

Cryptanalysis of a public key encryption scheme based on QC-LDPC and QC-MDPC codes

Vlad Dragoi^{*†} and Hervé Talé Kalachi[‡]

^{*} Faculty of Exact Sciences, “Aurel Vlaicu” University of Arad, Romania

Email: ^{*}vlad.dragoi@uav.ro

[†] Normandie University, France; UR, LITIS Mont-saint-aignan, France

Email: [†] vlad-florin.dragoi@univ-rouen.fr

[‡] ERAL, University of Yaounde 1

Email: [‡] hervekalachi@gmail.com

Abstract—This letter presents a cryptanalysis of the modified McEliece cryptosystem recently proposed by Moufek, Guenda and Gulliver [24]. The system is based on the juxtaposition of quasi-cyclic LDPC and quasi-cyclic MDPC codes. The idea of our attack is to find an alternative permutation matrix together with an equivalent LDPC code which allow the decoding of any cipher-text with a very high probability. We also apply a recent technique to determine weak keys [4] for this scheme. The results show that the probability of weak keys is high enough that this variant can be ruled out as a possible secure encryption scheme.

Index Terms—Post-quantum cryptography; McEliece cryptosystem; QC-LDPC and QC-MDPC codes.

I. INTRODUCTION

The problem of finding a practical solution for quantum resistant cryptography has become an urgent issue, mainly for two reasons: the existence of a quantum polynomial time algorithm [29] that breaks the actual RSA and ECC solutions and the improvements of classical algorithms for the discrete logarithm in small characteristic [2]. Among the possible candidates for post-quantum cryptography, code-based cryptography is one of the most promising solutions [6]. It is also one of the oldest public key encryption solutions thanks to McEliece’s idea [21].

In the original paper, McEliece proposed to use binary Goppa codes, which remain unbroken, to date. Other families of algebraic codes were proposed (see [25], [31], [14] ect). But they were successfully cryptanalyzed, mainly due to their algebraic structure [9], [22], [32]. Probabilistic codes were also considered: concatenated codes were analyzed in [28], [31]; convolutional codes were proposed in [17], but successfully cryptanalyzed in [16]. Polar codes were also proposed in [30] and cryptanalyzed in [3].

The emergence of all these attacks on several variants of the McEliece cryptosystem shows the importance and necessity of proposing new masking techniques. For example a code with an efficient decoding algorithm could be masked with the use of an arbitrary code in order to sustain the security of the scheme. This technique was proposed for the first time by Wieschebrink in [35] to avoid the Sidelnikov-Shestakov attack [32]. The idea is to use the juxtaposition of a Generalized Reed-Solomon code and a random code. This solution became famous since it was also used in the case of Reed-Muller codes

[12]. But these two variants were broken using the square code technique in [8], [26]. Nonetheless the idea was not totally abandoned since Wang [34] proposed to use the juxtaposition of a GRS code with a random code and then to multiply the generator matrix of the resulting code with a matrix that linearly combines the columns of the GRS and the random code. The main motivation is to obtain a random like code in the end. Another technique was introduced in [18] where the authors propose to use the “ $u|u + v$ ” construction with two GRS codes. This new trend also inspired researchers to use the juxtaposition of LDPC and MDPC codes [24]. This is the variant that we analyze in the present article.

a) *Our contribution:* We describe a key recovery attack against the modified McEliece cryptosystem based on the juxtaposition of LDPC and MDPC codes [24]. The idea of our attack is to find an alternative permutation matrix together with an equivalent LDPC code which allow the decoding of any cipher-text with a very high probability.

