a LPP

(a) disturbs feasibility (b) disturbs optimality
(c) may disturb both feasibility and optimality (d) none of these

74. Change in some column of the coefficients of a LPP

(a) disturbs feasibility (b) disturbs optimality
(c) may disturb both feasibility and optimality (d) none of these

75. Addition of variable and deletion of a constraint simultaneously to a LPP

(a) disturbs feasibility (b) disturbs optimality
(c) may disturb both feasibility and optimality (d) none of these

76. In a max LPP, if a constraint is added then the objective function value

(a) will decrease (b) will decrease or remains same
(c) will increase (d) nothing can be said

77. In a LPP the costs are changed and simultaneously a constraint is deleted, then in the optimal table

(a) only feasibility may be disturbed (b) only optimality is disturbed
(c) both feasibility and optimality may be disturbed (d) nothing can be said

78. Write true and false for each of the following statements

(a) Dual of dual is primal

(b) To solve a LP problem with some constraints of the type $\leq$ and some of the type $\geq$, it is must to use big M-method

(c) In standard form of a LPP all constraints must be of the type $\leq$

(d) A LPP with two constraints and three variables can be solved by the graphical method

(e) A LPP may have two optimal solutions one nondegenerate and other one degenerate

(f) In optimal table of a LPP, the relative cost for a nonbasic variable is the indication of alternate optimal solution. It is always possible to find alternate optimal basic feasible solution by permitting to enter this nonbasic variable into the basis

(g) In general, the dual of a LPP with m equality constraints contains m unrestricted variables. It is possible to have a dual which has less than m unrestricted variables

(h) The number of vertices of any closed bounded set can not be infinite

(i) A LPP has 7 variables and 5 constraints. It is possible to find the optimal solution of this LPP by the graphical method

(j) It is possible to construct examples in which primal and dual are both unbounded

(k) A LPP has an optimal solution. It is possible to get unbounded solution by changing the right hand side vector arbitrarily

(l) If a linear program is infeasible, then its dual must be unbounded.

(m) If a minimization LPP problem has a feasible vector, then its dual can never go to unbounded maximum value.

Suggestion. For (f), see Problem 42.

79. In a balanced transportation problem with m sources and n destinations, the number of basic variables is

(a) $m+n$ (b) $m+n+1$ (c) $m+n-1$ (d) none of these

80. If some constant is added to each cost c_{ij} of a row or column in a transportation matrix, then the the optimal value

(a) decreases (b) increases (c) may increase or decrease (d) remains same

81. In a balanced transportation problem with 3 sources and 3 destinations, the number of basic feasible solutions possible are

(a) 100 (b) 120 (c) 124 (d) 126

82. In a balanced transportation model, if u_i's and v_j's are the dual variables associated with rows and columns, then

(a) $u_i \geq 0$ (b) $v_j \geq 0$ (c) $u_i, v_j \geq 0$ (d) u_i, v_j unrestricted

83. In a balanced transportation problem with m sources and n destinations, the number of dual constraints will be

(a) $m+n$ (b) $m+n+1$ (c) $m+n-1$ (d) mn

84. In a balanced transportation problem with m sources and n destinations, the number of nonbasic basic variables will be

(a) mn (b) $(m-1)(n-1)$ (c) $m(n+1)$ (d) $(n+1)m$

85. In a balanced transportation problem with m sources and n destinations, the number of dual variables will be

(a) $m+n$ (b) $m+n+1$ (c) $m+n-1$ (d) none of these

86. In a transportation problem, one dual variable can be assigned arbitrary

(a) any real value (b) only zero value

(c) only values ≥ 0 (d) only values ≤ 0

87. For a balanced transportation problem with 2 sources and 3 destinations, the optimal solutions are $x_{11} = 10,\ x_{13} = 8,\ x_{22} = 4,\ x_{23} = 4$ and $x_{11} = 6,\ x_{13} = 12,\ x_{21} = 4,\ x_{22} = 4$, then $x_{11} = 8,\ x_{13} = 10,\ x_{21} = 2,\ x_{22} = 4,\ x_{23} = 2$

(a) is a solution but not optimal (b) may or may not be optimal solution

(c) is an optimal basic solution (d) is a nonbasic optimal solution

88. In TP one of the dual variable is assigned arbitrary value because

(a) solution is available immediately (b) one of the constraint in TP is redundant

(c) construction of loop becomes simple (d) none of these.

89. In balanced TP, the dual variables are unrestricted in sign because

(a) TP is a minimization problem (b) TP is with equality constraint

(c) all decision variables are integers (d) none of these.

90. In a balanced TP with two sources and three destinations and availabilities 30 at each source and demand 20 at each destination, the dual variables in the optimal table corresponding to sources and destinations are -1, 2 and 1, 2, 3, respectively. Then the optimal value is

(a) 90 (b) 110 (c) 80 (d) 150

91 In a transportation problem the value of the dual variables are not unique. If one dual variable is assigned two different values and remaining are computed as usual, then

(a) $z_{ij} - c_{ij}$ will also change (b) $z_{ij} - c_{ij}$ are unique (c) may or may not change

92. In an assignment problem with m jobs and m machines, the number of basic variables at zero level in a BFS is

(a) m (b) $m-1$ (c) $m+1$ (d) none of these

93. If some constant is added to each cost c_{ij} of the assignment matrix then the

(a) optimal solution changes (b) optimal solution remains same.

94. Which one of the following is not a deterministic model

(a) Linear programming problem (b) Transportation problem (c) CPM (d) PERT

95 In a minimization ILPP with x_1 and x_2 as the terminal nodes of Branch and bound algorithm at a particular stage are shown below

$x_0 = 15$	$x_0 = 13$	$x_0 = 10$	$x_0 = 16$	$x_0 = 15$	$x_0 = 13$
1, 2	1.2, 3	1, 2.3	2, 5	2, 7.2	2, 3.5
$N(1)$	$N(2)$	$N(3)$	$N(4)$	$N(5)$	$N(6)$

The next branching must be done from the node

(a) N(5) (b) N(3) (c) N(2) (d) none of these

96. In above problem the next branching corresponds to

(a) $x_1 \leq 1,\ x_2 \geq 2$ (b) $x_1 \leq 2,\ x_2 \geq 3$
(c) $x_1 \leq 7,\ x_2 \geq 7$ (d) none of these

97. State true or false

(a) The sum of two convex functions is convex

(b) The product of two convex functions is convex

(c) If in TP, two dual variables are assigned arbitrary value then the method will yield correct solution

(d) Assignment problem is not a linear model

(e) Assignment problem can be solved by using $u-v$ method

(f) A cyclic solution of an assignment problem with m machines and m jobs is also a solution of traveling salesman problem with m cities

(g) In traveling salesman problem we reduce the cost matrix first by row-wise and then by column-wise. If the process is reversed, then amount of reduction is always same

(h) In traveling salesman problem with n cities the number of possible tours are $(n-1)!$

(i) A maximum flow problem has always a unique solution

(j) The sum of two unimodal functions is unimodal

(k) A convex function is always unimodal

(l) The minimal spanning tree always gives unique solution

98. In an n-node square matrix the completion of Floyd's algorithm to find shortest route between any two nodes requires the number of comparisons:

(a) n^2 (b) $n^2(n-1)$ (c) $n(n-1)^2$ (d) $n(n+1)$

99. The following table shows the machine time (in days) for 5 jobs to be processed on two different machines M_1 and M_2 in the order M_1M_2:

Job	1	2	3	4	5
M_1	3	7	4	5	7
M_2	6	2	7	3	4

The optimal sequence of jobs to be processed on theses machines to minimize the total elapsed time is

(a) $1 \to 2 \to 3 \to 4 \to 5$ (b) $1 \to 4 \to 5 \to 2 \to 3$

(c) $1 \to 3 \to 5 \to 4 \to 2$ (d) $1 \to 5 \to 3 \to 4 \to 2$

100. The number of all possible optimal sequences to minimize the total elapsed time required to complete the following tasks is (each job is processed in the order $M_1M_3M_2M_4$).

