Decentralized Algorithms for Generalized Nash Equilibrium Seeking in Peer-to-peer Electricity Markets

Giuseppe Belgioioso

September 18, 2020

1 Problem statement

Consider the same problem statement in `problem_setup_v6.tex`.

2 Algorithms

2.1 Semi-decentralized

The algorithm presented in this section corresponds to [1, Alg. 6B] applied on `problem_setup_v6.tex`.

Consider the following choices for the step-sizes in Algorithm 1.

Assumption 1 (Step size selection) *For all* $i \in \mathcal{N}$*, set* $A_i = \mathrm{diag}(\alpha_i^{pi}, \alpha_i^{st}, \alpha_i^{mg}, \{\alpha_{(i,j)}^{tr}\}_{j \in \mathcal{N}_i}) \otimes I_H$*, where* $\alpha_i^{pi}, \alpha_i^{st} > 1$*,* $\alpha_i^{mg} > 3 + N \max_{h \in \mathcal{H}} d_h$*,* $\beta_{(i,j)} = \beta_{(j,i)} < \frac{1}{2}$*, for all* $j \in \mathcal{N}_i$

$A_{i,h} = \mathrm{diag}\left(\alpha_{i,h}^{\mathrm{dg}},\ \alpha_{i,h}^{\mathrm{st}},\ \alpha_{i,h}^{\mathrm{mg}},\ \{\alpha_{(i,j),h}^{\mathrm{tr}}\}_{j \in \mathcal{N}_i}\right)$*, for all* $h \in \mathcal{H}$*, with* $\alpha_{i,h}^{\mathrm{dg}},\ \alpha_{i,h}^{\mathrm{st}} > 0$ *and* $\alpha_{(i,j),h}^{\mathrm{tr}} > |\mathcal{N}_i|$*, for all* $j \in \mathcal{N}_i$*;* $\beta_{(i,j)} = \beta < \frac{1}{2}$ *for all* $j \in \mathcal{N}_i$*;* $\gamma < N$*. To complete.*

□

Semi-decentralized GWE seeking for P2P Energy Markets

Initialization: For all $i \in \mathcal{N}$, set locally:

(a) Initial conditions: $u_i(0) \in \mathcal{U}_i$, $\mu_{(i,j)}(0) = \mathbf{0}$ for all $j \in \mathcal{N}_i$, and $\lambda(0) \in \mathbb{R}^{2H}_{\geq 0}$.

(b) Step-sizes: A_i, $\{\beta_{ij}\}_{j\in\mathcal{N}_i}$,$\gamma$ as in Assumption 1.

Iterate until convergence:

Prosumers. For each prosumer $i \in \mathcal{N}$:

(1) Strategy update:

$$\begin{aligned}
a_i(k) &= \mathrm{col}\left(-\nu_y(k), -\nu_y(k), \left(\left[\begin{smallmatrix}1\\-1\end{smallmatrix}\right] \otimes I_H\right)^\top \lambda(k) + \rho(k), \left\{\mu_{(i,j)}(k)\right\}_{j\in\mathcal{N}_i}\right)\\
u_i(k+1) &= \begin{cases} \underset{\xi\in\mathbb{R}^{n_i}}{\mathrm{argmin}} & J_i\big(\xi, \sigma(k)\big) + a_i(k)^\top \xi + \frac{1}{2}\left\|\xi - u_i(k)\right\|^2_{A_i}\\ \quad \text{s.t.} & \xi \in \mathcal{U}_i \end{cases}\\
b_i(k+1) &= p_i^{\mathrm{d}} - p_i^{\mathrm{di}}(k+1) - p_i^{\mathrm{st}}(k+1)
\end{aligned}$$

(2) Communication with trading partners and DSO:

$$\begin{array}{llll}
\text{(a)} & \{p_i^{\mathrm{mg}}(k+1),\ b_i(k+1)\} & \longrightarrow & \text{DSO},\\
\text{(b)} & p_{(i,j)}(k+1) & \longrightarrow & \text{prosumer } j, \qquad \text{for all } j \in \mathcal{N}_i.
\end{array}$$

