11 TGR homeworks — December 12, 2018

11.1 Let $G = (V, E)$ be a connected undirected graph with the set ofedges E. A subset of edges $S \subseteq E$ is called a*mincut* if it is a minimal set of edges of G such that $G \setminus S$ is disconnected.

Let V_1, V_2 be two non-empty disjoint sets of vertices such that $V_1 \cup V_2 = V$. Then the *cut* $\langle V_1, V_2 \rangle$ is the set of edges $e = \{u, v\}$ for which $u \in V_1$ and $v \in V_2$.

Prove or disprove:

Every cut $\langle V_1, V_2 \rangle$ of a connected undirected graph G is a union of mincuts that are pairwise disjoint (i.e. no two mincuts share a common edge).

11.2 Let G be a connected undirected graph with the set of edges E.

Prove or disprove:

A subset $S \subseteq E$ is a mincut if and only if it si a minimal set of edges satisfying: for every spanning tree T of G we have $S \cap T \neq \emptyset$.

11.3 Let G be a connected undirected graph with the set of edges E. Denote by W_K the curcuits space and W_R the cut space of the graph G.

Prove or disprove:

A subset $S \subseteq E$ belongs to W_R if and only if for every K in W_K we have $S \cap K$ contains an even number of edges.