Decentralized Algorithms for Generalized Nash Equilibrium Seeking in Peer-to-peer Electricity Markets

Giuseppe Belgioioso

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1 Problem statement

Consider the same problem statement in `problem_setup_v7.tex`.

2 Algorithms

2.1 Semi-decentralized

The algorithm presented in this section corresponds to [1, Alg. 6B] applied on `problem_setup_v6.tex`.

Consider the following choices for the step-sizes in Algorithm 1.

Assumption 1 (Step-size) *Set the step sizes of prosumers and DSO as follows:*

(i) $\forall i \in \mathcal{N}$: *set* $A_i = \mathrm{diag}(\alpha_i^{pi}, \alpha_i^{st}, \alpha_i^{mg}, \{\alpha_{(i,j)}^{tr}\}_{j \in \mathcal{N}_i}) \otimes I_H$, *with* $\alpha_i^{pi}, \alpha_i^{st} > 1$, $\alpha_i^{mg} > 3 + N \max_{h \in \mathcal{H}} d_h^{mg}$, $\alpha_{(i,j)}^{tr} > 2$, $\forall j \in \mathcal{N}_i$, $\beta_{(i,j)}^{tr} = \beta_{(j,i)}^{tr} < \frac{1}{2}$, $\forall j \in \mathcal{N}_i$.

(ii) Set $A_{N+1} := \mathrm{diag}\left(\left\{\alpha_y^{\theta}, \alpha_y^{v}, \alpha_y^{tg}, \{\alpha_{(y,z)}^{p}, \alpha_{(y,z)}^{q}\}_{z \in \mathcal{B}_y}\right\}_{y \in \mathcal{B}}\right) \otimes I_H$, *with* $\alpha_y^{\theta}, \alpha_y^{v} > 0$, $\alpha_y^{tg} > 2$, $\alpha_{(y,z)}^{p} > 1$ *and* $\alpha_{(y,z)}^{q} > 0$, $\forall z \in \mathcal{B}_y$, $\forall y \in \mathcal{B}$. *Set* $\gamma^{mg} < \frac{1}{N}$, $\beta^{tg} < (|\mathcal{N}| + |\mathcal{B}|)^{-1}$ *and* $\beta_y^{pb} < (1 + 2|\mathcal{N}_y| + |\mathcal{B}_y|)^{-1}$, *for all* $y \in \mathcal{B}$. □