Job	1	2	3	4
Processing time on M_1	20	17	21	25
Processing time on M_2	10	7	8	5
Processing time on M_3	9	15	10	9
Processing time on M_4	25	5	9	25

(a) 2 (b) 4 (c) 16 (d) 24

101. The function $f(X) = 3x_1^2 - 2x_2^2 + x_3^2$ is

(a) positive definite (b) positive semi-definite (c) negative definite (d) indefinite

102. The function $f(X) = x_1^2 + x_2^2 + x_3^2 - 2x_1x_2$ is

(a) convex (b) strictly convex (c) concave (d) strictly concave

103. In dichotomous search technique with $\delta = 0.1$ and $L_0 = 1$, width of initial interval of uncertainty, after four experiments the width of interval of uncertainty is reduced to

(a) 0.325 (b) 0.375 (c) 0.425 (d) 0.475

104. In Fibonacci search technique with $n = 5$ and $L_0 = 1$, the measure of effectiveness is

(a) 0.001 (b) 0.01 (c) 0.1 (d) 0.125

105. Let L_4 and L_4' be the length of interval of uncertainty after four experiments in Fibonacci and Golden section search, respectively. Then

(a) $L_4 < L_4'$ (b) $L_4' < L_4$ (c) $L_4 = L_4'$ (d) none of these

106. The number of experiments in Fibonacci search that reduce the interval of uncertainty to $\approx 0.001L_0$ (L_0: initial interval of uncertainty) is

(a) 13 (b) 14 (c) 15 (d) 16

107. The number of experiments in Golden search that reduce the interval of uncertainty to $\approx 0.05571L_0$ (L_0: initial interval of uncertainty) is

(a) 7 (b) 8 (c) 9 (d) 10

108. For positive definite quadratic X^TAX in $n(> 3)$ variables, let X_1 be the initial approximation for minimum point when the conjugate gradient method is applied. Then the exact minimum occurs at

(a) X_{n-1} (b) X_n (c) X_{n+1} (d) none of these

109. Consider the following pay-off matrix

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & -2 & 2 \end{bmatrix}$$

If solved as game of pure strategies then the game value is

(a) -2 (b) 1 (c) 2 (d) none of these

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Answers

Problem Set 1

1. opt $z = 2x_1 + x_2' - x_3^+ + x_3^- + 1$

$$\begin{aligned}
\text{s.t. } & 2x_1 + x_2' - x_3^+ + x_3' + s_1 = 4 \\
& 3x_1 - 2x_2' - 3x_3^+ + 3x_3^- + s_2 = 5 \\
& x_1 - 3x_2' + 4x_3^+ - 4x_3^- - s_3 = 5 \\
& x_1 + x_2' + x_3^+ - x_3^- = 3 \\
& x_1, x_2', x_3^+, x_3^-, s_1, s_2, s_3 \geq 0
\end{aligned}$$

2. opt $z = 2x_1 - x_2' + x_3$

$$\begin{aligned}
\text{s.t. } & x_1 + x_2' + 2x_3 - s_1 = 2 \\
& u + v + s_2 = 4 \\
& 2x_1 - x_2' - x_3 - u + v = 0 \\
& 3x_1 + 2x_2' - 7x_3 + s_4 = 3 \\
& x_1, x_2', x_3, u, v, s_1, s_2, s_4 \geq 0
\end{aligned}$$

3. opt $z = x_1 + 2x_2' - x_3^+ + x_3^- + 2p$

$$\begin{aligned}
\text{s.t. } & x_1 + x_2' - x_3^+ + x_3^- + s_1 = 5 - p \\
& x_1 - 2x_2' - 3x_3^+ + 3x_3^- + s_2 = 4 + 2p \\
& 2x_1 + 3x_2' - 4x_3^+ + 4x_3^- - s_3 = 3 - 3p \\
& x_1 + x_2' + x_3^+ - x_3^- = 2 - p \\
& x_1, x_2', x_3^+, x_3^-, s_1, s_2, s_3 \geq 0.
\end{aligned}$$

The range of p is $-2 \leq p \leq 1$.

4. (a) This is not an LPP (b) min $z = x_1^+ + x_1^- + 2x_2^+ + 2x_2^- - x_3$

s.t. $x_1^+ - x_1^- + x_2^+ - x_2^- - x_3 \leq 9$

$x_1^+ - x_1^- - 2x_2^+ + 2x_2^- + 3x_3 = 11$

$x_1^+, x_1^-, x_2^+, x_2^-, x_3 \geq 0$

5. Assume $y = |\min\{x_1, x_2\}|$. The LPP is min $z = y_1 + y_2 - 2y$

s.t. $y_1 + y_2 - 2y + s_1 = 6$

$y_1 - 2y_2 + y - s_2 = 3$

$y_1, y_2, y, s_1, s_2 \geq 0$

6. Define $r = \dfrac{1}{6 + 3x_1 - x_2} > 0$. This implies $3x_1r - x_2r = 1 - 6r$.

Let $rx_j = y_j$, $j = 1, 2, 3$. The required LP model is

max $2y_1 + 5y_2 - 5y_3 - 3r$,

s.t. $3y_1 - y_2 = 1 - 6r$

$y_1 - y_2 \geq 0$

$7y_1 + 9y_2 + 10y_3 \leq 30r$

$y_1 \geq 0, y_2 \geq r, y_3 \geq 0$

7. x_j = number of units required of jth food, $j = 1, 2, \ldots, n$

min $z = \sum_{j=1}^{n} c_j x_j$

s.t. $\sum_{j=1}^{n} a_{ij} x_j \geq b_i$, $i = 1, 2, \ldots, m$

$x_j \geq 0$, $j = 1, 2, \ldots, n$.

8. x_i = number of units manufactured of ith product, $i = A, B$

max $z = 5x_A + 10x_B$

s.t. $x_A + 5x_B \leq 10000$

$3x_A + x_B \leq 7000$

$x_A, x_B \geq 0$ and are integers

9. x_1 = number of belts of type A, x_2 = number of belts of type B,

max $z = 3x_1 + 2x_2$

$$
\begin{aligned}
\text{s.t. } & 2x_1 + x_2 \le 1500 \\
& x_1 + x_2 \le 1000 \\
& x_1 \le 500 \\
& x_2 \le 800 \\
& x_1, x_2 \ge 0 \text{ and integers}
\end{aligned}
$$

10. m_1 = units of milk produced in Plant-I per day, m_2 = units of milk produced in Plant-II per day, b_1 = units of butter produced in Plant-I per day, b_2 = units of milk produced in Plant-II per day. Assume that one unit milk $\equiv$ 1000 liters and one unit butter $\equiv$ 100 kgs.

$$
\begin{aligned}
\min\ & z = (15m_1 + 28b_1 + 18m_2 + 26b_2)1000 \\
\text{s.t. } & m_1 + m_2 \ge 10 \\
& b_1 + b_2 \ge 8 \\
& 3m_1 + 2b_1 \le 16 \\
& b_1 + 1.5b_2 \le 16 \\
& m_1, m_2, b_1, b_2 \ge 0
\end{aligned}
$$

11. $\max\ z = 75x_1 + 50x_2$

$$
\begin{aligned}
\text{s.t. } & 25x_1 + 40x_2 \le 4400 \\
& 30x_1 + 15x_2 \le 3300 \\
& \tfrac{6}{19} \le \tfrac{x_2}{x_1} \le \tfrac{17}{8} \\
& x_1 \ge 0, x_2 \ge 0 \text{ and are integers}
\end{aligned}
$$