II. BACKGROUND ON CODING THEORY

Throughout the paper we denote by \mathbb{F}_2 the finite field with 2 elements and $M_{k,n}(\mathbb{F}_2)$ the set of $k \times n$ matrices with entries in \mathbb{F}_2 . The *Hamming weight* $\text{wt}(x)$ of a vector $x \in \mathbb{F}_2^n$ is the number of non-zero coordinates of x . A *binary linear code* \mathcal{C} of length n and dimension k is a k -dimensional linear subspace of \mathbb{F}_2^n . A *generator matrix* of \mathcal{C} is any $k \times n$ matrix $G \in M_{k,n}(\mathbb{F}_2)$ with rows that generate \mathcal{C} . The dual \mathcal{C}^\perp of \mathcal{C} is the $n - k$ -dimensional linear subspace defined by

$$\mathcal{C}^\perp = \left\{ z \in \mathbb{F}_2^n : \forall c \in \mathcal{C}, \sum_i c_i z_i = 0 \right\}.$$

A parity check matrix of \mathcal{C} is a generator of \mathcal{C}^\perp .

Definition 1. A (n, k, ω) -code is a binary linear code defined by a $k \times n$ parity-check matrix ($k < n$) where each row has weight ω . When $n \rightarrow \infty$ we define

- An *LDPC* code is a (n, k, ω) -code with $\omega = O(1)$ [11].
- An *MDPC* code is a (n, k, ω) -code with $\omega = O(\sqrt{n})$ [23].

Furthermore we will use the $(n, k, \omega_{\text{ldpc}})$ notation, respectively $(n, k, \omega_{\text{mdpc}})$. The original decoding technique for LDPC codes is the well-known bit flipping algorithm [11]. This

technique is known to provide an error-correction capability which increases linearly with the length of the code, but decreases with the weight of the parity-checks. Therefore MDPC codes suffer from a degradation in decoding performance, compared with LDPC codes. A full description of the bit flipping algorithm can be found in [7], [11], [23].

Remark 1. Since the performance and the correctness of the bit flipping algorithm depend on the density of the parity-check matrix, any equivalent parity-check matrix that respects the density condition enables a correct decoding algorithm.

III. MOUFEK, GUENDA AND GULLIVER'S CRYPTOSYSTEM

In [24] Moufek, Guenda and Gulliver proposed a McEliece type cryptosystem, based on LDPC and MDPC codes. The scheme is composed of three algorithms: key generation $\text{KeyGen}(\cdot)$, encryption $\text{Encrypt}(\cdot)$ and decryption $\text{Decrypt}(\cdot)$.

a) $\text{KeyGen}(n, k, t_1, t_2) = (\text{pk}, \text{sk})$:

- Pick a generator matrix G_1 of a $(n_1, k, \omega_{\text{ldpc}})$ LDPC code \mathcal{C}_1 that can correct t_1 errors, and a generator matrix G_2 of a $(n_2, k, \omega_{\text{mdpc}})$ MDPC code denoted \mathcal{C}_2 that can correct t_2 errors.
- Pick at random S in $\text{GL}_k(\mathbb{F}_2)$ and an $n \times n$ permutation matrix P , where $n = n_1 + n_2$.
- Compute $G_{\text{pub}} \stackrel{\text{def}}{=} SGP$, where $G = (G_1 \mid G_2)$.
- Return $\text{pk} = (G_{\text{pub}}, t_1, t_2)$ and $\text{sk} = (S, G_1, G_2, P)$.

b) $\text{Encrypt}(m, \text{pk}) = z$:

- Randomly generate $e = (e_1 \mid e_2) \in \mathbb{F}_2^n$ with $e_1 \in \mathbb{F}_2^{n_1}$, $e_2 \in \mathbb{F}_2^{n_2}$, $\text{wt}(e_1) = t_1$ and $\text{wt}(e_2) = t_2$.
- Compute $z = mG_{\text{pub}} + e$.

c) $\text{Decrypt}(z, \text{sk}) = m$:

- Compute $z^* = zP^{-1}$ and decode using the bit flipping algorithm for \mathcal{C}_1 and \mathcal{C}_2 . The output is $m^* \in \mathbb{F}_2^k$.
- Return the message m^*S^{-1} .

