Decentralized Algorithms for Generalized Nash Equilibrium Seeking in Peer-to-peer Electricity Markets

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1 Problem statement

Consider the same problem statement in `problem_setup_v6.tex`.

2 Algorithms

2.1 Semi-decentralized

The algorithm presented in this section corresponds to [1, Alg. 6B] applied on `problem_setup_v6.tex`.

Consider the following choices for the step-sizes in Algorithm 1.

Assumption 1 (Step-size) *Set the step sizes of prosumers and DSO as follows:*

(i) $\forall i \in \mathcal{N}$: *set* $A_i = \mathrm{diag}(\alpha_i^{pi}, \alpha_i^{st}, \alpha_i^{mg}, \{\alpha_{(i,j)}^{tr}\}_{j \in \mathcal{N}_i}) \otimes I_H$, *with* $\alpha_i^{pi}, \alpha_i^{st} > 1$, $\alpha_i^{mg} > 3 + N \max_{h \in \mathcal{H}} d_h$, $\beta_{(i,j)}^{tr} = \beta_{(j,i)}^{tr} < \frac{1}{2}$, $\forall j \in \mathcal{N}_i$.

(ii) Set $A_{N+1} := \mathrm{diag}\left(\left\{\alpha_y^{\theta}, \alpha_y^{v}, \alpha_y^{tg}, \{\alpha_{(y,z)}^{p}, \alpha_{(y,z)}^{q}\}_{z \in \mathcal{B}_y}\right\}_{y \in \mathcal{B}}\right) \otimes I_H$, *with* $\alpha_y^{\theta}, \alpha_y^{v} > 0$, $\alpha_y^{tg} > 2$, $\alpha_{(y,z)}^{p} > 1$ *and* $\alpha_{(y,z)}^{q} > 0$, $\forall z \in \mathcal{B}_y$, $\forall y \in \mathcal{B}$. *Set* $\gamma^{mg} < \frac{1}{N}$, $\beta^{tg} < (|\mathcal{N}| + |\mathcal{B}|)^{-1}$ *and* $\beta_y^{pb} < (1 + |\mathcal{N}_y| + |\mathcal{B}_y|)^{-1}$, *for all* $y \in \mathcal{B}$. □

Algorithm 1 Semi-decentralized GWE seeking for P2P Energy Markets

repeat
  for all prosumer $i \in \mathcal{N}$ **do**
    procedure Strategy update
      $a_i(k) = \mathrm{col}\big(-\mu_y^{\mathrm{pb}}(k), -\mu_y^{\mathrm{pb}}(k), \left[\begin{smallmatrix} I_H \\ -I_H \end{smallmatrix}\right]^\top \lambda^{\mathrm{mg}}(k) + \mu^{\mathrm{tg}}(k), \left\{\mu_{(i,j)}^{\mathrm{tr}}(k)\right\}_{j \in \mathcal{N}_i}\big)$ ▷ aux. vector
      $u_i(k+1) = \begin{cases} \underset{\xi \in \mathbb{R}^{n_i}}{\operatorname{argmin}} & J_i\big(\xi, \sigma(k)\big) + a_i(k)^\top \xi + \frac{1}{2}\left\|\xi - u_i(k)\right\|_{A_i}^2 \\ \text{s.t.} & \xi \in \mathcal{U}_i \end{cases}$
    end procedure
    procedure Communication with trading partners and DSO
      $b_i(k+1) = p_i^{\mathrm{d}} - p_i^{\mathrm{di}}(k+1) - p_i^{\mathrm{st}}(k+1)$ ▷ aux. vector
      $p_i^{\mathrm{mg}}(k+1), b_i(k+1) \longrightarrow \mathrm{DSO}$,
      for all prosumer $j \in \mathcal{N}_i$ **do**
        $p_{(i,j)}^{\mathrm{tr}}(k+1) \longrightarrow$ prosumer j
      end for
    end procedure
    procedure Dual variable (reciprocity constraints) update
      for all $j \in \mathcal{N}_i$ **do**
        $c_{(i,j)}^{\mathrm{tr}}(k+1) = p_{(i,j)}^{\mathrm{tr}}(k+1) + p_{(j,i)}^{\mathrm{tr}} x(k+1)$ ▷ aux. vector
        $\mu_{(i,j)}(k+1) = \mu_{(i,j)}(k) + \beta_{ij}^{\mathrm{tr}}\left(2c_{(i,j)}^{\mathrm{tr}}(k+1) - c_{(i,j)}^{\mathrm{tr}}(k)\right)$
      end for
    end procedure
  end for