12. x_i = number of master rolls cut on the pattern p_i, $i = 1, 2, \ldots, 8$
$\min\ z = x_1 + x_2 + \cdots + x_8$

$$
\begin{aligned}
\text{s.t. } & 5x_1 + 3x_2 + 3x_3 + 2x_4 + x_5 \ge 200 \\
& x_2 + 2x_5 + x_6 + x_7 \ge 90 \\
& x_3 + x_6 + 2x_8 \ge 350 \\
& x_4 + x_7 \ge 850 \\
& x_1, x_2, \ldots, x_8 \ge 0 \text{ and integers}
\end{aligned}
$$

13. x_i = Number of parent metallic sheets cut on the pattern p_i, $i = 1, 2, 3$.

$$\min z = x_1 + x_2 + x_3$$
$$\text{s.t. } 10x_1 + 6x_2 + 2x_3 \geq 2500$$
$$x_2 + 2x_3 \geq 1500$$
$$x_1, x_2 \geq 0 \text{ and are integers}$$

14. x_1 = number of tables and x_2 = number of chairs to be manufactured

$$\max\ z = 9x_1 + 6x_2$$
$$\text{s.t. } 30x_1 + 20x_2 \leq 381$$
$$10x_1 + 5x_2 \leq 117$$
$$x_1, x_2 \geq 0 \text{ and integers}$$

15. $$\max\ z = 15x_1 + 25x_2$$
$$\text{s.t. } 3x_1 + 4x_2 = 100$$
$$2x_1 + 3x_2 \leq 70$$
$$x_1 + 2x_2 \leq 30$$
$$x_1 \geq 0, x_2 \geq 3$$

16. Let x_1, x_2, x_3 be the number of models I, II, III, respectively to be manufactured.

$$\max\ z = 30x_1 + 20x_2 + 60x_3$$
$$\text{s.t. } 6x_1 + 3x_2 + 2x_3 \leq 4200$$
$$2x_1 + 3x_2 + 5x_3 \leq 2000$$
$$4x_1 + 2x_2 + 7x_3 \leq 3000$$
$$x_1 \geq 200,\ x_2 \geq 200,\ x_3 \geq 150 \text{ and integers}$$

17. x_{ij} = number of units of product i processed on machine j, $i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$

$$\min\ z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}x_{ij}$$
$$\text{s.t. } a_{i1}x_{11} + a_{i2}x_{12} + \cdots + a_{in}x_{1n} \leq b_i$$
$$x_{1j} + x_{2j} + \cdots + x_{mj} \geq d_j$$
$$x_{ij} \geq 0;\ i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n.$$

18. x_{ij} = the amount (in tons) of the ith commodity given the placement at jth position, $i = A, B, C$, $j = 1$ (for forward), 2 (for centre), 3 (for after)

$$\max z = 60(x_{A1} + x_{A2} + x_{A3}) + 80(x_{B1} + x_{B2} + x_{B3}) \\ + 50(x_{C1} + x_{C2} + x_{C3})$$

$$\begin{aligned}
\text{s.t. } & x_{A1} + x_{A2} + x_{A3} \leq 6000 \\
& x_{B1} + x_{B2} + x_{B3} \leq 4000 \\
& x_{C1} + x_{C2} + x_{C3} \leq 2000 \\
& x_{A1} + x_{B1} + x_{C1} \leq 2000 \\
& x_{A2} + x_{B2} + x_{C2} \leq 3000 \\
& x_{A3} + x_{B3} + x_{C3} \leq 1500 \\
& 60x_{A1} + 50x_{B1} + 25x_{C1} \leq 100000 \\
& 60x_{A2} + 50x_{B2} + 25x_{C2} \leq 135000 \\
& 60x_{A3} + 50x_{B3} + 25x_{C3} \leq 30000 \\
& x_{ij} \geq 0
\end{aligned}$$

19. min $z = y$

$$\begin{aligned}
\text{s.t. } & 20x_{1a} + 25x_{2a} + 20x_{3a} - y \geq 0 \\
& 25x_{1b} + 20x_{2b} + 5x_{3b} - y \geq 0 \\
& x_{1a} + x_{1b} \leq 100 \\
& x_{2a} + x_{2b} \leq 150 \\
& x_{3a} + x_{3b} \leq 200 \\
& x_{ij}, y \geq 0,\ i = 1, 2, 3;\ j = 1, b
\end{aligned}$$

20. The first constraint $6x_1 + 3x_2 + 2x_2$ in answer to Problem 16 is replaced by

$$x_1 + 2x_2 \leq 700$$

$$x_2 + 3x_3 \leq 1400$$

21. max $z = \min\left\{\sum_{i=1}^m a_{i1}x_{i1},\ \frac{1}{2}\sum_{i=1}^m a_{i2}x_{i2},\ \ldots,\ \frac{1}{n}\sum_{i=1}^m a_{in}x_{in}\right\}$

$$\begin{aligned}
\text{s.t. } & x_{i1} + x_{i2} + \cdots + x_{in} \leq b_i \\
& x_{ij} \geq 0,\ j = 1, 2, \ldots, n
\end{aligned}$$

22. Let x_j be the number of waiters recruited on jth day, $j = 1, 2, 3, 4, 5$

$$\min\ z = x_1 + x_2 + x_3 + x_4 + x_5$$

$$\text{s.t. } x_1 + x_4 + x_5 \geq 25$$

$$x_1 + x_2 + x_5 \geq 35$$

$$x_1 + x_2 + x_3 \geq 40$$

$$x_2 + x_3 + x_4 \geq 30$$

$$x_3 + x_4 + x_5 \geq 20$$

$$0 \leq x_j \leq 30 \text{ and are integers}$$

23. x_{ij} = number of buses of type i allocated to city j; $i = 1, 2, 3$, $j = 1, 2, 3, 4$. s_j = number of passenger not served for the cities $j = 1, 2, 3, 4$

$$\begin{aligned}\min z = {} & 3000x_{11} + 2200x_{12} + 2400x_{13} + 1500x_{14} \\ & + 3600x_{21} + 3000x_{22} + 3300x_{23} + 2400x_{24} \\ & + 3500x_{31} + 4500x_{32} + 3600x_{33} + 2000x_{34} \\ & + 40s_1 + 50s_2 + 45s_3 + 70s_4,\end{aligned}$$

subject to

$$\sum_{j=1}^{4} x_{1j} \leq 5, \quad \sum_{j=1}^{4} x_{2j} \leq 8, \quad \sum_{j=1}^{4} x_{3j} \leq 10$$

$$300x_{11} + 280x_{21} + 250x_{31} + s_1 = 1000$$

$$200x_{12} + 210x_{22} + 250x_{32} + s_2 = 2000$$

$$200x_{13} + 210x_{23} + 200x_{33} + s_3 = 900$$

$$100x_{14} + 140x_{24} + 100x_{34} + s_4 = 1200$$

$$\text{all } x_{ij} \geq 0, \text{ and } s_j \geq 0$$

24. x_{iA} = amount invested in year i under Scheme A; x_{iB} = amount invested in year i under Scheme B

$$\max\ z = 2.4x_{2B} + 1.6x_{3A}$$

$$\text{s.t. } x_{1A} + x_{1B} \leq 2,00,000$$

$$x_{2A} + x_{2B} \leq 1.6x_{1A}$$

$$x_{3A} \leq 2.4x_{1B} + 1.6x_{2A}$$

$$x_{iA}, x_{iB} \geq 0, \ i = 1, 2, 3$$

25. Define x_1 = number of units of P_1 made on regular time; x_2 = number of units of P_2 made on regular time; x_3 = number of units of P_1 made on overtime; x_4 = number of units of P_2 made on overtime; x_5 = number of units of P_1 made on regular time on M_1 and overtime on M_2; x_6 = number of units of P_2 made on regular time on M_1 and overtime on M_2.