Algorithm 1 Semi-decentralized GWE seeking for P2P Energy Markets

Iterate until convergence
  for all prosumer $i \in \mathcal{N}$ **do**
    primal update ▷ power generated, stored, from the grid, traded
      $a_i(k) = \mathrm{col}\big(-\mu_y^{\mathrm{pb}}(k), -\mu_y^{\mathrm{pb}}(k), \left[\begin{smallmatrix} I_H \\ -I_H \end{smallmatrix}\right]^\top \lambda^{\mathrm{mg}}(k) + \mu^{\mathrm{tg}}(k), \left\{\mu_{(i,j)}^{\mathrm{tr}}(k)\right\}_{j \in \mathcal{N}_i}\big)$ ▷ aux. vector
      $u_i(k+1) = \begin{cases} \underset{\xi \in \mathbb{R}^{n_i}}{\mathrm{argmin}} & J_i\big(\xi, \sigma^{\mathrm{mg}}(k)\big) + a_i(k)^\top \xi + \frac{1}{2}\left\|\xi - u_i(k)\right\|_{A_i}^2 \\ \text{s.t.} & \xi \in \mathcal{U}_i \end{cases}$ ▷ quadratic progr.
    end
    communication ▷ to DSO and trading partners
      $b_i(k+1) = p_i^{\mathrm{d}} - p_i^{\mathrm{di}}(k+1) - p_i^{\mathrm{st}}(k+1)$ ▷ local load unbalance of prosumer i
      $p_i^{\mathrm{mg}}(k+1), b_i(k+1) \longrightarrow$ DSO, ▷ forward to DSO
      for all prosumer $j \in \mathcal{N}_i$ **do**
        $p_{(i,j)}^{\mathrm{tr}}(k+1) \longrightarrow$ prosumer j ▷ forward local trade to prosumer j
      end for
    end
    dual update ▷ reciprocity constraints
      for all $j \in \mathcal{N}_i$ **do**
        $c_{(i,j)}^{\mathrm{tr}}(k+1) = p_{(i,j)}^{\mathrm{tr}}(k+1) + p_{(j,i)}^{\mathrm{tr}}(k+1)$ ▷ aux. vector
        $\mu_{(i,j)}^{\mathrm{tr}}(k+1) = \mu_{(i,j)}(k) + \beta_{ij}^{\mathrm{tr}}\left(2c_{(i,j)}^{\mathrm{tr}}(k+1) - c_{(i,j)}^{\mathrm{tr}}(k)\right)$ ▷ reflected dual ascent
      end for
    end
  end for
  DSO update
    primal update ▷ physical variables
      $a_{N+1}(k) = \mathrm{col}\left(\left\{\mathbf{0}, \mathbf{0}, -\mu^{\mathrm{tg}}(k) - \mu_y^{\mathrm{pb}}(k), \{-\mu_y^{\mathrm{pb}}(k), \mathbf{0}\}_{z \in \mathcal{B}_y}\right\}_{y \in \mathcal{B}}\right)$ ▷ aux. vector
      $u_{N+1}(k+1) = \mathrm{proj}_{\mathcal{U}_{N+1}}\left(u_{N+1}(k) - A_{N+1}a_{N+1}(k)\right)$ ▷ solved via Algorithm 2
    end
    aggregation update
      $\sigma^{\mathrm{mg}}(k+1) = \sum_{i \in \mathcal{N}} p_i^{\mathrm{mg}}(k+1)$ ▷ aggregate grid-to-prosumers power
      $\sigma^{\mathrm{tg}}(k+1) = \sum_{y \in \mathcal{B}} p_y^{\mathrm{tg}}(k+1)$ ▷ aggregate grid-to-buses power
    end
    dual update
      $b_{N+1}(k+1) = \left[\begin{smallmatrix} 1 \\ -1 \end{smallmatrix}\right] \otimes \left(2\sigma^{\mathrm{mg}}(k+1) - \sigma^{\mathrm{mg}}(k)\right) - \left[\begin{smallmatrix} \overline{p}^{\mathrm{mg}}\mathbf{1}_H \\ -\underline{p}^{\mathrm{mg}}\mathbf{1}_H \end{smallmatrix}\right]$ ▷ aux. vector
      $\lambda^{\mathrm{mg}}(k+1) = \mathrm{proj}_{\mathbb{R}_{\geq 0}^{2H}}\left(\lambda^{\mathrm{mg}}(k) + \gamma^{\mathrm{mg}} b_{N+1}(k+1)\right)$ ▷ grid constraints
      for all buses $y \in \mathcal{B}$ **do**
        $c_y^{\mathrm{pb}}(k+1) = p_y^{\mathrm{pd}} + \sum_{i \in \mathcal{N}_y} b_i(k+1) - p_y^{\mathrm{tg}}(k+1) - \sum_{z \in \mathcal{B}_y} p_{(y,z)}^{\ell}(k+1)$ ▷ aux. vector
        $\mu_y^{\mathrm{pb}}(k+1) = \mu_y^{\mathrm{pb}}(k) + \beta_y^{\mathrm{pb}}(2c_y^{\mathrm{pb}}(k+1) - c_y^{\mathrm{pb}}(k))$ ▷ local power balance of bus y
      end for
      $c^{\mathrm{tg}}(k+1) = \sigma^{\mathrm{mg}}(k+1) - \sigma^{\mathrm{tg}}(k+1)$ ▷ aux. vector
      $\mu^{\mathrm{tg}}(k+1) = \mu^{\mathrm{tg}}(k) + \beta^{\mathrm{tg}}(2c^{\mathrm{tg}}(k+1) - c^{\mathrm{tg}}(k))$ ▷ grid-to-buses constraints
    end
    communication ▷ broadcast
      $\{\sigma(k+1), \lambda^{\mathrm{mg}}(k+1), \mu^{\mathrm{tg}}(k+1)\} \longrightarrow \mathcal{N}$ ▷ to all prosumers
      for all buses $y \in \mathcal{B}$ **do**
        $\mu_y^{\mathrm{pb}}(k+1) \longrightarrow \mathcal{N}_y$ ▷ to all prosumers on bus y
      end for
    end
  end
end

