

1 TGR homeworks — October 3rd, 2018

1.1 Find an example of a simple graph G without loops which is smallest among those that satisfy following properties: They have 6 vertices of degree 3, other vertices have degree at most 2, and they have 12 edges.

Prove that your example is the smallest one.

1.2 Given a simple undirected graph $G = (V, E)$ without loops and with n vertices. Prove or disprove:

$$\sum_{v \in V} d(v)^2 = \sum_{\{x, y\} \in E} (d(x) + d(y)).$$

1.3 Let G be a simple connected undirected graph without loops with $n > 2$ vertices. Denote by C_1 a longest path, and C_2 a next longest path in G (they might be of the same length). Prove or disprove:

1. C_1 and C_2 share a common vertex.
2. C_1 and C_2 share a common edge.

Hence, either prove the above assertion, or find a contra example.