$$\max\ z = 8x_1 + 10x_2 + 4x_3 + 8x_4 + 6x_5 + 9x_6$$

$$\text{s.t. } 5x_1 + 4x_2 + 5x_5 + 4x_6 \leq 120 \text{ (regular time of } M_1)$$

$$5x_3 + 4x_4 + \leq 50 \text{ (overtime of } M_1)$$

$$3x_1 + 6x_2 \leq 150 \text{ (regular time on } M_2)$$

$$3x_3 + 6x_4 + 3x_5 + 6x_6 \leq 40 \text{ (overtime on } M_2)$$

$$x_j \geq 0,\ j = 1, 2, \ldots 6$$

26. $\max\ z = (x_1 + x_{11} + x_{12})p_1 + (y_1 + y_{11} + y_{12})p_2$

$$\text{s.t. } 24x_1 + 36y_1 - 0.75x_{11} - 0.75y_{11} - 0.33y_{12} \leq G_1$$

$$8x_1 + 12y_1 - 0.5x_{11} - 0.5y_{11} \leq G_2$$

$$100x_1 + 50y_1 - 0.9x_{11} - 0,9y_{11} - 0.6y_{12} \leq P_1$$

$$x_1, x_{11}, x_{12}, y_1, y_{11}, y_{12} \geq 0$$

27. x_1 = number of units of P_1, x_2 = number of units of P_2
$\max\ z = 10x_1 + 30x_2$

$$\text{s.t. } -0.6x_1 + 0.4x_2 \leq 0$$

$$x_1 \leq 100$$

$$x_1 + 2x_2 \leq 120$$

$$x_1, x_2 \geq 0$$

28. $\max\ z = 4000(x_{1A} + x_{1B} + x_{1C}) + 3000(x_{2A} + x_{2B} + x_{2C}) + 4000(x_{3A} + x_{3B} + x_{3C})$
subject to

Availability of acreage for each crop

$$x_{1A} + x_{1B} + x_{1C} \leq 700$$

$$x_{2A} + x_{2B} + x_{2C} \leq 800$$

$$x_{3A} + x_{3B} + x_{3C} \leq 300$$

Availability of usable acreage in each farm

$x_{1A} + x_{2A} + x_{3A} \leq 400$

$x_{1B} + x_{2B} + x_{3B} \leq 600$

$x_{1C} + x_{2C} + x_{3C} \leq 300$

Water available (in acre feet) constraints

$5x_{1A} + 4x_{2A} + 3x_{3A} \leq 1500$

$5x_{1B} + 4x_{2B} + x_{3C} \leq 2000$

$5x_{1C} + 4x_{2C} + x_{3C} \leq 900$

To ensure that the percentage of usable acreage is same for each farm

$\frac{x_{1A} + x_{1B} + x_{1C}}{400} = \frac{x_{2A} + x_{2B} + x_{2C}}{600} = \frac{x_{3A} + x_{3B} + x_{3C}}{300}$ or

$3(x_{1A} + x_{1B} + x_{1C}) = 2(x_{2A} + x_{2B} + x_{2C})$

$x_{2A} + x_{2B} + x_{2C} = 2(x_{3A} + x_{3B} + x_{3C})$

$x_{ij} \geq 0 \; i = 1, 2, 3; \; j = A, B, C$

29. max $z = \sum_{j=1}^{8} u_j (1 - p_{1j})^{x_{ij}} p_{2j}^{x_{2j}} p_{3j}^{x_{3j}}$, subject to

$\sum_{j=1}^{8} x_{ij} \leq a_i, \quad i = 1, 2, 3$

$\sum_{i=1}^{3} x_{ij} \geq b_j, \quad j = 1, 2, \ldots, 8$

$x_{ij}, p_{ij} \geq 0,$

where p_{ij} is the probability that target j will be undamaged by weapon i and x_{ij} is the number of weapons i assigned for target j.

Problem Set 2

1. (a) Not convex; (b) convex; (c) not convex; (d) convex; (e) not convex; (f) not convex

2. min $z = 2x_1 + x_2,$

s.t. $x_1 + x_2 \geq 1$

$x_1 + 2x_2 \geq 2$

$x_1, x_2 \geq 0$

7. The equivalent system with inequalities is

$$x + 1 + x + 2 \leq 1$$

$$2x_1 - 4x_3 \leq -5$$

$$-3x_1 - x_2 + 4x_3 \leq 4$$

10. (a) Triangle with vertices at $(0, -1)$, $(2, 0)$, $(1, 2)$; (b) $\mathbb{R}^2$; (c) Disc $\{(x_1, x_2) : x_1^2 + x_2^2 \leq 1\}$; (d) quadrilateral with vertices as these points.

12. (a) $(0, 2, 0, 0, 0)$; (b) $(-6, 14, 0, 0, 0)$; (c) for x_3 and x_4 as basic variables: infinite solutions; (d) if we take x_1 and x_5 as basic variables: nonexisting solution

13. $(1, 2, 0, 0, 0, 0)$, $(0, 5/3, 4/3, 0, 0)$, $(3, 0, 0, 0, 8)$

17. No

19. $\binom{n-p}{m-p}$

20. Problem 18 ensures the existence of three bases corresponding to the degenerate BFS $(0, 2, 0, 0)$. Two bases are given in Example 2, Section 2.3 and the third one is

$$\begin{bmatrix} 1 & -8 \\ -1 & -1 \end{bmatrix}$$

21. $(-11, 5, 2, 8)$

24. (a) $x_1 = 2$, $x_2 = 0$, $z = -2$; (b) Unbounded solution

25. The optimum will not exist until the objective function is constant.

29. min $z = -x_1 - x_2 + 14$,

$$\begin{aligned} \text{s.t. } -3x_1 + x_2 &\leq 3 \\ x_2 &\leq 7 \\ x_1 - x_2 &\leq 2 \\ x_1, x_2 &\geq 0 \end{aligned}$$

Optimal solution: $x_1 = 9$, $x_2 = 7$, $z = -2$

30. $(1, 1, 0, 0)$, $(1, 0, 1, 0)$, $(1, 0, 0, 1)$, $(0, 7/2, 0, -3/2)$, $(0, 0, -7/2, -3/2)$

Problem Set 3

2. (a) $x_1 = 2/3,\ x_2 = 7/3,\ \ z = 16/3$; (b) $x_1 = 2,\ x_2 = 2,\ \ z = 14$; (c) $x_1 = 0,\ x_2 = 1,\ \ z = 1$; (d) $x_1 = 0,\ x_2 = 30,\ \ z = 18$

3.
$$\begin{aligned} \max\ & z = -4x_1 + x_2 \\ \text{s.t.}\ & 7x_1 - 3x_2 \leq 3 \\ & -2x_1 + x_2 \leq 1 \\ & x_1, x_2 \geq 0 \end{aligned}$$

4. Basic solution $(4, 0, -6, 0)$, but not feasible

5. Unbounded solution

6. $x_1 = 0, x_2 = -2,\ z = 6$

8. (i) x_3, x_1 and s_3 are the basic variables in that order; (ii) The LPP is

$$\begin{aligned} \max\ \ & z = 6x_1 + 2x_2 + 10x_3 + 2x_4, \\ \text{s.t.}\ & x_2 + 2x_3 \leq 5 \\ & 3x_1 - x_2 + x_3 + x_4 \leq 10 \\ & x_1 + x_3 + x_4 \leq 8 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

10. Unbounded solution

11. (a) $x_1 = 0,\ x_2 = 2,\ s_1 = 1,\ s_2 = 0,\ s_3 = 7$
(b) $x_1 = 1/8,\ x_2 = 9/4,\ s_1 = 11/8,\ s_2 = s_3 = 0$

12. (a) $(0, 2, 1, 0, 1)$ (b) $(1/8, 9/4, 11/8, 0, 0)$

13. $x_1 = x_2 = 0,\ x_3 = x_4 = 1,\ x_5 = 3,\ z = 5$

15. The solution space is unbounded in x_2 direction

16. $x_1 = 7,\ x_2 = -1,\ \ z = 22$

17. Optimal solution: $x_1 = 22/5,\ x_2 = 31/5,\ x_3 = 0\ \ z = 8$

18. x_1 must be preferred if $\theta_1(z_1 - c_1) < \theta_2(z_2 - c_2)$ for min problem, and the reverse inequality is considered for max problem