Remark 2. During the decryption, we have $z^* = mS(G_1 \mid G_2) + (e_1^* \mid e_2^*)$ with $(e_1^* \mid e_2^*) = eP^{-1}$ and $e_1^* \in \mathbb{F}_2^{n_1}$. The authors of this scheme propose to use the decoding capability of both LDPC and MDPC codes to find e_1^* and e_2^* . However, we emphasize that one can obtain $\text{wt}(e_1^*) > t_1$ or $\text{wt}(e_2^*) > t_2$. This can imply a failure in the decoding algorithm of \mathcal{C}_1 or \mathcal{C}_2 . But both situations cannot occur simultaneously since these would then imply that $\text{wt}(eP^{-1}) > t$. It is also important to remark that obtaining e_1^* or e_2^* is sufficient to recover mS and thus m .

IV. CRYPTANALYSIS OF THE MOUFEK, GUENDA AND GULLIVER'S SCHEME.

We propose here a key recovery attack against the previous cryptosystem. One of the key points in our attacks is the following proposition.

Proposition 1. *A parity check matrix of the public code is*

$$H' = HP = \begin{pmatrix} H_{\text{ldpc}} & 0 \\ 0 & H_{\text{mdpc}} \\ A & B \end{pmatrix} P,$$

where $H_{\text{ldpc}} \in \mathcal{M}_{n_1-k, n_1}(\mathbb{F}_2)$ is the low weight parity-check matrix of the LDPC code, $H_{\text{mdpc}} \in \mathcal{M}_{n_2-k, n_2}(\mathbb{F}_2)$ the low

weight parity-check matrix of the MDPC code, $(A \mid B) \in \mathcal{M}_{k, n_1+n_2}(\mathbb{F}_2)$ a full-rank matrix such that $G_1A^t + G_2B^t = 0$ and $A, B \neq 0$.

Proof. The proof is obvious since we have $H'G_{\text{pub}}^t = 0$. \square

This proposition shows the existence of a sufficient number of codewords with weights ω_{ldpc} in the dual of the public code. We represent this set of codewords by the matrix $H^* \in \mathcal{M}_{k^*, n_1+n_2}(\mathbb{F}_2)$, each row of H^* being an element of the set and conversely. Since these codewords of weights ω_{ldpc} contain the rows of $(H_{\text{ldpc}} \mid 0)P$, we have $\text{rank}(H^*) \geq n_1 - k$. One can expect to have $\text{rank}(H^*) > n_1 - k$ but our practical experiments always gave an equality, that is to say $\text{rank}(H^*) = n_1 - k$. In the sequel we can suppose that $k^* = n_1 - k$. If $k^* > n_1 - k$, one can select only $n_1 - k$ rows of H^* that are linearly independent.

The attack starts with a search for codewords of weights ω_{ldpc} .

a) **Search of codewords of weights ω_{ldpc}** : This step aims to find the rows of the matrix $H^* \in \mathcal{M}_{k^*, n_1+n_2}(\mathbb{F}_2)$. Assuming without loss of generality that an adversary knows the value of the parameter ω_{ldpc} , this step can be achieved by applying any of the ISD variants such as, for example, Dumer's algorithm [10]. This issue is discussed in detail in Section V, where a complexity analysis of our attack is given. We emphasize that ω_{ldpc} can be easily guessed during this step by starting with $\omega_{\text{ldpc}} = 1$ and increasing up to the value that satisfies the condition $\text{rank}(H^*) = n_1 - k$.

Remark 3. At the end of this step we have managed to build the matrix $H^* \in \mathcal{M}_{k^*, n_1+n_2}(\mathbb{F}_2)$ that generates the same code as $(H_{\text{ldpc}} \mid 0)P$ and therefore has n_2 zero columns.

Proposition 2. *There exists a permutation matrix P^* such that $H^*P^* = (H_1^* \mid 0)$ with $H_1^* \in \mathcal{M}_{k^*, n_1}(\mathbb{F}_2)$. Such a matrix can be computed with complexity $O(n)$ and satisfies*

$$\begin{aligned} \bullet & (H_{\text{ldpc}} \mid 0)PP^* = (H_{\text{ldpc}}P_1 \mid 0), \\ \bullet & PP^* = \begin{pmatrix} P_1 & 0 \\ 0 & P_2 \end{pmatrix}. \end{aligned}$$

P_1 and P_2 being permutation matrices of sizes n_1 and n_2 respectively.

