7 TGR homeworks — November 14th, 2018

7.1 Given a tree G with $n \geq 4$ vertices such that every vertex of degree greater than 1 has the same degree d. For which pairs of numbers n and d such tree exists? In the case such tree exists, determine the number of vertices of degree 1 of G (with respect to n and d).

Thoroughly justify your answer. Draw at least two such trees for $d > 2$ and $n > 7$.

7.2 Given a simple undirected graph G without loops, having n vertices, m edges and k components of connectivity.

Prove or disprove:

a) $n - k \leq m \leq \frac{(n-k)(n-k+1)}{2}$.

b) If G does not have circuits of odd length then

$$n - k \leq m \leq \begin{cases} \frac{(n-k)(n-k+2)}{2} & \text{for } n-k \text{ even;} \\ \frac{(n-k+1)^2}{2} & \text{for } n-k \text{ odd.} \end{cases}$$

7.3 Prove or disprove: *Given an k edge connected simple graph without loops which has n vertices and m edges. Then it holds that*

$$m \geq \frac{k \cdot n}{2}.$$