21. $x_1 = 1,\ x_2 = 0,\ x_3 = 1,\ x_4 = 0,\ \ z = -5/4$

Problem Set 4

1. $C = -b$ and A is skew-symmetric matrix

2. Yes, take (x_1, x_2) as the starting BFS

3. Dual:

$$\begin{aligned} \max\ \ & z = -y_1 - 2y_2 - 2y_3 \\ \text{s.t.}\ \ & -y_1 + y_2 - y_3 \le -2 \\ & -2y_1 + y_3 \le -3 \\ & y_2 - 2y_3 \le -1 \\ & y_1, y_2, y_3 \ge 0 \end{aligned}$$

Optimal solution of the primal $x_1 = 0,\ x_2 = 1/2,\ x_3 = 5/4,\ \ x_0 = -11/4$

4. Optimal solution of primal: $x_1 = 1/8,\ x_2 = 9/4,\ \ x_0 = 7/4$; Optimal solution of the dual: $y_1 = 0,\ y_2 = 7/4,\ y_3 = 1/4,\ \ y_0 = 7/4$.

5. Dual:

$$\begin{aligned} \min\ \ & y_0 = 14y_1 + 17y_2 - 19y_3 + 100 \\ \text{s.t.}\ \ & -3y_1 - 5y_2 + 2y_3 \le -9 \\ & 8y_1 - 2y_2 - 4y_2 \le 6 \\ & -5y_1 + 6y_2 = -4 \\ & y_1, y_3 \ge 0,\ y_2 \text{ unrestricted} \end{aligned}$$

6. Primal:

$$\begin{aligned} \min\ & z = -2x_1 - 3x_2 \\ \text{s.t.}\ \ & x_1 - x_2 \ge 3 \\ & -x_1 + x_2 \ge 2 \\ & x_1, x_2 \ge 0 \end{aligned}$$

Dual:

$$\begin{aligned} \max\ \ & z' = 3y_1 + 2y_2 \\ \text{s.t.}\ \ & y_1 - y_2 \le -2 \\ & -y_1 + y_2 \le -3 \\ & y_1, y_2 \ge 0 \end{aligned}$$

7. $x_1 = 0,\ x_2 = 5,\ \ x_0 = 15$; Dual:

$$\begin{aligned} \min \ \ & y_0 = -3y_1 + 5y_2 \\ \text{s.t.} \ \ & 2y_1 + 3y_2 \geq 2 \\ & -y_1 + y_2 \geq 3 \\ & y_1, y_2 \geq 0 \end{aligned}$$

10. Converse is also true

11.
$$\begin{aligned} \min \ \ & x_0 = 2x_1 + 3x_2 + 4x_3 \\ \text{s.t.} \ \ & x_2 + x_3 \geq -2 \\ & -x_1 + 3x_3 \geq 3 \\ & -2x_1 - 3x_2 \geq 4 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

13. yes, converse is also true

15.
$$\begin{aligned} \min \ \ & y_0 = 5y_1 + 10y_2 + 8y_3 \\ \text{s.t.} \ \ & 3y_2 + y_3 \geq 6 \\ & y_1 - y_2 \geq 2 \\ & 2y_1 + y_2 + y_3 \geq 10 \\ & y_2 + y_3 \geq 0 \\ & Y - 1, y_2, y_3 \geq 0 \end{aligned}$$

Optimal solution: $y_1 = 4,\ y_2 = 2,\ y_3 = 0,\ \ y_0 = 40$

16. $x_1 = -4,\ x_2 = 4,\ \ x_0 = 0$

17. $y_1 = 3,\ y_2 = -1,\ \ y_0 = 5$

18. (b) $y_1 = 1,\ y_2 = -4/3,\ \ y_0 = 11/3$

19. Optimal solution is $x_1 = 1.286,\ x_2 = 0.476,\ x_3 = 0,\ \ z = 2.095$

20. No feasible solution

21. Optimal of primal: $x_1 = 0,\ x_2 = 0,\ x_3 = 3.57,\ x_4 = 1.43,\ x_5 = 0,\ x_6 = 0.86,\ \ x_0 = 2.71$, Optimal of dual: $y_1 = -0.36,\ y_2 = 0.07,\ y_3 = 0.93,\ \ y_0 = 2.71$

22. max $y_0 = b^TY + \ell^TY' - u^TY''$, subject to $A^TY + \mathbf{I}Y' - \mathbf{I}Y'' \le C$, where $Y = (y_1, y_2, \ldots, y_m)$ unrestricted, $Y' = (y', y', \ldots y'_n) \ge 0$, $Y'' = (y''_1, y''_2, \ldots, y''_n) \ge 0$ and $\mathbf{I}$ is identity matrix of order n

23. max $y_0 = y_{m+1}$, subject to $A^TY + ey_{m+1} \le C$, where $Y = (y_1, y_2, \ldots, y_m)^T$ and y_{m+1} are unrestricted

Problem Set 5

1. $x_1 = 71/10$, $x_2 = 0$, $x_3 = 0$, $x_4 = 13/10$, $x_5 = 0$, $x_6 = 2/5$, $z = 71/10$

2. Infeasible solution (no solution)

3. $x_1 = 5/3$, $x_2 = 0$, $x_0 = 10/3$

4. $x_1 = 0$, $x_2 = 14$, $x_3 = 9$, $z = -9$

5. $x_1 = 4$, $x_2 = 15/4$, $x_3 = 0$, $z = 123/14$

6. $y_1 = 3$, $y_2 = 5$, $y_3 = 2$, $z = 34$

7. A-type belts = 200, B-type belts 800, maximum profit = 2200

8.
$$\begin{aligned} \max\ & z = 6x_1 + 7.50x_2 - 0.5x_3^- \\ \text{s.t.}\ & 10x_1 + 12x_2 + x_3^+ - x_3^- = 2500 \\ & 150 \le x_1 \le 200 \\ & x_2 \le 45 \\ & \text{all var } \ge 0 \end{aligned}$$

(a) Optimal solution $x_1 = 200$, $x_2 = 45$, $x_3^+ = 0$, $x_3^+ = 40$ and $z = \$1517.50$

(b) Optimal solution $x_1 = 196$, $x_2 = 45$, $x_3^+ = 0$, $x_3^+ = 0$ and $z = \$1513.50$, hence no overtime is recommended by the new solution.

9. $x_1 = 47/20$, $x_2 = 1/10$, $x_3 = 27/10$, $x_4 = 6/4$, $z = 479/10$

11. Taking $Q = 2^6 = 65$, $n = 6$ and $M > 0$ (large), the Karmarkar's standard form is

$$\begin{aligned}
\min\ & y_0 = 65y_1 + 65y_2 + 65y_3 - 195y_4 + 390y_5 + 260y_6 + My_9 \\
\text{s.t. } & y_1 + y_2 + 3y_4 - y_5 + 2y_6 - 6y_7 = 0 \\
& y_2 + y_3 - y_4 + 4y_5 + y_6 - 3y_7 - 3y_8 = 0 \\
& y_1 + y_3 - 2y_4 + y_5 + y_5 + 5y_6 - 5y_7 - y_8 \\
& y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 - 64y_8 + 57y_9 = 0 \\
& y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 + y_8 + y_9 = 1 \\
& \text{all var } \geq 0
\end{aligned}$$

12. With $\alpha = 1/\sqrt{6}$, $x^1 = (4/9, 5/18, 5/18)$, $\min = 5/9$. Continuing this iterative process, the Karmarkar's algorithm will stop at the optimal solution $x^1 = (1, 0, 0)$, $\min = 0$

