# 4 TGR homeworks — October 24th, 2018

**4.1** Prove or disprove:
*A directed graph is strongly connected if and only if it can be reprezented as a union of cycles $C_1, C_2, \ldots, C_k$ such that every two cycles $C_i$ a $C_{i+1}$ , $i = 1, \ldots, k-1$, share a common vertex.*

Either prove the statement, or find an contra example.

**4.2** Given a simple directed graph without loops and with $n$ vertices and $m$ edges. Prove or disprove:
*If $G$ is connected but not strongly connected then*

$$n - 1 \leq m \leq (n-1)^2.$$

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**4.3** Prove: *Every undirected graph $G$ without loops have an orientation which is an acyclic graph.*

**4.4** Find an example of a simple directed graph without loops with the smallest number of vertices that has two transitive reductions of different number of edges.

Justify why it has the smallest possible number of vertices.