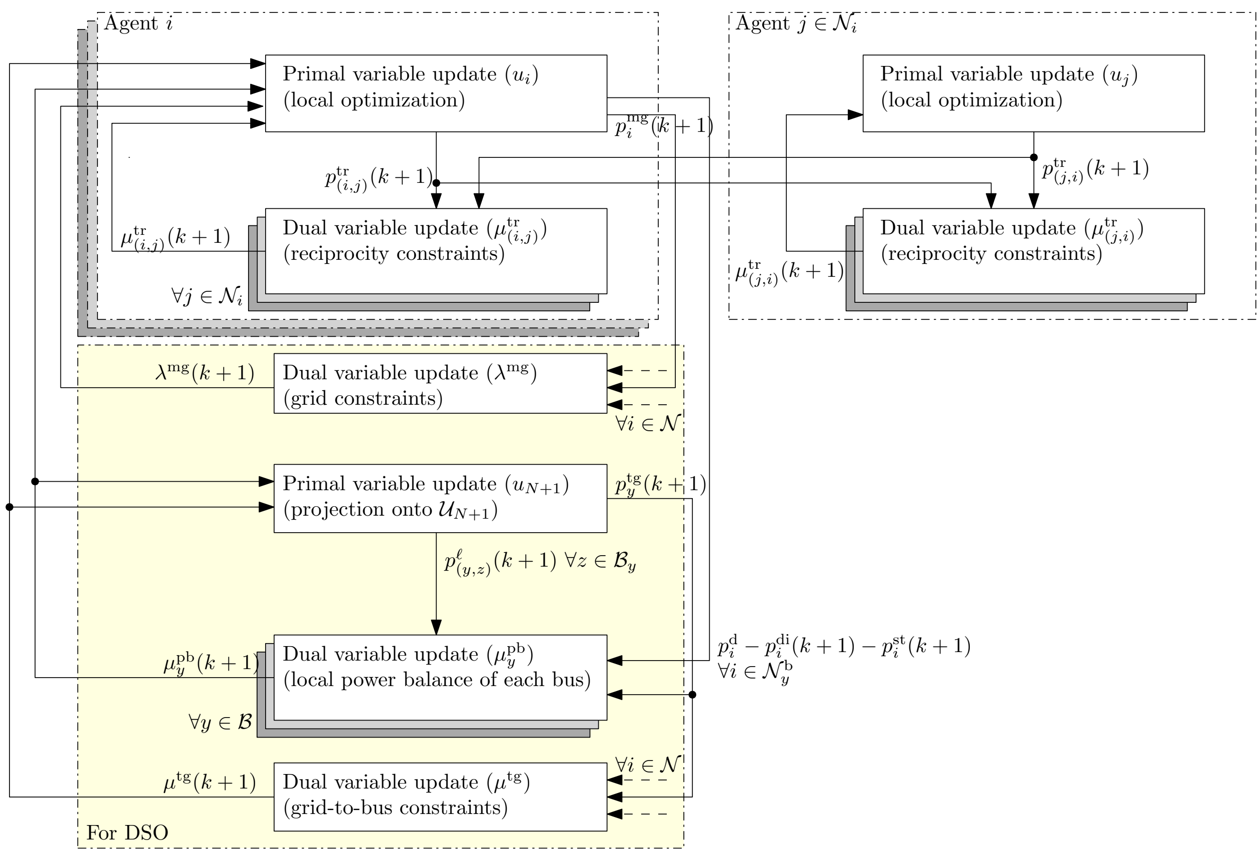

Figure 1: Flowchart of the iterations in Algorithm 1 for agent $i \in \mathcal{N}$ and DSO. It also shows the flow of information between agent i and DSO as well as between agent i and its trading partner j.