Problem Set 6

1. For same optimal solution the variation in costs satisfies the inequality $\underline{\delta} \leq \delta \leq \overline{\delta}$, where

$$\underline{\delta} = \max_j \left\{ \max_k \left(\frac{z_j - c_j}{\alpha_k^j},\ \alpha_k^j,\ j \in \overline{N} \right), -\infty \right\},$$

$$\overline{\delta} = \min_j \left\{ \min_k \left(\frac{z_j - c_j}{\alpha_k^j},\ \alpha_k^j,\ j \in \overline{N} \right), +\infty \right\}$$

2. For same optimal basis the variation δ in b satisfies the inequality $\underline{\delta} \leq \delta \leq \overline{\delta}$, where $\underline{\delta} = \max_k \left\{ \max_i \left\{ \frac{X_{B_i}}{-\beta_{ik}},\ \beta_{ik} > 0 \right\}, -\infty \right\}$,

$$\overline{\delta} = \min_k \left\{ \min_i \left\{ \frac{X_{B_i}}{-\beta_{ik}},\ \beta_{ik} < 0 \right\}, +\infty \right\}$$

4. (a) $c_1 \in [1, 6]$ (b) $b_2 \in [5/3, 10]$

6. (a) $x_1 = 7/2,\ x_2 = 1/2,\ \ x_0 = 17/2$ (b) $x_1 = 12/5,\ x_2 = 6/5,\ \ x_0 = 42/5$ (c) $x_1 = 1,\ x_2 = 1,\ \ x_0 = 5$ or $x_1 = 5/2,\ x_2 = 0,\ \ x_0 = 5$

7. (a) $x_1 = 0,\ x_2 = 2,\ x_3 = 0,\ \ x_0 = -4$ (b) $x_1 = 0,\ x_2 = 6,\ x_3 = 4,\ \ x_0 = -8$ (c) Unbounded solution

9. $x_1 = x_2 = 0,\ x_3 = 8,\ \ z = -8$, Dual variables $y_1 = 1,\ y_2 = 0,\ \ y_0 = 8$; (a) optimal solution remains unchanged (b) optimal solution remains unchanged (c) $x_1 = x_2 = 0,\ x_3 = 4,\ \ x_0 = -4$

(d) Increase right side of the first constraint. Since $C_B^T B^{-1} = (-1, 0)$, the increase in right side of the first constraint will further reduce the objective function value, it is profitable to increase right side of the first constraint. Increase in right side of second constraint will cause no effect on the objective function value.

10. (i) Optimal solution $x_1 = 0,\ x_2 = 8,\ x_3 = 20/3,\quad x_0 = 20/3,$ $x_0 = 680/3$; (ii) $c_1 > 44/3$; (iii) $c_2 = 56/3$; (iv) $x_1 = 0,\ x_2 = 43/5,\ x_3 = 17/3,\quad x_0 = 686/3$ when $b_1 = 103$, and $x_1 = 0,\ x_2 = 12,\ x_3 = 0,\quad x_0 = 240$, when $b_1 = 200$

11. $x_1 = 7/5,\ x_2 = 2/5,\quad z = 4;\qquad x_1 = x_2 = x_3 = 1,\quad z' = 6$

12. (a) Deletion of first constraint renders the LPP with unbounded solution. Similar is the case when third constraint is deleted (b) The deletion of second constraint causes no effect on the optimal solution

13. $x_1 = 20,\ x_2 = 0,\quad x_0 = 80$

Problem Set 7

1. $x_{12} = 5,\ x_{13} = 20,\ x_{21} = 20,\ x_{24} = 10,\ x_{32} = 15,\ x_{34} = 5,\ x_{43} = 10,\ x_{45} = 25,\quad x_0 = 590$

3. $x_{13} = 50, x_{21} = 30, x_{23} = 120, x_{31} = 70, x_{32} = 130, x_{43} = 30;$ $x_0 = 1460$

4. $x_{12} = 50, x_{22} = 50, x_{23} = 100, x_{31} = 100, x_{33} = 100, x_{42} = 30;$ $x_0 = 1500$

5. $x_{12} = 15, x_{13} = 13, x_{21} = 5, x_{23} = 7, x_{24} = 6, x_{25} = 5, x_{34} = 19;$ $x_0 = 148$

6. N-W rule: 1765, LCM: 1800, VAM: 1695, modified VAM: 1645. Clearly, modified VAM gives better initial basic feasible solution.

7. $\binom{m(n+1)}{m+n}$

8. (a) No (b) up to 4 (c) yes, $x_{12} = 12,\ x_{14} = 9,\ x_{1\text{dummy}} = 1,\ x_{23} = 10,\ x_{31} = 5,\ x_{3\text{dummy}} = 3,\quad x_0 = 89$

9. 270

12. (a) Not an optimal schedule (b) 4 (c) no change

13. $x_{13} = 20, x_{21} = 30, x_{24} = 10, x_{32} = x_{33} = 25$; $x_0 = 495$

14. $W_1 \to M_4,\ W_2 \to M_3,\ W_3 \to M_2,\ W_4 \to M_1,\ W_5 \to M_5,\ W_5 \to M_6$

15. $M_1 \to J_1,\ M_2 \to J_2,\ M_3 \to J_3,\ M_4 \to J_4$; $x_0 = 19$

16. $1 \to 1,\ 2 \to 2,\ 3 \to 3,\ 4 \to 4$, total cost: \$15,000

17. $M_1 \to A,\ M_2 \to C,\ M_3 \to B,\ M_4 \to D$, minimum cost of installation: 34

21. The formulation of the problem is

$$\min \quad x_0 = \sum_{i=1}^{l}\sum_{j=1}^{m}\sum_{k=1}^{n} c_{ijk}x_{ijk}$$

$$\text{s.t.} \quad \sum_{j=1}^{m} x_{ijk} \geq a_{ik}, \quad i = 1, 2, \ldots, l; \quad k = 1, 2, \ldots, n \quad \text{(demand)}$$

$$\sum_{i=1}^{l} x_{ijk} \geq b_{jk}, \quad j = 1, 2, \ldots, m; \quad k = 1, 2, \ldots, n \quad \text{(demand)}$$

$$\sum_{k=1}^{n} x_{ijk} \leq d_{ij}, \quad i = 1, 2, \ldots, l; \quad j = 1, 2, \ldots, m \quad \text{(supply)}$$

$$x_{ijk} \geq 0 \quad \text{for all } i,\ j, \text{ and } k.$$

Problem Set 8

1. (a) Connect nodes 1 and 2, 2 and 3, 3 and 4, 3 and 5, 3 and 6, 5 and 7, minimum length = 14 units (b) shortest path: $1 \to 2 \to 5 \to 7$, shortest distance = 10 units, alternate shortest path: $1 \to 3 \to 5 \to 7$

2. (a) $1 \to 3 \to 2 \to 5 \to 6 \to 4 \to 7$, distance 11 units (b) $7 \to 6 \to 5 \to 2 \to 3 \to 1$, distance 11 units (c) $2 \to 5 \to 6 \to 4 \to 7$, distance 7 units

3. max flow 25 units and optimal flow in arcs $1 \to 2$: 8 units, $1 \to 3$: 13, $1 \to 5$: 4, $2 \to 4$: 2, $2 \to 5$: 6, $3 \to 4$: 3, $3 \to 5$: 10, $4 \to 5$: 5

(a) surplus capacities: $1 \to 3$ 1 unit, $2 \to 4$ 2 units $3 \to 4$ 3 units;

(b) flow through node 2 = 8 units, through node 3 = 13 units, through node 4 = 5 units;

(c) can be increased along $3 \to 5$ because there is surplus capacity at node 1, increase along $4 \to 5$ is also possible;

(d) alternative optimal solution: max flow 25 units and optimal flow in arcs $1 \to 2$: 7, $1 \to 3$: 14, $1 \to 5$: 4, $2 \to 4$: 3, $2 \to 5$: 6, $3 \to 2$: 4, $3 \to 4$: 3, $3 \to 5$: 10, $4 \to 5$: 5

4. maximum flow = 110 million bbl/day; (a) Refinery 1 = 20 million bbl/day, Refinery 2 = 80 million bbl/day, Refinery 10 million bbl/day; (b) Terminal 7 = 60 million bbl/day, Terminal 8 = 50million bbl/day; (c) Pump 4 = 30 million bbl/day, Pump 5 = 50 million bbl/day, Pump 6 = 70 million bbl/day

5. maximum flow = 100 million bbl/day; Pump 4 = 30 million bbl/day, Pump 5 = 40 million bbl/day, Pump 6 = 60 million bbl/day.