  Update DSO
    procedure Aggregations update
      $\sigma^{\mathrm{mg}}(k+1) = \sum_{i \in \mathcal{N}} p_i^{\mathrm{mg}}(k+1) + \sum_{i \in \mathcal{B}} p_k^{\mathrm{pd}}$
      $\sigma^{\mathrm{tg}}(k+1) = \sum_{y \in \mathcal{B}} p_y^{\mathrm{tg}}(k+1)$
    end procedure
    procedure Physical variables update
      $a_{N+1}(k+1) = \mathrm{col}\left(\left\{\mathbf{0}, \mathbf{0}, -\mu^{\mathrm{tg}}(k), \{\mu_y^{\mathrm{pb}}(k), \mathbf{0}\}_{z \in \mathcal{B}_y}\right\}_{y \in \mathcal{B}}\right)$ ▷ aux. vector
      $u_{N+1}(k+1) = \mathrm{proj}_{\mathcal{U}_{N+1}}\left(u_{N+1}(k) - A_{N+1} a_{N+1}(k)\right)$
    end procedure
    procedure Dual variable (grid constraints) update
      $\lambda^{\mathrm{mg}}(k+1) = \mathrm{proj}_{\mathbb{R}_{\geq 0}^{2H}}\left(\lambda^{\mathrm{mg}}(k) + \gamma^{\mathrm{mg}}\left(\left[\begin{smallmatrix} 1 \\ -1 \end{smallmatrix}\right] \otimes \left(2\sigma^{\mathrm{mg}}(k+1) - \sigma^{\mathrm{mg}}(k)\right) - \left[\begin{smallmatrix} \overline{p}^{\mathrm{mg}} \mathbf{1}_H \\ -\underline{p}^{\mathrm{mg}} N \mathbf{1}_H \end{smallmatrix}\right]\right)\right)$
    end procedure
    procedure Dual variable (local power balance of bus y)
      for all bus $y \in \mathcal{B}$ **do**
        $c_y^{\mathrm{pb}}(k+1) = p_y^{\mathrm{pd}} + \sum_{i \in \mathcal{N}_y} b_i(k+1) - p_y^{\mathrm{tg}}(k+1) - \sum_{z \in \mathcal{B}_y} p_{(y,z)}^{\ell}(k+1)$ ▷ aux. vector
        $\mu_y^{\mathrm{pb}}(k+1) = \nu_u(k) + \beta_y^{\mathrm{pb}}(2c_y^{\mathrm{pb}}(k+1) - c_y^{\mathrm{pb}}(k))$
      end for
    end procedure
    procedure Dual variable (trading with the main grid)
      $c^{\mathrm{tg}}(k+1) = \sigma^{\mathrm{mg}}(k+1) - \sigma^{\mathrm{tg}}(k+1)$ ▷ aux. vector
      $\mu^{\mathrm{tg}}(k+1) = \mu^{\mathrm{tg}}(k) + \beta^{\mathrm{tg}}(2c^{\mathrm{tg}}(k+1) - c^{\mathrm{tg}}(k))$
    end procedure
    procedure Broadcast to prosumers
      $\{\sigma(k+1), \lambda^{\mathrm{mg}}(k+1), \mu^{\mathrm{tg}}(k+1)\} \longrightarrow$ All prosumers $(\mathcal{N})$
      for all bus $y \in \mathcal{B}$ **do**
        $\mu_y^{\mathrm{pb}}(k+1) \longrightarrow$ Prosumers on bus y $(\mathcal{N}_y)$
      end for
    end procedure
  End
until **convergence**

References

[1] G. Belgioioso and S. Grammatico, "Semi-decentralized generalized nash equilibrium seeking in monotone aggregative games," *arXiv preprint arXiv:2003.04031*, 2020.