

Decentralized Algorithms for Generalized Nash Equilibrium Seeking in Peer-to-peer Electricity Markets

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1 Problem statement

Consider the same problem statement in `problem_setup_v6.tex`.

2 Algorithms

2.1 Semi-decentralized

The algorithm presented in this section corresponds to [1, Alg. 6B] applied on `problem_setup_v6.tex`.

Consider the following choices for the step-sizes in Algorithm 1.

Assumption 1 (Step-size) *Set the step sizes of prosumers and DSO as follows:*

- (i) $\forall i \in \mathcal{N}$: set $A_i = \text{diag}(\alpha_i^{pi}, \alpha_i^{st}, \alpha_i^{mg}, \{\alpha_{(i,j)}^{tr}\}_{j \in \mathcal{N}_i}) \otimes I_H$, with $\alpha_i^{pi}, \alpha_i^{st} > 1$, $\alpha_i^{mg} > 3 + N \max_{h \in \mathcal{H}} d_h^{mg}$, $\alpha_{(i,j)}^{tr} > |\mathcal{N}_i|$, $\forall j \in \mathcal{N}_i$, $\beta_{(i,j)}^{tr} = \beta_{(j,i)}^{tr} < \frac{1}{2}$, $\forall j \in \mathcal{N}_i$.
- (ii) Set $A_{N+1} := \text{diag} \left(\left\{ \alpha_y^\theta, \alpha_y^v, \alpha_y^{tg}, \{\alpha_{(y,z)}^p, \alpha_{(y,z)}^q\}_{z \in \mathcal{B}_y} \right\}_{y \in \mathcal{B}} \right) \otimes I_H$, with $\alpha_y^\theta, \alpha_y^v > 0$, $\alpha_y^{tg} > 2$, $\alpha_{(y,z)}^p > 1$ and $\alpha_{(y,z)}^q > 0$, $\forall z \in \mathcal{B}_y$, $\forall y \in \mathcal{B}$. Set $\gamma^{mg} < \frac{1}{N}$, $\beta^{tg} < (|\mathcal{N}| + |\mathcal{B}|)^{-1}$ and $\beta_y^{pb} < (1 + |\mathcal{N}_y| + |\mathcal{B}_y|)^{-1}$, for all $y \in \mathcal{B}$. \square

Algorithm 1 Semi-decentralized GWE seeking for P2P Energy Markets

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1: Iterate until convergence
2:   for all prosumer  $i \in \mathcal{N}$  do
3:     primal update ▷ power generated, stored, from the grid, traded
4:      $a_i(k) = \text{col} \left( -\mu_y^{\text{pb}}(k), -\mu_y^{\text{pb}}(k), \begin{bmatrix} I_H \\ -I_H \end{bmatrix}^\top \lambda^{\text{mg}}(k) + \mu^{\text{tg}}(k), \left\{ \mu_{(i,j)}^{\text{tr}}(k) \right\}_{j \in \mathcal{N}_i} \right)$  ▷ aux. vector
5:      $u_i(k+1) = \begin{cases} \underset{\xi \in \mathbb{R}^{n_i}}{\text{argmin}} & J_i(\xi, \sigma(k)) + a_i(k)^\top \xi + \frac{1}{2} \|\xi - u_i(k)\|_{A_i}^2 \\ \text{s.t.} & \xi \in \mathcal{U}_i \end{cases}$  ▷ quadratic progr.
6:   end
7:   communication ▷ to DSO and trading partners
8:    $b_i(k+1) = p_i^{\text{d}} - p_i^{\text{di}}(k+1) - p_i^{\text{st}}(k+1)$  ▷ local load unbalance of prosumer  $i$ 
9:    $p_i^{\text{mg}}(k+1), b_i(k+1) \rightarrow \text{DSO}$  ▷ forward to DSO
10:  for all prosumer  $j \in \mathcal{N}_i$  do
11:     $p_{(i,j)}^{\text{tr}}(k+1) \rightarrow \text{prosumer } j$  ▷ forward local trade to prosumer  $j$ 
12:  end for
13: end
14: dual update ▷ reciprocity constraints
15:   for all  $j \in \mathcal{N}_i$  do
16:     $c_{(i,j)}^{\text{tr}}(k+1) = p_{(i,j)}^{\text{tr}}(k+1) + p_{(j,i)}^{\text{tr}}x(k+1)$  ▷ aux. vector
17:     $\mu_{(i,j)}(k+1) = \mu_{(i,j)}(k) + \beta_{ij}^{\text{tr}} \left( 2c_{(i,j)}^{\text{tr}}(k+1) - c_{(i,j)}^{\text{tr}}(k) \right)$  ▷ reflected dual ascent
18:   end for
19: end
20: end for
21: DSO update
22:   primal update ▷ physical variables
23:    $a_{N+1}(k+1) = \text{col} \left( \{ \mathbf{0}, \mathbf{0}, -\mu^{\text{tg}}(k), \{ \mu_y^{\text{pb}}(k), \mathbf{0} \}_{z \in \mathcal{B}_y} \}_{y \in \mathcal{B}} \right)$  ▷ aux. vector
24:    $u_{N+1}(k+1) = \text{proj}_{\mathcal{U}_{N+1}} (u_{N+1}(k) - A_{N+1}a_{N+1}(k))$  ▷ solved via Algorithm 2
25:   end
26:   aggregation update
27:    $\sigma^{\text{mg}}(k+1) = \sum_{i \in \mathcal{N}} p_i^{\text{mg}}(k+1)$  ▷ aggregate grid-to-prosumers power
28:    $\sigma^{\text{tg}}(k+1) = \sum_{y \in \mathcal{B}} p_y^{\text{tg}}(k+1)$  ▷ aggregate grid-to-buses power
29:   end
30:   dual update
31:    $b_{N+1}(k+1) = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \otimes (2\sigma^{\text{mg}}(k+1) - \sigma^{\text{mg}}(k)) - \begin{bmatrix} \bar{p}^{\text{mg}} \mathbf{1}_H \\ -\underline{p}^{\text{mg}} N \mathbf{1}_H \end{bmatrix}$  ▷ aux. vector
32:    $\lambda^{\text{mg}}(k+1) = \text{proj}_{\mathbb{R}_{\geq 0}^{2H}} (\lambda^{\text{mg}}(k) + \gamma^{\text{mg}} b_{N+1}(k+1))$  ▷ grid constraints
33:   for all buses  $y \in \mathcal{B}$  do
34:     $c_y^{\text{pb}}(k+1) = p_y^{\text{pd}} + \sum_{i \in \mathcal{N}_y} b_i(k+1) - p_y^{\text{tg}}(k+1) - \sum_{z \in \mathcal{B}_y} p_{(y,z)}^\ell(k+1)$  ▷ aux. vector
35:     $\mu_y^{\text{pb}}(k+1) = \underline{v}_y(k) + \beta_y^{\text{pb}} (2c_y^{\text{pb}}(k+1) - c_y^{\text{pb}}(k))$  ▷ local power balance of bus  $y$ 
36:   end for
37:    $c^{\text{tg}}(k+1) = \sigma^{\text{mg}}(k+1) - \sigma^{\text{tg}}(k+1)$  ▷ aux. vector
38:    $\mu^{\text{tg}}(k+1) = \mu^{\text{tg}}(k) + \beta^{\text{tg}} (2c^{\text{tg}}(k+1) - c^{\text{tg}}(k))$  ▷ grid-to-buses constraints
39: end
40: communication ▷ broadcast
41:    $\{\sigma(k+1), \lambda^{\text{mg}}(k+1), \mu^{\text{tg}}(k+1)\} \rightarrow \mathcal{N}$  ▷ to all prosumers
42:   for all buses  $y \in \mathcal{B}$  do
43:     $\mu_y^{\text{pb}}(k+1) \rightarrow \mathcal{N}_y$  ▷ to all prosumers on bus  $y$ 
44:   end for
45: end
46: end
47: end

