Decentralized Algorithms for Generalized Nash Equilibrium Seeking in Peer-to-peer Electricity Markets

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1 Problem statement

Consider the same setup of the ECC 2020 paper.

2 Algorithms

2.1 Semi-decentralized

The algorithm presented in this section corresponds to [1, Alg. 1] applied on the setup of the ECC 2020 paper.

Consider the following choices for the step-sizes in Algorithm 1.

Assumption 1 (Step size selection) *For all* $i \in \mathcal{N}$*, set* $A_i = \mathrm{blkdiag}(A_{i,1}, \ldots, A_{i,H})$*, where* $A_{i,h} = \mathrm{diag}\left(\alpha^{\mathrm{dg}}_{i,h},\ \alpha^{\mathrm{st}}_{i,h},\ \alpha^{\mathrm{mg}}_{i,h},\ \{\alpha^{\mathrm{tr}}_{(i,j),h}\}_{j\in\mathcal{N}_i}\right)$*, for all* $h \in \mathcal{H}$*, with* $\alpha^{\mathrm{dg}}_{i,h},\ \alpha^{\mathrm{st}}_{i,h} > 0$ *and* $\alpha^{\mathrm{tr}}_{(i,j),h} > |\mathcal{N}_i|$*, for all* $j \in \mathcal{N}_i$*;* $\beta_{(i,j)} = \beta < \frac{1}{2}$ *for all* $j \in \mathcal{N}_i$*;* $\gamma < N$

□

Algorithm 1 Semi-decentralized GWE seeking for P2P Energy Markets

Initialization: For all $i \in \mathcal{N}$, set locally:

(a) Initial conditions: $u_i(0) \in \mathcal{U}_i$, $\mu_{(i,j)}(0) = \mathbf{0}$ for all $j \in \mathcal{N}_i$, and $\lambda(0) \in \mathbb{R}^{2H}_{\geq 0}$.

(b) Step-sizes: A_i, $\{\beta_{ij}\}_{j\in\mathcal{N}_i}$,$\gamma$ as in Assumption 1.

Iterate until convergence:

Local. For each agent $i \in \mathcal{N}$:

(1) Strategy update:

$$a_i(k) = A_i^{-1}\left(\left(\left[\begin{smallmatrix}1\\-1\end{smallmatrix}\right] \otimes S_i^{\mathrm{mg}}\right)^\top \lambda(k) + \sum_{j\in\mathcal{N}_i} \left(S_{(i,j)}^{\mathrm{tr}}\right)^\top \mu_{(i,j)}(k)\right)$$

$$u_i(k+1) = \begin{cases} \underset{\xi\in\mathbb{R}^{n_i}}{\operatorname{argmin}} & J_i\big(\xi,\sigma(k)\big) + \frac{1}{2}\left\|\xi - u_i(k) + a_i(k)\right\|^2_{A_i} \\ \text{s.t.} & \xi \in \mathcal{U}_i \end{cases}$$

(2) Communication with Neighboring Agents and Central Coordinator (CC):

$$\begin{array}{llcll} \text{(a)} & S^{\mathrm{mg}} u_i(k+1) & \longrightarrow & \text{CC}, & \\ \text{(b)} & S_{(i,j)}^{\mathrm{tr}} u_i(k+1) & \longrightarrow & j, & \text{for all } j \in \mathcal{N}_i. \end{array}$$

(3) Dual variables (reciprocity constraints) update, $\forall j \in \mathcal{N}_i$:

$$\begin{aligned} b_{(i,j)}(k+1) &= S_{(i,j)}^{\mathrm{tr}} u_i(k+1) + S_{(j,i)}^{\mathrm{tr}} u_j(k+1), \\ \mu_{(i,j)}(k+1) &= \mu_{(i,j)}(k) + \beta_{ij}\left(2b_{(i,j)}(k+1) - b_{(i,j)}(k)\right). \end{aligned}$$

Central.

(1) Average strategy update

$$\sigma(k+1) = \frac{1}{N}\sum_{i=1}^{N} S_i^{\mathrm{mg}}\, u_i(k+1)$$

(2) Dual variable (grid constraints) update:

$$\lambda(k+1) = \operatorname{proj}_{\mathbb{R}^{2H}_{\geq 0}}\left(\lambda(k) + \gamma\left(\left[\begin{smallmatrix}1\\-1\end{smallmatrix}\right] \otimes \left(2\sigma(k+1) - \sigma(k)\right) - \left[\begin{smallmatrix}\overline{p}^{\mathrm{mg}}\mathbf{1}_H\\ -\underline{p}^{\mathrm{mg}} N\mathbf{1}_H\end{smallmatrix}\right]\right)\right).$$

(3) Broadcast to all the prosumers

$$\sigma(k+1),\ \lambda(k+1) \quad \longrightarrow \quad \text{Prosumers}$$

2.2 Fully-distributed

The algorithm presented in this section is a variation of [2, Alg. 3] for the setup considered in the ECC 2020 paper.