2.2 Projection onto $\mathcal{U}_{N+1}$

Next, we propose an iterative method to solve line 24 in Algorithm 1, namely, to compute the projection onto $\mathcal{U}_{N+1}$. First, let us define the sets $C_1 := (17) \cap (18) \cap (19) \cap (20)$ and $C_2 = (15) \cap (16)$ and recall that the decision vector of the DSO reads as $u_{N+1} = \mathrm{col}\left(\{\theta_y, v_y, p_y^{\mathrm{tg}}, \{p^{\ell}_{(y,z)}, q^{\ell}_{(y,z)}\}_{z\in\mathcal{B}_y}\}_{y\in\mathcal{B}}\right)$. The projection onto C_1 can be characterize in closed-form as follows:

$$\mathrm{proj}_{C_1}(u_{N+1}) = \mathrm{col}\left(\{\theta_y^+, v_y^+, {p_y^{\mathrm{tg}}}^+, \{{p^{\ell}_{(y,z)}}^+, {q^{\ell}_{(y,z)}}^+\}_{z\in\mathcal{B}_y}\}_{y\in\mathcal{B}}\right),$$

where, for all $y \in \mathcal{B}$,

$$\begin{aligned}
\theta_y^+ &= \begin{cases} \underline{\theta}_y, & \text{if } \theta_y < \underline{\theta}_y \\ \overline{\theta}_y, & \text{if } \theta_y > \overline{\theta}_y \\ \theta_y, & \text{otherwise} \end{cases}, \\
v_y^+ &= \begin{cases} \underline{v}_y, & \text{if } v_y < \underline{v}_y \\ \overline{v}_y, & \text{if } v_y > \overline{v}_y \\ v_y, & \text{otherwise} \end{cases} \\
{p_y^{\mathrm{tg}}}^+ &= \begin{cases} p_y^{\mathrm{tg}}, & \text{if } y \in \mathcal{B}^{\mathrm{mg}} \\ 0, & \text{otherwise} \end{cases} \\
(p^{\ell}_{(y,z),h})^+ &= \frac{\overline{s}_{(y,z)}}{\max\left\{\|\mathrm{col}(p^{\ell}_{(y,z),h}), q^{\ell}_{(y,z),h})\|, \overline{s}_{(y,z)}\right\}} p^{\ell}_{(y,z),h}, && \forall z \in \mathcal{B}_y, \forall h \in \mathcal{H} \\
(q^{\ell}_{(y,z),h})^+ &= \frac{\overline{s}_{(y,z)}}{\max\left\{\|\mathrm{col}(p^{\ell}_{(y,z),h}), q^{\ell}_{(y,z),h})\|, \overline{s}_{(y,z)}\right\}} q^{\ell}_{(y,z),h}, && \forall z \in \mathcal{B}_y, \forall h \in \mathcal{H}
\end{aligned}$$

The projection onto C_2 can be computed by solving a quadratic programming (e.g. via lsqlin, quadprog, osqp, etc...) with appropriate matrices.

Algorithm 2 Douglas–Rachford splitting to compute the projection of x onto $\mathcal{U}_{N+1} = C_1 \cap C_2$

1: **Iterate until convergence**
2: $\quad z(k) = \mathrm{proj}_{C_1}(\frac{1}{2}\xi(k) + \frac{1}{2}x)$
3: $\quad \xi(k+1) = \xi(k) + \lambda\left(\mathrm{proj}_{C_2}(2z(k) - \xi(k)) - z(k)\right), \quad$ with $\lambda \in (0, 2)$
4: **end**

2.3 Fully-distributed

Consider the following choices for the step-sizes.

