

## 2 TGR homeworks — October 10th, 2018

**2.1** Prove or disprove:

*Given a simple undirected graph  $G$  without loops and with  $n > 2$  vertices. If the smallest degree  $\delta(G)$  satisfies*

$$\delta(G) \geq \frac{n-1}{2},$$

*then  $G$  is connected.*

**2.2** Given a simple undirected graph  $G$  without loops. Prove or disprove:

*If  $G$  has radius at most  $k$  and the greatest degree  $\Delta(G)$  is at least 3, then  $G$  had less than*

$$\frac{d}{d-2}(d-1)^k$$

*vertices, where  $d = \Delta(G)$ .*

**2.3** Let  $G$  be a simple undirected graph without loops with  $\delta(G) > 1$  ( $\delta(G)$  is the smallest degree of a vertex in  $G$ ). Prove or disprove:

1. *In  $G$  there exists a path of length  $\delta(G)$  (i.e. a path with  $\delta(G)$  edges).*
2. *In  $G$  there exists a circuit of length at least  $\delta(G) + 1$ .*

Hence, either prove the above assertion, or find contra examples.