Proof. The existence of P^* comes directly from Remark 3 which also provides $(H_{\text{ldpc}} \mid 0)PP^* = (H_{\text{ldpc}}P_1 \mid 0)$. Furthermore, this last equality with the fact that P_1 and PP^* are permutation matrices imply that $PP^* = \begin{pmatrix} P_1 & 0 \\ 0 & P_2 \end{pmatrix}$ where P_2 is also a permutation matrix. To finish, given the matrix H^* , a matrix P^* can be easily computed by identifying the n_2 zero columns of H^* . \square

This proposition shows that a cryptanalysis is able to find an alternative permutation matrix P^* together with a parity check matrix H_1^* of an equivalent LDPC code that can correct the same number of errors as the secret one. In the sequel, we are going to show that the pair (P^*, H_1^*) is sufficient to decode any cipher-text with a high probability.

b) **Decryption with (P^*, H_1^*) :** We show here the way to decrypt any cipher-text with P^* and H^* . Let $e^* = eP^* = (e_1^* | e_2^*)$ and $z^* = zP^* = (z_1^* | z_2^*)$ with $e_1^*, z_1^* \in \mathbb{F}_2^{n_1}$. So

$$z^* = (mG_{\text{pub}} + e)P^* = mS(G_1P_1 | G_2P_2) + (e_1^* | e_2^*).$$

This implies that $z_1^* = mSG_1P_1 + e_1^*$. Since SG_1P_1 generates an LDPC code with parity-check matrix H_1^* , we can recover e_1^* using the bit-flipping algorithm, assuming that $\text{wt}(e_1^*) \leq t_1$.

In the next paragraph we prove that the probability that $\text{wt}(e_1^*) \leq t_1$ is asymptotically close to 1 when the length of the codes goes to infinity.

Theorem 1. For $i \in \{1, 2\}$ let n_i, t_i be integers such that $t_i < n_i$ with $n_1 = \gamma n_2$ and $t_1 > \gamma t_2$, where $\gamma \geq 1$. Let $x = (x_1 | x_2)$ be a random vector over $\mathbb{F}_2^{n_1+n_2}$ with $\text{wt}(x) = t_1 + t_2$, where $x_i \in \mathbb{F}_2^{n_i}$ for $i \in \{1, 2\}$. Then we have

$$\Pr(\text{wt}(x_1) > t_1) < t_2 \frac{\binom{n_1}{t_1+1} \binom{n_2}{t_2-1}}{\binom{n_1+n_2}{t_1+t_2}}.$$

Proof. First notice that we have

$$\forall 0 \leq i \leq t_1 + t_2, \Pr(\text{wt}(x_1) = i) = \frac{\binom{n_1}{i} \binom{n_2}{t_1+t_2-i}}{\binom{n_1+n_2}{t_1+t_2}}. \quad (1)$$

Using the latter probability we obtain by a simple computation

$$\frac{\Pr(\text{wt}(x_1) = i)}{\Pr(\text{wt}(x_1) = i+1)} = \frac{(i+1)(n_2 - t_1 - t_2 + i + 1)}{(n_1 - i)(t_1 + t_2 - i)}.$$

Hence for any $i \geq t_1 + 1$ we obtain $\frac{\Pr(\text{wt}(x_1) = i)}{\Pr(\text{wt}(x_1) = i+1)} \geq \frac{(t_1 + 2)(n_2 - t_2 + 2)}{(n_1 - t_1)t_2}$. Replacing $n_1 = \gamma n_2$ and $t_1 > \gamma t_2$ in the latter fraction we obtain that

$$\forall i \geq t_1 + 1 \text{ we have } \Pr(\text{wt}(x_1) = i) > \Pr(\text{wt}(x_1) = i+1) \quad (2)$$

