5 TGR homeworks — October 31st, 2018

5.1 Prove or disprove: *Given a connected simple undirected graph G without loops with $n > 3$ vertices. Assume that G does not contain $K_{1,3}$ Then there are vertices x and y in G joined by an edge and such that $G \setminus \{x, y\}$ is also connected. ($G \setminus \{x, y\}$ is the subgraph of G where both vertices x and y are removed., not only the edge $\{x, y\}$.)*

($K_{1,3}$ is the complete bipartite graph with sides with 1 and 3 vertices).

5.2 Given a connected simple undirected graph $G = (V, E)$ without loops with $n \geq 3$ vertices. Let x and y be two vertices of G for which $\{x, y\} \notin E$ and $d(x) + d(y) \geq n$.

Prove or disprove: *G contains a Hamiltonian circuit if and only if so does $G + \{x, y\}$.*

($G + \{x, y\}$ has the same set of vertices, and the sef of edges is $E \cup \{\{x, y\}\}$.)

5.3 Prove or disprove: *Given a simple undirected graph G without loops and with $n > 2$ vertices. Assume that G satisfies the following condition:*

$$\textit{for every } \{u, v\} \notin E(G) \textit{ we have } d(u) + d(v) \geq n,$$

then the sequence of degrees satisfies

$$\textit{for every } 1 \leq k < \frac{n}{2} \textit{ it is } d_k > k.$$

(We assume that the sequence of degrees is non-decreasing, i.e. that $d_1 \leq d_2 \leq \ldots \leq d_n$.)