Assumption 2 (Step size selection) *Set the step sizes of prosumers and DSO as follows:*

(i) $\forall i \in \mathcal{N}$*: set* $A_i = \mathrm{diag}(\alpha_i^{pi}, \alpha_i^{st}, \alpha_i^{mg}, \{\alpha_{(i,j)}^{tr}\}_{j \in \mathcal{N}_i}) \otimes I_H$*, with* $\alpha_i^{pi}, \alpha_i^{st} > 1$*,* $\alpha_i^{mg} > 3 + N \max_{h \in \mathcal{H}} d_h^{mg}$*,* $\alpha_{(i,j)}^{tr} > 2$*,* $\forall j \in \mathcal{N}_i$*,* $\beta_{(i,j)}^{tr} = \beta_{(j,i)}^{tr} < \frac{1}{2}$*,* $\forall j \in \mathcal{N}_i$*.*

(ii) $\forall y \in \mathcal{B}$*: set* $A_y := \mathrm{diag}\left(\alpha_y^\theta, \alpha_y^v, \alpha_y^{tg}, \{\alpha_{(y,z)}^p, \alpha_{(y,z)}^q\}_{z \in \mathcal{B}_y}\right) \otimes I_H$*, with* $\alpha_y^\theta, \alpha_y^v > 2|\mathcal{B}_y|(\|B\| + \|G\|)$*,* $\alpha_y^{tg} > 2$*,* $\alpha_{(y,z)}^p > 2$ *and* $\alpha_{(y,z)}^q > 1$*,* $\forall z \in \mathcal{B}_y$*,* $\forall y \in \mathcal{B}$*. Set* $\delta_y^{mg} = \delta^{mg} < \frac{1}{2}$*,* $\delta_y^{tg} = \delta^{tg} < \frac{1}{2}$*. Set* $\gamma_y^{mg} < (|\mathcal{N}_y| + |\mathcal{B}_y|)^{-1}$*. Set* $\beta_y^{tg} < (2|\mathcal{N}_y| + |\mathcal{B}_y|)^{-1}$ *and* $\beta_y^{pb} < (1 + 2|\mathcal{N}_y| + |\mathcal{B}_y|)^{-1}$*. Set* $\beta_{(z,y)}^{p,c} = \beta_{(y,z)}^p < (2\|B_{(y,z)}\| + 2\|G_{(y,z)}\| + 2)^{-1}$*,* $\beta_{(z,y)}^{q,c} = \beta_{(y,z)}^q < (2\|B_{(y,z)}\| + 2\|G_{(y,z)}\| + 2)^{-1}$*,* $\forall z \in \mathcal{B}_y$*. Set* $\beta_{(z,y)}^{p,c} = \beta_{(y,z)}^{p,c} < \frac{1}{2}$*,* $\beta_{(z,y)}^{q,c} = \beta_{(y,z)}^{q,c} < \frac{1}{2}$*,* $\forall z \in \mathcal{B}_y$*.*

$\square$

- the decision variable of bus y is $u_y := \left(\theta_y, v_y^{\mathrm{v}}, p_y^{\mathrm{tg}}, \{p_{(y,z)}^\ell, q_{(y,z)}^\ell\}_{z \in \mathcal{B}_y}\right) \in \mathbb{R}^d$, with $d := (3 + 2|\mathcal{B}_y|)24$.
- Let us introduce the local sets of constraints of bus y, i.e., $\mathcal{U}_y := \left\{u \in \mathbb{R}^d \mid (17) - (20) \text{ are satisfied}\right\}$.
- We recast the power flow equation (15) as

$$
\begin{aligned}
0 &= 2B(\theta_y - \theta_z) - 2G(v_y - v_z) - (p_{(y,z)}^\ell - p_{(z,y)}^\ell) \\
0 &= p_{(y,z)}^\ell + p_{(z,y)}^\ell
\end{aligned}
$$

- Similarly, we recast the power flow equation (16) as

$$
\begin{aligned}
0 &= 2G(\theta_y - \theta_z) + 2B(v_y - v_z) - (q_{(y,z)}^\ell - q_{(z,y)}^\ell) \\
0 &= q_{(y,z)}^\ell + q_{(z,y)}^\ell
\end{aligned}
$$

GB: It is convenient for deriving an effective distributed algorithm. Intuitively, the dual variables of such constraints will evolve (almost) identically on bus y and z.