From (2) and (1) we deduce the desired result. \square

When t_1 and t_2 are linear in the code length we obtain the following asymptotic approximation

Corollary 1. Let $n_2 = n$, $n_1 = \gamma n$ and $t_1 = \alpha n$, $t_2 = \beta n$ with $\beta < \alpha \leq 1/2$. Then when $n \rightarrow \infty$ we have

$$\Pr(\text{wt}(x_1) > t_1) < c_{\alpha, \beta, \gamma} \sqrt{n} 2^{-n \left((\gamma+1)h(\alpha + \frac{\beta-\alpha}{\gamma+1}) - \gamma h(\alpha) - h(\beta) \right)},$$

where $c_{\alpha, \beta, \gamma}$ is a constant and h is the binary entropy function.

Proof. Apply the Stirling approximation for factorials and expand the series to obtain the result. \square

V. COMPLEXITY ANALYSIS AND NUMERICAL RESULTS.

a) **Complexity analysis:** The work factor of computing an alternative private key (P^*, H^*) is given by the work factor of the low weight codewords search algorithm plus the computation required to find P^* . The first step can be done using any of the ISD variants such as Dummer (D-ISD, [10]), May, Meurer and Thomae (MMT-ISD, [19]) or Becker, Joux, May and Meurer (BJMM-ISD, [5]) or May and Ozerov (MO-ISD [20]). These algorithms have a time

TABLE I

THE PROBABILITY $\Pr(\text{wt}(x_1) > t_1)$ FOR $t_1 = n_1/20$ AND $t_2 = n_2/40$ WITH $n_1 = 4k$ AND $n_2 = 2k$. THE FIRST ROW IS THE EXACT VALUE FOR THE PROBABILITY AND THE SECOND ROW IS THE UPPER BOUND FROM THEOREM 1, NAMELY $\text{ub}_{k, t_1, t_2} = t_2 \binom{n_1}{t_1+1} \binom{n_2}{t_2-1} / \binom{n_1+n_2}{t_1+t_2}$. THE THIRD ROW IS THE ASYMPTOTIC VALUE OF THE UPPER BOUND FROM COROLLARY 1, I.E. $\text{ub}_{k, \alpha, \beta, \gamma} = \sqrt{n} 2^{-n \left((\gamma+1)h(\alpha + \frac{\beta-\alpha}{\gamma+1}) - \gamma h(\alpha) - h(\beta) \right)}$.

k	6851	8261	9857
$\log_2(\Pr(\text{wt}(x_1) > t_1))$	-117	-141	-166
$\log_2(\text{ub}_{k, t_1, t_2})$ Thm1	-110	-133	-159
$\log_2(\text{ub}_{k, \alpha, \beta, \gamma})$ Cor1	-106	-129	-155

complexity equal to $O(e^{-\omega \ln(1-k/n)(1+o(1))})$, as long as $\omega = o(n)$ when $n \rightarrow \infty$ [33]. Hence, in our case computing the matrix H^* requires a work factor asymptotically equal to $O(e^{-\omega_{\text{ldpc}} \ln(k/(n_1+n_2))(1+o(1))})$, when $n_1 + n_2$ tends to infinity. Regarding the complexity of computing P^* , it requires $n_1 + n_2$ basic operations (here we consider binary additions of length k binary vectors). Thus the complexity of our attack, denoted by $\text{WF}_{\mathcal{A}}(n_1, n_2, k, \omega_{\text{ldpc}})$ is in the worst case dominated by the cost of the best ISD variant.

b) **Numerical results:** We analyze the effective cost of our attack on some practical parameters. Firstly we considered suggested values given in [24], more precisely $n_1 + n_2 = 16128$. As for the co-dimension we analyzed three different cases, $n_1 + n_2 - k \in \{8064, 10080, 12096\}$. Hence the complexity of finding the codewords of weight 15 in the dual of the public code (i.e. $\omega_{\text{ldpc}} = 15$), using the BJMM-ISD variant equals $2^{6.13}, 2^{10.97}$ respectively $2^{19.20}$. The computations were done using a PariGP implementation similar to that in [27]. The probability $\Pr(\text{wt}(x_1) > t_1)$ (for these parameters) is given in Table I. These results show that the parameters proposed in [24] are too vulnerable to be considered in practice.