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2.2 Compute the projection onto \mathcal{U}_{N+1}

Next, we propose an iterative method to solve line 24 in Algorithm 1, namely, to compute the projection onto \mathcal{U}_{N+1} . First, let us define the sets $\mathcal{C}_1 := (17) \cap (18) \cap (19)$ and $C_2 = (15) \cap (16)$ and recall that the decision vector of the central coordinator reads as $u_{N+1} = \text{col} \left(\{\theta_y, v_y, p_y^{\text{tg}}, \{p_{(y,z)}^\ell, q_{(y,z)}^\ell\}_{z \in \mathcal{B}_y}\}_{y \in \mathcal{B}} \right)$. The projection onto C_1 can be characterize in closed-form as follows:

$$\text{proj}_{C_1}(u_{N+1}) = \text{col} \left(\{\theta_y^+, v_y^+, p_y^{\text{tg}+}, \{p_{(y,z)}^{\ell+}, q_{(y,z)}^{\ell+}\}_{z \in \mathcal{B}_y}\}_{y \in \mathcal{B}} \right),$$

where, for all $y \in \mathcal{B}$,

$$\begin{aligned} \theta_y^+ &= \begin{cases} \underline{\theta}_y, & \text{if } \theta_y < \underline{\theta}_y \\ \bar{\theta}_y, & \text{if } \theta_y > \bar{\theta}_y \\ \theta_y, & \text{otherwise} \end{cases}, & \forall y \in \mathcal{B} \\ v_y^+ &= v_y & \forall y \in \mathcal{B} \\ p_y^{\text{mg}+} &= p_y^{\text{mg}} \\ (p_{(y,z),h}^\ell)^+ &= \frac{\bar{s}_{(y,z)}}{\max \left\{ \|\text{col}(p_{(y,z),h}^\ell, q_{(y,z),h}^\ell)\|, \bar{s}_{(y,z)} \right\}} p_{(y,z),h}^\ell, & \forall z \in \mathcal{B}_y, \forall h \in \mathcal{H} \\ (q_{(y,z),h}^\ell)^+ &= \frac{\bar{s}_{(y,z)}}{\max \left\{ \|\text{col}(p_{(y,z),h}^\ell, q_{(y,z),h}^\ell)\|, \bar{s}_{(y,z)} \right\}} q_{(y,z),h}^\ell, & \forall z \in \mathcal{B}_y, \forall h \in \mathcal{H} \end{aligned}$$

The projection onto C_2 can be computed by solving a quadratic programming (e.g. via lsqlin, quadprog, osqp, etc...) with appropriate matrices.

Algorithm 2 Douglas–Rachford splitting to compute the projection of x onto $\mathcal{U}_{N+1} = C_1 \cap C_2$

1: **Iterate until convergence**

2: $z(k) = \text{proj}_{C_1} \left(\frac{1}{2} \xi(k) + \frac{1}{2} x \right)$

3: $\xi(k+1) = \xi(k) + \lambda \left(\text{proj}_{C_2} (2z(k) - \xi(k)) - z(k) \right)$, with $\lambda \in (0, 2)$

4: **end**

References

- [1] G. Belgioioso and S. Grammatico, “Semi-decentralized generalized nash equilibrium seeking in monotone aggregative games,” *arXiv preprint arXiv:2003.04031*, 2020.