Algorithm 3 Fully-decentralized GWE seeking for P2P Energy Markets

```
Iterate until convergence
    for all Bus y ∈ B (in parallel) do
        for all prosumer i ∈ N_y (in parallel) do
            Prosumer i update
                primal update        ▷ power generated, stored, from the grid, traded
                communication        ▷ with bus y operator and trading partners j ∈ N_i
                dual update          ▷ reciprocity constraints
            end prosumer i update
        end for
        DSO update (local bus y unit)
            primal update            ▷ local physical variables of bus y
            aggregation update       ▷ total grid-to-pros. power and load unbalance on bus y
            auxiliary update         ▷ for consensus of the dual variables
            dual update              ▷ physical constraints
            communication            ▷ with prosumers on bus y and neighbouring buses j ∈ B_y
        end
    end for
end
```

Algorithm 4 Prosumer i update

primal update $\triangleright$ power generated, stored, from the grid, traded

$\quad a_i(k) = \mathrm{col}\left(-\mu_y^{\mathrm{pb}}(k), -\mu_y^{\mathrm{pb}}(k), \bar{\lambda}_y^{\mathrm{mg}}(k) + \mu_y^{\mathrm{tg}}(k), \left\{\mu_{(i,j)}^{\mathrm{tr}}(k)\right\}_{j\in\mathcal{N}_i}\right)$ $\triangleright$ aux. vector

$$u_i(k+1) = \begin{cases} \underset{\xi\in\mathbb{R}^{n_i}}{\mathrm{argmin}} & J_i\left(\xi, \sum_{y\in\mathcal{B}} \sigma_y^{\mathrm{mg}}(k)\right) + a_i(k)^\top \xi + \frac{1}{2}\left\|\xi - u_i(k)\right\|_{A_i}^2 \\ \text{s.t.} & \xi \in \mathcal{U}_i \end{cases}$$ $\triangleright$ quadratic progr.

end

communication

$\quad b_i(k+1) = p_i^{\mathrm{d}} - p_i^{\mathrm{di}}(k+1) - p_i^{\mathrm{st}}(k+1)$ $\triangleright$ local load unbalance of prosumer i

$\quad p_i^{\mathrm{mg}}(k+1), b_i(k+1) \longrightarrow$ Bus y, $\triangleright$ forward to Bus y

$\quad$ **for all** prosumer $j \in \mathcal{N}_i$ **do**

$\quad\quad p_{(i,j)}^{\mathrm{tr}}(k+1) \longrightarrow$ prosumer j $\triangleright$ forward local trade to prosumer j

$\quad$ **end for**

end

dual update $\triangleright$ reciprocity constraints

$\quad$ **for all** $j \in \mathcal{N}_i$ **do**

$\quad\quad c_{(i,j)}^{\mathrm{tr}}(k+1) = p_{(i,j)}^{\mathrm{tr}}(k+1) + p_{(j,i)}^{\mathrm{tr}}(k+1)$ $\triangleright$ aux. vector

$\quad\quad \mu_{(i,j)}^{\mathrm{tr}}(k+1) = \mu_{(i,j)}(k) + \beta_{ij}^{\mathrm{tr}}\left(2c_{(i,j)}^{\mathrm{tr}}(k+1) - c_{(i,j)}^{\mathrm{tr}}(k)\right)$ $\triangleright$ reflected dual ascent

$\quad$ **end for**

end

Algorithm 5 DSO bus y update

primal update ▷ physical variables

  $a_y^\theta(k) = 4\sum_{z\in\mathcal{B}_y}\left(B_{(y,z)}^\top \mu_{(y,z)}^p(k) + G_{(y,z)}^\top \mu_{(y,z)}^q(k)\right)$ ▷ aux. vector

  $a_y^{\mathrm{v}}(k) = 4\sum_{z\in\mathcal{B}_y}\left(B_{(y,z)}^\top \mu_{(y,z)}^q(k) - G_{(y,z)}^\top \mu_{(y,z)}^p(k)\right)$ ▷ aux. vector