One might generate a more resistant set of parameters for the scheme, which fact we illustrate in Table II. However, it is important to consider the weak keys approach, a recent technique introduced in [4], where the authors use the Extended Euclidean algorithm in order to recover a private key given a public key. We compute the probability of weak keys for the Moufek et al. variant with reasonable parameters in Table II.

TABLE II

THE PROPORTION OF WEAK KEYS AND THE COMPLEXITY OF OUR ATTACK AGAINST THE MOUFEK ET AL. MCÉLIECE VARIANT USING A $(4k, k, \omega_{\text{ldpc}})$ LDPC CODE AND A $(2k, k, \omega_{\text{mdpc}})$ MDPC CODE.

$(k, \omega_{\text{ldpc}})$	(6851, 36)	(8261, 44)	(9857, 54)
$\log(\text{WF}_{\mathcal{A}}(4k, 2k, k, \omega_{\text{ldpc}}))$	80.9	101.6	127.4
Proportion of weak keys	$2^{-7.3}$	$2^{-10.6}$	2^{-14}

Remark 4. In the first place we remark that the key size for the [24] scheme is considerably greater than for similar schemes such as [1], [23].

Notice from Table II that the odds of generating weak keys are too big to imagine that the scheme can be protected against this type of attack. Indeed, we find that the scheme might be secured against the weak keys approach by increasing the values of the ω_{ldpc} . But in order to obtain reasonable secure parameters this solution is equivalent to replacing the LDPC

code with an MDPC code and thus is of no interest compared with the MDPC McEliece variant [23].

Remark 5. It is also worth mentioning the recent reaction attacks against the QC-MDPC scheme [13] and the QC-LDPC scheme [15]. This technique can be used to recover the structure of the MDPC code in the case of Guenda's et al. variant. Nonetheless we remark from Theorem 1 and Corollary 1 that the weight of x_2 is likely to be much bigger than the error capacity of the MDPC code. Hence, one might not be able to retrieve the initial message unless it uses the LDPC code.

However, the reaction attack remains highly interesting in similar constructions, namely in the case of the direct sum or Plotkin sum of LDPC and MDPC codes. Indeed, in these cases attacking the LDPC code with our technique is not sufficient for retrieving the initial message, and thus revealing the structure of the MDPC code is necessary.

VI. CONCLUSION

We have proposed a successful cryptanalysis of the McEliece variant in [24]. Our attack exploits the structure of the dual of the public code and its complexity is dominated by the low weight search algorithm on this dual. The attack is entirely based on finding the structure of the LDPC code, regardless of the nature of the second code. As a consequence, our result can be applied even if the MDPC code is replaced by another code.

We notice that this variant is also vulnerable to the weak keys approach [4], since the proportion of weak keys is not negligible. Hence, one can consider that the McEliece variant [24] is too vulnerable to be practical. We also emphasize that similar constructions, such as the direct sum or Plotkin sum of MDPC and LDPC codes can be attacked by combining our technique with the latest reaction attacks.

