10 TGR homeworks — December 5th, 2018

10.1 Prove that in every undirected graph G it holds that

$$\alpha_1(G) \leq \beta_0(G),$$

where $\alpha_1(G)$ is the number of edges contained in a maximum matching in G, and $\beta_0(G)$ is the number of vertices a vertex cover of G with the least number of vertices has.

10.2 Prove that in every bipartite graph G it holds that

$$\alpha_1(G) = \beta_0(G),$$

Hint: Use the König theorem for the number of edges in a maximum matching in a bipartite graph.

10.3 Let G be a simple undirected graph without loops. Prove that in some optimal coloring (i.e. a coloring with $\chi(G)$ colors) every neighborough $N(v)$ of any vertex v has the same color if and only if G is a bipartite graph.