Problem Set 9

1. Denote (1,2)=A, (1,3)=D, (2,4)=B, (3,5)=E, (4,5)=C and join appropriate nodes in ascending order

2. Critical path: $BEGIJ$

3. Take $A = (1,2)$, $B = (1,3)$, $C = (1,4)$, $D = (2,3)$, $G = (3,4)$, $I = (3,5)$, $E = (2,6)$, $F = (2,7)$, $H = (4,6)$, dummy activity = $(6,7)$, $J = (6,9)$, $M = (7,8)$, $O = (8,9)$, dummy activity $(5,8)$, $L = (5,9)$, $K = (5,10)$, $P = (9,10)$ and join appropriate nodes in ascending order

4. A is initial node and H is terminal node, Also, introduce dummy activity from E to F. Critical path: $A \to B \to D \to E \to F \to H$, Normal duration 29 days

5. Critical path $ADFI$, 67 days

6. Critical Path: $1 \longrightarrow 2 \longrightarrow 5 \longrightarrow 6 \longrightarrow 7 \longrightarrow 8 \longrightarrow 10 \longrightarrow 12 \longrightarrow 13$. The normal duration is 320 hours

7. Critical path: $ABEG$ (dummy activity connects D and E), Normal duration = 26 days

8. The most economical schedule is for 21 days with total cost = 9535

Problem Set 10

1. $1 \to 3 \to 5 \to 4 \to 2$, Total elapsed time = 28 days, idle time for $M_1 = 2$ days, idle time for $M_2 = 6$ days

2. $1 \to 4 \to 3 \to 7 \to 2 \to 6 \to 5$, Total elapsed time = 485 hours, idle time for $M_1 = 20$ hours, idle time for $M_2 = 105$ hours

3. $A \to D \to C \to G \to B \to F \to E$, Total elapsed time = 85 hours, idle time for $M_1 = 12$ hours, idle time for $M_2 = 9$ hours

4. No method is applicable

5. (a) $3 \to 6 \to 5 \to 7 \to 2 \to 4 \to 1$, alternative sequence $3 \to 5 \to 7 \to 2 \to 6 \to 4 \to 1$, Total elapsed time = 54 hours (b) $1 \to 4 \to 3 \to 6 \to 2 \to 5 \to 7$, Total elapsed time = 88 hours

6. $D \to A \to C \to B$

8. Job 1 precedes job 2 on machine M_1, job 1 precedes job 2 on machine M_2, job 1 precedes job 2 on machine M_3, job 2 precedes on machine 1; Total elapsed time $22 + 10 + 1 = 33$ hours

9. $4 \to 1 \to 3 \to 6 \to 5 \to 2$, Total elapsed time = 159 minutes, idle time for $A = 17$ minutes, idle time for $B = 3$ minutes

Problem Set 11

1. $x_1 = 1,\ x_2 = 3$, min value = 4.

2. $x_1 = 15,\ x_2 = 2$, min value = 106

4. Optimal tour $1 \longrightarrow 2 \longrightarrow 4 \longrightarrow 3 \longrightarrow 1$, Cost of travel = 14

5. Optimal tour $1 \longrightarrow 4 \longrightarrow 2 \longrightarrow 3 \longrightarrow 1$, Travel cost = 13

6. Optimal tour $1 \longrightarrow 3 \longrightarrow 4 \longrightarrow 2 \longrightarrow 1$, Travel cost = 170

9. x_i = amount of the food item i to be loaded, $i = 1, 2, 3, 4, 5$.

$$\begin{aligned} &\max 40x_1 + 50x_2 + 60x_3 + 55x_4 + 60x_5 \\ &\text{ s.t. } 20x_1 + 30x_2 + 40x_3 + 55x_4 + 80x_5 \le 105 \\ &\qquad x_i = 0 \text{ or } 1 \end{aligned}$$

Problem Set 12

2. $x = 0, y = 2$, min value: 2

3. $y_1 = 18/3,\ y_2 = 12/13$ min value: $30/13$

4. $y_1 = 1, y_2 = 1, y_3 = 18$ max value: 326, For alternate solutions any two variables at unity level and third variable at 18.

5. $y_1 = 4, y_2 = 5, y_3 = 1,\ y_0 = 38$

7. $x_1 = 7,\ x_2 = 11,\ z_{\max} = 106$

8. $x_1 = 2,\ x_2 = 6,\ z_{\max} = 36$

Problem Set 13

1. (a) Indefinite (b) positive semi-definite

2. (a) $\{(x_1, x_2) : x_1 > 1/3,\ x_2 > 3 - 7x_1\}$ (b) $\{(x_1, x_2) : x_1 > 0,\ x_1x_2 > 1\}$

3. Not convex

4. Minimum points: $X_1 = (2, 2, 1), X_2 = (2, -2, 1),\ X_3 = (2.8, 0, 1.4)$ is not an extreme point

9. Optimal solution $x_1 = 2/3,\ x_2 = 14/9,\ \ z_{\max} = 22/9$.

10. Optimal solution $x_1 = 0,\ x_2 = 1,\ \ f_{\max} = 1$

11. Infeasible solution

12. Optimal solution $x_1 = 4/11,\ x_2 = 3/11,\ \ x_0 - 5/11$.

13. $x_A = 0,\ x_B = 85/2;\ \ f(X) = 80325/4$, x_i = number of units of product i, $i = A, B$

15. min $z = 3x_1^2 + 3x_2^2 + 8x_3$

s.t. $x_1 - x_3 = 200$

$x_2 + x_3 = 200$

$x_1, x_2, x_3 \geq 0$

$x_1 = 748/3$, $x_2 = 752/3$, $x_3 = 148/3$, where x_1 = number of units produced in January, x_2 = number of units produced in February, x_3 = number of units sold at the end of January.

Problem Set 14

4. (a) 34 (b) 20 (c) 16 (d) 16

5. (a) $X_{\min} = 1.49975$, $f_{\min} = 0.943$ (b) $X_{\min} = 1.845$, $f_{\min} = 0.25$ (c) $X_{\min} = 1.932$, $f_{\min} = 0.0.26$

6. For both methods with $n = 8$, $X_{\min} = 0.01$, $f_{\min} = 2.01$

10. Minimum point $X_3 = (-1, 3/2)^T$ and $f_{\min} = 0.25$

11. Minimum point $X_2 = (4/3, 0)^T$ and $f_{\min} = 248/27$, only one iteration is required, since $\nabla f(X_2) = (0, 0)^T$.

Problem Set 15

1. (a) $\sqrt{2}$ (b) 1

5. minimum cost $= 2c_4V$; each side $= [2c_4V/(c_1 + c_2 + c_3)]^{1/3}$ $L = [Vc_1^2/(\pi c_2^2 N)]^{1/3}$, $R = [Vc_2/(\pi c_1 N)]^{1/3}$, L and R are length and radius of the pipe, respectively

10. $100[1 - n(1 + 0.01k)]$

11. $x_0 = (2^{11/4}/5^{5/8})c_1^{5/8}c_2^{1/8}c_3^{1/4}$, $D = 5x_0/8c_1$, $Q = 4c_3/x_0$

12. $P_2 = \left(P_1^2 P_4\right)^{1/3}$, $P_3 = \left(P_1 P_4^2\right)^{1/3}$

13. $D = (5c_2/c_1)^{1/6}$

Problem Set 16

1. $x_1 = 25/3$, $x_2 = 0$, $x_3 = 5/3$, $s_2^+ = 25/3$. Since $s_1^- = s_1^+ = 0$ and $s_3^+ = 0$, the first and third goals are fully satisfied. However, the employment level exceeds by 25/3, i.e., 833 employees.