Consider the following mixing matrix.

Assumption 2 *For all $k \in \mathbb{N}$, the matrix $W = [w_{i,j}]$ satisfies the following conditions:*

(i) (Edge utilization) Let $i, j \in \mathcal{I}$, $i \neq j$. If $(i,j) \in \mathcal{E}_k$, $w_{i,j} \geq \epsilon$, for some $\epsilon > 0$; $w_{i,j} = 0$ otherwise;

(ii) (Positive diagonal) For all $i \in \mathcal{I}$, $w_{i,i} > \epsilon$;

(iii) (Double-stochasticity) $W\mathbf{1} = \mathbf{1}$, $\mathbf{1}^\top W = \mathbf{1}^\top$. □

Assumption 2 is strong but typical for multiagent coordination and optimization [3]. For an undirected graph it can be fulfilled, for example, by using Metropolis weights:

$$w_{i,j} = \begin{cases} (\max\{|\mathcal{N}_i|, |\mathcal{N}_j|\})^{-1} & \text{if } (i,j) \in \mathcal{E}, \\ 0 & \text{if } (i,j) \notin \mathcal{E}, \\ 1 - \sum_{\ell \in \mathcal{N}_i} w_{i,\ell} & \text{if } i = j. \end{cases} \tag{1}$$

Consider the following choices for the step-sizes in Algorithm 2.

Assumption 3 (Step size selection) *For all $i \in \mathcal{N}$, set $A_i = \mathrm{blkdiag}(A_{i,1}, \ldots, A_{i,H})$, where $A_{i,h} = \mathrm{diag}\left(\alpha^{\mathrm{dg}}_{i,h},\ \alpha^{\mathrm{st}}_{i,h},\ \alpha^{\mathrm{mg}}_{i,h},\ \{\alpha^{\mathrm{tr}}_{(i,j),h}\}_{j \in \mathcal{N}_i}\right)$, for all $h \in \mathcal{H}$, with $\alpha^{\mathrm{dg}}_{i,h}$, $\alpha^{\mathrm{st}}_{i,h} > 0$, $\alpha^{mg}_{i,h} > q^{mg}_h + 1$ and $\alpha^{\mathrm{tr}}_{(i,j),h} > |\mathcal{N}_i|$, for all $j \in \mathcal{N}_i$; $\beta_{(i,j)} = \beta < \frac{1}{2}$ for all $j \in \mathcal{N}_i$; $\gamma < 1$.*

□

Assumption 4 (Kransol'skii–Mann step) *The sequence $(\delta(k))_{k \in \mathbb{N}}$ satisfies the following conditions:*

(i) (non-increasing) $0 \leq \delta(k)^{k+1} \leq \delta(k) \leq 1$, for all $k \geq 0$;

(ii) (non-summable) $\sum_{k=0}^{\infty} \delta(k) = \infty$;

(iii) (square-summable) $\sum_{k=0}^{\infty} \delta(k)^2 < \infty$. □

Remark 1 (Vanishing KM steps seem not necessary in practice) *For the numerical studies, we can use $\delta(k) = 1$, for all $k \in \mathbb{N}$.*

Algorithm 2 Fully-Distributed GWE seeking for P2P Energy Markets

Initialization: For all $i \in \mathcal{N}$, set locally:

(a) Initial conditions: $u_i(-1), u_i(0), \tilde{u}_i(-1) \in \mathcal{U}_i$, $\mu_{(i,j)}(0) = \mathbf{0}$ for all $j \in \mathcal{N}_i$, $\lambda_i(0) \in \mathbb{R}^{2H}_{\geq 0}$, $\sigma_i(0) = x_i(0)$, $z_i(0) = \lambda_i(0)$, $y_i(0) = 2\tilde{u}_i(-1) - u_i(-1) - \frac{1}{N}\begin{bmatrix} \overline{p}^{\mathrm{mg}}\mathbf{1}_H \\ -\underline{p}^{\mathrm{mg}}N\mathbf{1}_H \end{bmatrix}$.

(b) Step-sizes: A_i, $\{\beta_{ij}\}_{j\in\mathcal{N}_i}$ and γ_i as in Assumption 3; $\{\delta_i(k)\}_{k\in\mathbb{N}}$ as in Assumption 4.