  $a_y(k) = \mathrm{col}\left(a_y^\theta(k), a_y^{\mathrm{v}}(k), -\mu_y^{\mathrm{tg}}(k) - \mu_y^{\mathrm{pb}}(k), \left\{\mu_{(y,z)}^{p,c}(k) - \mu_y^{\mathrm{pb}}(k) - \mu_{(y,z)}^p(k), \mu_{(y,z)}^{q,c} - \mu_{(y,z)}^q(k)\right\}_{z\in\mathcal{B}_y}\right)$

  $u_y(k+1) = \mathrm{proj}_{\mathcal{U}_y}\left(u_y(k) - A_y a_y(k)\right)$ ▷ solved via Algorithm 2

end

aggregation update

  $\sigma_y^{\mathrm{mg}}(k+1) = \sum_{i\in\mathcal{N}_y} p_i^{\mathrm{mg}}(k+1)$ ▷ aggregate grid-to-prosumers power on bus y

  $b_y(k+1) = \sum_{i\in\mathcal{N}_y} b_i(k+1)$ ▷ aggregate load unbalance on bus y

end

auxiliary update

  $w_y^{\mathrm{mg}}(k+1) = w_y^{\mathrm{mg}}(k) + \delta_y^{\mathrm{mg}}\left(|\mathcal{N}_y|\lambda_y^{\mathrm{mg}}(k) - \sum_{z\in\mathcal{B}_y}\lambda_y^{\mathrm{mg}}(z)\right)$ ▷ consensus on λ_y^{mg}'s

  $w_y^{\mathrm{tg}}(k+1) = w_y^{\mathrm{tg}}(k) + \delta_y^{\mathrm{tg}}\left(|\mathcal{N}_y|\lambda_y^{\mathrm{tg}}(k) - \sum_{z\in\mathcal{B}_y}\lambda_y^{\mathrm{tg}}(z)\right)$ ▷ consensus on λ_y^{tg}'s

end

dual update(global grid constraints)

  $c_y^{\mathrm{mg}}(k+1) = \begin{bmatrix}1\\-1\end{bmatrix}\otimes\sigma_y^{\mathrm{mg}}(k+1) - |\mathcal{B}|^{-1}\begin{bmatrix}\bar{p}^{\mathrm{mg}}\mathbf{1}_H\\ -\underline{p}^{\mathrm{mg}}\mathbf{1}_H\end{bmatrix} + w_y^{\mathrm{mg}}(k+1)$ ▷ aux. vector

  $\lambda_y^{\mathrm{mg}}(k+1) = \mathrm{proj}_{\mathbb{R}^{2H}_{\geq 0}}\left(\lambda_y^{\mathrm{mg}}(k) + \gamma_y^{\mathrm{mg}}(2c_y^{\mathrm{mg}}(k+1) - c_y^{\mathrm{mg}}(k))\right)$ ▷ grid constr.

  $c_y^{\mathrm{tg}}(k+1) = \sigma_y^{\mathrm{mg}}(k+1) - p_y^{\mathrm{tg}}(k+1) + w_y^{\mathrm{tg}}(k+1)$ ▷ aux. vector

  $\mu_y^{\mathrm{tg}}(k+1) = \mu_y^{\mathrm{tg}}(k) + \beta_y^{\mathrm{tg}}(2c^{\mathrm{tg}}(k+1) - c^{\mathrm{tg}}(k))$ ▷ grid-to-buses constraints

end

dual update (power balance on bus y)

  $c_y^{\mathrm{pb}}(k+1) = p_y^{\mathrm{pd}} + b_y(k+1) - p_y^{\mathrm{tg}}(k+1) - \sum_{z\in\mathcal{B}_y} p_{(y,z)}^\ell(k+1)$ ▷ aux. vector