REFERENCES

- [1] Marco Baldi. *QC-LDPC Code-Based Cryptography 2014*. Springer, Berlin, Heidelberg.
- [2] Razvan Barbulescu, Pierrick Gaudry, Antoine Joux, and Emmanuel Thomé. A heuristic quasi-polynomial algorithm for discrete logarithm in finite fields of small characteristic. In *Nguyen, P.Q. Oswald, E. (eds.) EUROCRYPT 2014 LNCS*, vol. 8441, pp. 1–16, Springer, Berlin, Heidelberg.
- [3] Magali Bardet, Julia Chautet, Vlad Dragoi, Ayoub Otmani, and Jean-Pierre Tillich. Cryptanalysis of the McEliece public key cryptosystem based on polar codes. In *Post-Quantum Cryptography 2016*, LNCS, Fukuoka, Japan. Springer International Publishing, Cham.
- [4] Magali Bardet, Vlad Dragoi, Jean-Gabriel Luque, and Ayoub Otmani. Weak keys for the quasi-cyclic MDPC public key encryption scheme. In *AFRICACRYPT 2016, Fes, Morocco, Proceedings*, pages 346–367. Springer International Publishing, Cham.
- [5] Anja Becker, Antoine Joux, Alexander May, and Alexander Meurer. Decoding random binary linear codes in $2^{n/20}$: How $1+1=0$ improves information set decoding. In *EUROCRYPT 2012*, LNCS Springer, Berlin, Heidelberg.
- [6] Daniel J. Bernstein, Johannes Buchmann, and Erik Dahmen, editors. *Post-Quantum Cryptography*. Springer-Verlag, 2009.
- [7] Julia Chautet and Nicolas Sendrier. Worst case QC-MDPC decoder for McEliece cryptosystem. In *IEEE Int. Symposium on Inf. Theory, ISIT 2016, Barcelona, Spain*, pages 1366–1370.
- [8] Alain Couvreur, Philippe Gaborit, Valérie Gauthier-Umaña, Ayoub Otmani, and Jean-Pierre Tillich. Distinguisher-based attacks on public-key cryptosystems using Reed-Solomon codes. *Des. Codes Cryptogr.*, 73(2):641–666, 2014.
- [9] Alain Couvreur, Irene Márquez-Corbella, and Ruud Pellikaan. A polynomial time attack against algebraic geometry code based public key cryptosystems. In *Proc. IEEE Int. Symposium Inf. Theory - ISIT 2014*, pages 1446–1450.
- [10] Ilya Dumer. On minimum distance decoding of linear codes. In *Proc. 5th Joint Soviet-Swedish Int. Workshop Inform. Theory*, pages 50–52, Moscow, 1991.
- [11] R. G. Gallager. *Low Density Parity Check Codes*. M.I.T. Press, Cambridge, Massachusetts, 1963.
- [12] Cheikh Thiécoumba Gueye and El Hadji Modou Mboup. Secure cryptographic scheme based on modified Reed Muller codes. *Int. Journal of Security and Its Applications*, 7(3):55–64, 2013.
- [13] Guo Qian and Johansson Thomas and Stankovski Paul. A key recovery attack on MDPC with CCA security using decoding errors, *ASIACRYPT 2016*, Vietnam, 2016, Proceedings, Part I 22, pages 789–815, Springer, Berlin, Heidelberg.
- [14] Heeralal Janwa and Oscar Moreno. McEliece public key cryptosystems using algebraic-geometric codes. *Des. Codes Cryptogr.*, 8(3):293–307, 1996.
- [15] Tomáš Fabšič, Viliam Hromada, Paul Stankovski, Pavol Zając, Qian Guo, and Thomas Johansson. A reaction attack on the QC-LDPC McEliece cryptosystem. In *Post-Quantum Cryptography 2017, Utrecht, The Netherlands*, pages 51–68, Springer, Cham.
- [16] Grégory Landais and Jean-Pierre Tillich. An efficient attack of a McEliece cryptosystem variant based on convolutional codes. In P. Gaborit, editor, *Post-Quantum Cryptography'13*, volume 7932 of LNCS, pages 102–117, June 2013. Springer, Berlin, Heidelberg.
- [17] Carl Löndahl and Thomas Johansson. A new version of McEliece PKC based on convolutional codes. In *Inf. and Communications Security, ICICS, 2012*, volume 7168 of LNCS, pages 461–470. Springer, Berlin, Heidelberg.