4. (b) The formulation of the problem is

$$\begin{aligned} \min\ S = {} & P_1(s_1^- + s_1^+) + P_2 s_2^+ + P_3 s_3^+ + P_4 s_4^- + P_5 s_5^+ + P_6 s_6^+ \\ & + P_7 s_7^- + P_8 s_8^+ \end{aligned}$$

s.t.

$$\begin{aligned} & 2x_1 + 5x_2 + 4x_3 + d_1 = 600 \\ & 3x_1 + 7x_2 + 5x_3 + d_2 = 500 \\ & 2x_1 + 5x_2 + 6x_3 + s_1^- - s_1^+ = 450 \\ & 4x_1 + 3x_2 + 3x_3 + s_2^- - s_2^+ = 500 \\ & 2x_1 + 3x_2 + 4x_3 + s_3^- - s_3^+ = 600 \\ & 3x_1 + 3x_2 + 5x_3 + s_4^- - s_4^+ = 500 \\ & 3x_1 + 4x_2 + 7x_3 + s_5^- - s_5^+ = 900 \\ & 5x_1 + 3x_2 + 3x_3 + s_6^- - s_6^+ = 800 \\ & 3x_1 + 6x_2 + 4x_3 + s_7^- - s_7^+ = 400 \\ & 3x_1 + 2x_2 + 2x_3 + s_8^- - s_8^+ = 500 \\ & d_1, d_2, x_j, s_i^-, s_i^+ \geq 0, j = 1, 2, 3; i = 1, 2, \ldots, 8. \end{aligned}$$

The optimal solution is $x_1 = 91.66$, $x_2 = 0$, $x_3 = 44.44$, $d_1 = 238.88$, $d_2 = 2.78$, $s_3^- = 238.88$, $s_4^- = 2.78$, $s_5^- = 313.88$, $s_6^- = 208.33$, $s_7^+ = 52.77$, $s_8^- = 136.11$, $S = 2.78$. The algorithm requires 4 iterations to find the optimal solution of the problem whereas, the number of iterations required by the lexicographic minimization method using LINDO software is 8.

6. The optimal solution is $x_1 = 27/13, x_2 = 0, x_3 = 3/26, N_g = 4$. The goal constraints with the respective priorities being assigned are:

$$\begin{aligned} x_1 - x_2 - x_3 \quad & \leq 2 \\ x_1 + x_2 + 2x_3 \quad & \leq 3 \\ 4x_1 - x_2 + 6x_3 \quad & = 9 \\ 3x_1 - 2x_2 - 2x_3 \quad & \geq 4 \\ -2x_1 - x_2 + x_3 \quad & \geq 2 \quad P_5 \\ x_1, x_2, x_3 \geq 0 & \end{aligned}$$

Here the goal constraints (1), (2), (3) and (4) may be assigned any priorities from P_1 to P_4.

Problem Set 17

1. $x_1 = 1/2$, $x_2 = 1/2$, value of game $= 2.5$; $y_1 = 1/2$, $y_2 = 1/2$, $y_3 = 0$

3. $x_1 = 1/2$, $x_2 = 1/2$, value of game $= 0$; $y_1 = 2/3$, $y_2 = 1/3$, $y_3 = 0$

4. $x_1 = 2/3$, $x_2 = 1/3$, value of game $= 4/3$; $y_1 = 2/3$, $y_2 = 1/3$

5. $x_1 = 0$, $x_1 = 2/3$, value of game $= 7/3$; $y_1 = 8/9$, $y_2 = 1/9$, $y_3 = 0$

6. $p = (5, \infty)$, $q = (-\infty, 5)$

7. x_1, x_2 and x_3 are the investments on Deposits, Bonds and Stocks, respectively. (a) $x_1 = 1,000,000$, $x_2 = 0$, $x_3 = 0$, value of the game $= 8,000$; (b) $x_1 = 0$, $x_2 = 87,500$, $x_3 = 12,500$, value of the game $= 8,750$; (c) $x_1 = 0$, $x_2 = 1,000,000$, $x_3 = 0$; value of the game $= 1,000$.

8. Let x_1 and x_2 correspond to frequency the Australian and West Indies teams use lineup-i and lineup-j, respectively, $i = 1, 2, 3$ and $j = 1, 2, 3$. (a) $x_1 = 0$, $x_2 = 2/5$, $x_3 = 3/5$, $y_1 = 4/5$, $y_2 = 0$, $y_3 = 1/5$, value of the game $= 23/50$; (b) $x_1 = 23/49$, $x_2 = 9/49$, $x_3 = 17/49$, value of the game $= 22/49$

Problem Set 18

1. $x_{11} = 10,\ x_{12} = 0,\ x_{13} = 15,\ x_{14} = 0,\ x_{15} = 5,\ x_{21} = 0,\ x_{22} = 0,\ x_{23} = 10, x_{24} = 30,\ x_{25} = 0,\ x_{31} = 10,\ x_{32} = 20,\ x_{33} = 0,\ x_{34} = 0,\ x_{35} = 0.$

   This solution yields values of 14, 20 and 21 for the first, second and third secondary objective functions, respectively. The solution also happens to be the optimal solutions for the second and third secondary objective functions.

Appendix: Answers to Objective Type Questions

1. (a) **2.** (a) **3.** (a) **4.** (c) **5.** (c) **6.** (b) **7.** (b) **8.** (a) **9.** (c) **10.** (c) **11.** X_1 **12.** (a) **13.** (d) **14.** X_2 **15.** (c) **16.** (c) **17.** (c) **18.** (b) **19.** (d) **20.** (b) **21.** (a) **22.** (b) **23.** (a) **24.** (b) **25.** (d) **26.** (c) **27.** (c) **28.** (d) **29.** (c) **30.** (a) **31.** (b) **32.** (a) **33.** (c) **34.** (b) **35.** (b) **36.** (d) **37.** (c) **38.** (c) and (b) depending whether the rule $\theta_j(z_j - c_j)$ is followed **39.** (c) **40** -5 **41.** (a) **42.** (b) **43.** (a) **44.** (d) **45.** (c) **46.** (b) **47.** (a) **48.** (c) **49.** (a) **50.** (b) **51.** $(0, -1)^T$ **52.** $y_1 = -16/5, y_2 = -11/10$ **53.** (b) **54.** (a) **55.** (b) **56.** (c) **57.** (c) **58.** (c) **59.** (b) **60.** (c) **61.** (a) **62.** (a) **63.** (a) **64.** (d) **65.** (a) **66.** 3 **67.** $8x_1 + 13x_2$ **68.** $(13, 8)^T$ **69.** $(3, 2)^T$**70.** $x_1 = 2, x_2 = 1$ **71.** $x_1 = 4, x_2 = 0$ **72.** (d) **73.** (b) **74.** (b) **75.** (c) **76.** (a) **77.** (c) **78.** (a) true (b) False (c) False (d) True (e) True (f) True, if solution space is bounded; False, if solution space is unbounded (g) True (h) False (i) True (j) False (k) False (l) False (m) True **79.** (c) **80** (c) **81.** (d) **82.** (d) **83.** (d) **84.** (b) **85.** (a) **86.** (a) **87.** (d) **89.** (b) **90.** (d) **91.** (a) **92.** (b) **93.** (b) **94.** (d) **95.** (b **96.** (b) **97.** (a) True (b) False (c) False (d) False (e) True (f) True (g) false (h) true (i) False (j) True (k) True (l) False **98.** (c) **99.** (c) **100.** (a) **101.** (d) **102.** (b) **103.** (a) **104.** (d) **105.** (a) **106.** (c) **107.** (b) **109.** (b)

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