(3) Dual variables (reciprocity constraints) update, $\forall j \in \mathcal{N}_i$:

$$\begin{aligned}
c_{(i,j)}(k+1) &= p^{\mathrm{tr}}_{(i,j)}(k+1) + p^{\mathrm{tr}}_{(j,i)}x(k+1),\\
\mu_{(i,j)}(k+1) &= \mu_{(i,j)}(k) + \beta_{ij}\left(2c_{(i,j)}(k+1) - c_{(i,j)}(k)\right).
\end{aligned}$$

DSO.

(1) Aggregate strategy update

$$\sigma(k+1) = \sum_{i\in\mathcal{N}} p_i^{\mathrm{mg}}(k+1) + \sum_{i\in\mathcal{B}} p_k^{\mathrm{pd}}$$

(2) Dual variable (grid constraints) update:

$$\lambda(k+1) = \mathrm{proj}_{\mathbb{R}^{2H}_{\geq 0}}\left(\lambda(k) + \gamma\left(\left[\begin{smallmatrix}1\\-1\end{smallmatrix}\right] \otimes (2\sigma(k+1) - \sigma(k)) - \left[\begin{smallmatrix}\overline{p}^{\mathrm{mg}}\mathbf{1}_H\\ -\underline{p}^{\mathrm{mg}}N\mathbf{1}_H\end{smallmatrix}\right]\right)\right).$$

(3) Update of the physical variables (define $\mathcal{U}_{N+1} := (15) \cap (16) \cap (17) \cap (18) \cap (19)$)

$$\begin{aligned}
a_{N+1}(k+1) &= \mathrm{col}\left(\left\{\mathbf{0}, \mathbf{0}, -\rho(k), \{\nu_y(k), \mathbf{0}\}_{z\in\mathcal{B}_y}\right\}_{y\in\mathcal{B}}\right)\\
u_{N+1}(k+1) &= \mathrm{proj}_{\mathcal{U}_{N+1}}\left(u_{N+1}(k) - A_{N+1}a_{N+1}(k)\right)
\end{aligned}$$

(4) Dual variable (local power balance of bus y). For all $y \in \mathcal{B}$:

$$
\begin{aligned}
d_y(k+1) &= p_y^{\mathrm{pd}} + \sum_{i \in \mathcal{N}_y} b_i(k+1) - p_y^{\mathrm{tg}}(k+1) - \sum_{z \in \mathcal{B}_y} p_{(y,z)}^{\ell}(k+1) \\
\nu_u(k+1) &= \nu_u(k) + \delta_y(2d_y(k+1) - d_y(k)),
\end{aligned}
$$

(5) Dual variable (trading with the main grid).

$$
\begin{aligned}
\hat{p}^{\mathrm{tg}}(k+1) &= \sigma(k+1) - \sum_{y \in \mathcal{B}^{\mathrm{mg}}} p_y^{\mathrm{tg}}(k+1), \\
\rho(k+1) &= \rho(k) + \eta(2\hat{p}^{\mathrm{tg}}(k+1) - \hat{p}^{\mathrm{tg}}(k)),
\end{aligned}
$$

(6) Broadcast to the prosumers

$$
\begin{array}{rlcl}
 & \{\sigma(k+1),\, \lambda(k+1), \rho(k+1)\} & \longrightarrow & \text{All prosumers } (\mathcal{N}), \\
(\forall y \in \mathcal{B}): & \nu_y(k+1) & \longrightarrow & \text{Prosumers on bus } y\ (\mathcal{N}_y),
\end{array}
$$

References

[1] G. Belgioioso and S. Grammatico, "Semi-decentralized generalized nash equilibrium seeking in monotone aggregative games," *arXiv preprint arXiv:2003.04031*, 2020.