- [18] Irene Márquez-Corbella and Jean-Pierre Tillich. Using Reed-Solomon codes in the $(u|u+v)$ construction and an application to cryptography. In *Proc. IEEE Int. Symposium Inf. Theory - ISIT*, pages 930–934, 2016.
- [19] Alexander May, Alexander Meurer, and Enrico Thomae. Decoding random linear codes in $O(2^{0.054n})$. In *ASIACRYPT 2011*, volume 7073 of LNCS, pages 107–124. Springer, Berlin, Heidelberg.
- [20] Alexander May and Ilya Ozerov. On computing nearest neighbors with applications to decoding of binary linear codes. In *EUROCRYPT 2015*, volume 9056 of LNCS, pages 203–228. Springer, Berlin, Heidelberg.
- [21] Robert J. McEliece. *A Public-Key System Based on Algebraic Coding Theory*, pages 114–116. Jet Propulsion Lab, 1978. DSN Progress Report 44.
- [22] Lorenz Minder and Amin Shokrollahi. Cryptanalysis of the Sidelnikov cryptosystem. In *EUROCRYPT 2007*, volume 4515 of LNCS, pages 347–360, Barcelona, Spain. Springer, Berlin, Heidelberg.
- [23] Rafael Misoczki, Jean-Pierre Tillich, Nicolas Sendrier, and Paulo S. L. M. Barreto. MDPC-McEliece: New McEliece variants from moderate density parity-check codes. In *Proc. IEEE Int. Symposium Inf. Theory - ISIT*, pages 2069–2073, 2013.
- [24] Hamza Moufek, Kenza Guenda, and T Aaron Gulliver. A new variant of the McEliece cryptosystem based on QC-LDPC and QC-MDPC codes. *IEEE Communications Letters*, 21(4):714–717, 2017.
- [25] Harald Niederreiter. Knapsack-type cryptosystems and algebraic coding theory. *Problems of Control and Information Theory*, 15(2):159–166, 1986.
- [26] Ayoub Otmani and Hervé Talé Kalachi. Square code attack on a modified sidelnikov cryptosystem. In *Codes, Cryptology, and Information Security 2015*, pages 173–183. Springer, Cham.
- [27] Christiane Peters. Information-set decoding for linear codes over \mathbb{F}_q . In *Post-Quantum Cryptography 2010*, volume 6061 of LNCS, pages 81–94. Springer, Berlin, Heidelberg.
- [28] Nicolas Sendrier. On the concatenated structure of a linear code. *Appl. Algebra Eng. Commun. Comput. (AAECC)*, 9(3):221–242, 1998.
- [29] P.W. Shor. Algorithms for quantum computation: Discrete logarithms and factoring. In S. Goldwasser, editor, *FOCS*, pages 124–134, 1994.
- [30] Sujan Raj Shrestha and Young-Sik Kim. New McEliece cryptosystem based on polar codes as a candidate for post-quantum cryptography. In *2014 14th Int. Symp. on Comm. and Inf. Tech. (ISCIT)*, pages 368–372. IEEE, 2014.
- [31] Vladimir Michilovich Sidelnikov. A public-key cryptosystem based on Reed-Muller codes. *Discrete Math. Appl.*, 4(3):191–207, 1994.
- [32] Vladimir Michilovich Sidelnikov and S.O. Shestakov. On the insecurity of cryptosystems based on generalized Reed-Solomon codes. *Discrete Math. Appl.*, 1(4):439–444, 1992.

- [33] Rodolfo Canto Torres and Nicolas Sendrier. Analysis of information set decoding for a sub-linear error weight. In *Post-Quantum Cryptography 2016, Fukuoka, Japan, February 24-26, Proceedings*, pages 144–161. Springer, Cham.
- [34] Yongge Wang. Quantum resistant random linear code based public key encryption scheme rlce. In *Proc. IEEE Int. Symposium Inf. Theory - ISIT*, pages 2519–2523, 2016.
- [35] Christian Wieschebrink. Two NP-complete problems in coding theory with an application in code based cryptography. In *Proc. IEEE Int. Symposium Inf. Theory - ISIT*, pages 1733–1737, 2006.