  $\mu_y^{\mathrm{pb}}(k+1) = \mu_y^{\mathrm{pb}}(k) + \beta_y^{\mathrm{pb}}(2c_y^{\mathrm{pb}}(k+1) - c_y^{\mathrm{pb}}(k))$ ▷ power balance of bus y

end

communication ▷ broadcast

  $\bar{\lambda}_y^{\mathrm{mg}}(k+1) = \begin{bmatrix}I_H\\-I_H\end{bmatrix}^\top \lambda_y^{\mathrm{mg}}(k+1)$

  $\{\bar{\lambda}_y^{\mathrm{mg}}(k+1), \mu_y^{\mathrm{tg}}(k+1), \mu_y^{\mathrm{pb}}(k+1)\} \longrightarrow \mathcal{N}_y$ ▷ to all prosumers on bus y

  $\{\lambda_y^{\mathrm{mg}}(k+1), \mu_y^{\mathrm{tg}}(k+1)\} \longrightarrow \mathcal{B}_y$ ▷ to the neighbour buses

  for all neighbour bus $z\in\mathcal{B}_y$ **do**

    $\{\theta_y(k+1), v_y(k+1), \mu_{(y,z)}^p(k+1), \mu_{(y,z)}^q(k+1)\} \longrightarrow$ bus z ▷ forward to bus z

  end for

end

dual update (power flow equations)

  for all bus $z\in\mathcal{B}_z$ (in parallel) **do**

    $b_{(y,z)}^p(k+1) = p_{(y,z)}^\ell(k+1) - p_{(z,y)}^\ell(k+1)$ ▷ aux. vectors

    $c_{(y,z)}^p(k+1) = 2B_{(y,z)}(\theta_y(k+1) - \theta_z(k+1)) - 2G_{(y,z)}(v_y(k+1) - v_z(k+1)) - b_{(y,z)}(k+1)$

    $\mu_{(y,z)}^p(k+1) = \mu_{(y,z)}^p(k) + \beta_{(y,z)}^p(2c_{(y,z)}^p(k+1) - c_{(y,z)}^p(k))$ ▷ power flow eq.1

    $d_{(y,z)}^p(k+1) = p_{(y,z)}^\ell(k+1) + p_{(z,y)}^\ell(k+1)$ ▷ aux. vector

    $\mu_{(y,z)}^{p,c}(k+1) = \mu_{(y,z)}^{p,c}(k) + \beta_{(y,z)}^{p,c}\left(2d_{(y,z)}^p(k+1) - d_{(y,z)}^p(k)\right)$ ▷ physical reciprocity constr.

    $b_{(y,z)}^q(k+1) = q_{(y,z)}^\ell(k+1) - q_{(z,y)}^\ell(k+1)$ ▷ aux. vectors

    $c_{(y,z)}^q(k+1) = 2G_{(y,z)}(\theta_y(k+1) - \theta_z(k+1)) + 2B_{(y,z)}(v_y(k+1) - v_z(k+1)) - b_{(y,z)}^q(k+1)$

    $\mu_{(y,z)}^q(k+1) = \mu_{(y,z)}^q(k) + \beta_{(y,z)}^q(2c_{(y,z)}^q(k+1) - c_{(y,z)}^q(k))$ ▷ power flow eq.2

    $d_{(y,z)}^q(k+1) = q_{(y,z)}^\ell(k+1) + q_{(z,y)}^\ell(k+1)$ ▷ aux. vector

    $\mu_{(y,z)}^{q,c}(k+1) = \mu_{(y,z)}^{q,c}(k) + \beta_{(y,z)}^{q,c}\left(2d_{(y,z)}^q(k+1) - d_{(y,z)}^q(k)\right)$ ▷ physical reciprocity constr.

  end for

end

References

[1] G. Belgioioso and S. Grammatico, "Semi-decentralized generalized nash equilibrium seeking in monotone aggregative games," *arXiv preprint arXiv:2003.04031*, 2020.

[2] ——, "A distributed proximal-point algorithm for nash equilibrium seeking in generalized potential games with linearly coupled cost functions," in *2019 18th European Control Conference (ECC)*. IEEE, 2019, pp. 1–6.