Iterate until convergence:

Local. For each agent $i \in \mathcal{N}$:

(1) Communication with neighboring agents

$$\left\{S^{\mathrm{mg}}u_i(k+1),\, \sigma_i(k),\, y_i(k),\, z_i(k)\right\} \quad \longrightarrow \quad j,\ \text{for all } j \in \mathcal{N}_i,$$

(2) Distributed averaging

$$\hat{\sigma}_i(k) = \textstyle\sum_{j=1}^N w_{i,j}\sigma_j(k), \quad \hat{y}_i(k) = \sum_{j=1}^N w_{i,j}y_j(k), \quad \hat{z}_i(k) = \sum_{j=1}^N w_{i,j}z_j(k),$$

(3) Strategy update

$$d_i(k) = A_i^{-1}\left(\left(\left[\begin{smallmatrix}1\\-1\end{smallmatrix}\right] \otimes S_i^{\mathrm{mg}}\right)^\top \hat{z}_i(k) + \sum_{j\in\mathcal{N}_i}\left(S^{\mathrm{tr}}_{(i,j)}\right)^\top \mu_{(i,j)}(k)\right)$$

$$\tilde{u}_i(k) = \begin{cases} \underset{\xi\in\mathbb{R}^{n_i}}{\operatorname{argmin}} & J_i\big(\xi, \hat{\sigma}_i(k)\big) + \frac{1}{2}\left\|\xi - u_i(k) + d_i(k)\right\|^2_{A_i} \\ \text{s.t.} & \xi \in \mathcal{U}_i \end{cases}$$

(4) Dual variables (reciprocity constraints) update, $\forall j \in \mathcal{N}_i$:

$$\tilde{\mu}_{(i,j)}(k) = \mu_{(i,j)}(k) + \beta_{ij}\left(2S^{\mathrm{tr}}_{(i,j)}\tilde{u}_i(k) - S^{\mathrm{tr}}_{(i,j)}u_i(k) + 2S^{\mathrm{tr}}_{(j,i)}\tilde{u}_j(k) - S^{\mathrm{tr}}_{(j,i)}u_j(k)\right).$$

(5) Dynamic tracking of the estimate y_i:

$$y_i(k+1) = \hat{y}_i(k) + \big(2\tilde{u}_i(k) - u_i(k)\big) - \big(2\tilde{u}_i(k-1) - u_i(k-1)\big),$$

(6) Dual variable (grid constraints) update:

$$\tilde{\lambda}_i(k) = \operatorname{proj}_{\mathbb{R}^{2H}_{\geq 0}}\left(\lambda_i(k) + \gamma_i\left(\left[\begin{smallmatrix}1\\-1\end{smallmatrix}\right] \otimes y_i(k+1) - \lambda_i(k) + \hat{z}_i(k)\right)\right),$$

(7) Krasnosel'skii-Mann process

$$\begin{aligned} u_i(k+1) &= u_i(k) + \delta_i(k)\big(\tilde{u}_i(k) - u_i(k)\big), \\ \mu_{(i,j)}(k+1) &= \mu_{(i,j)}(k) + \delta_i(k)\big(\tilde{\mu}_{(i,j)}(k) - \mu_{(i,j)}(k)\big), \quad \forall j \in \mathcal{N}_i, \\ \lambda_i(k+1) &= \lambda_i(k) + \delta_i(k)\big(\tilde{\lambda}(k) - \lambda_i(k)\big), \end{aligned}$$

(8) Dynamic tracking of the estimates σ_i and z_i:

$$\begin{aligned} \sigma_i(k+1) &= \hat{\sigma}(k) + u_i(k+1) - u_i(k), \\ z_i(k+1) &= \hat{z}_i(k) + \lambda_i(k+1) - \lambda_i(k) \end{aligned}$$

References

[1] G. Belgioioso and S. Grammatico, "Projected-gradient algorithms for generalized equilibrium seeking in aggregative games arepreconditioned forward-backward methods," in *2018 European Control Conference (ECC)*. IEEE, 2018, pp. 2188–2193.

[2] G. Belgioioso, A. Nedić, and S. Grammatico, "Distributed generalized Nash equilibrium seeking in aggregative games on time-varying networks, *Available at:* `https://arxiv.org/abs/1907.00191`," 2019.

[3] K. Margellos, A. Falsone, S. Garatti, and M. Prandini, "Distributed constrained optimization and consensus in uncertain networks via proximal minimization," *IEEE Transactions on Automatic Control*, vol. 63, no. 5, pp. 1372–1387, 2018.