Understanding and Forecasting Stock Market Volatility through Wavelet Decomposition, Statistical Learning and…

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Understanding and Forecasting Stock Market Volatility through Wavelet Decomposition, Statistical Learning and Econometric Methods

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Abstract: Volatility in stock markets evokes varying responses from market participants. While some perceive it as opportunity to make money, others perceive it as a threat and start unwinding their positions. In today's globalized environment, increased volatility reflects not only the domestic macroeconomic state, but also global uncertainty. While volatility in the stock market as a whole can be influenced by events like oil price shocks, increase in rates of interest in the US and domestic elections, volatility in individual stock prices can be due to perceived growth prospects of the company or the sector, company specific news or policy announcements that can affect a company/sector. In this study, associations, causal influence among three volatility indicators namely, CBOE VIX, INDIA VIX and Historic Volatility (HV) have been carefully examined, and predictive models for forecasting have been developed. An integrated framework incorporating Wavelet decomposition, statistical predictive modeling and standard econometric methods is presented to accomplish the research objectives.

Keywords: INDIA VIX, CBOE VIX, HV, Discrete Wavelet Transformation, Haar Wavelet, ARIMA, ANN, Random Forest, Volatility Spillover, Granger Causality Test, Vector Auto Regression.

1. INTRODUCTION

The global environment today is financially integrated, and progress in information technology has enabled any investor anywhere to invest in financial assets of other countries. This global spread in investments is also present in financial institutions. While this has led to portfolio diversification and opportunities for enhancement of returns, it has also exposed economies to volatility in other economies. In particular, foreign institutional investors (FIIs) are large players in the Indian stock market, and any macroeconomic event in any part of the world causes reallocation of FII funds, leading to volatility in the Indian stock market. Given today's environment, it is thus all the more important to focus on volatility, understand the reasons behind it, and also try to generate efficient forecasts. It is this context that Datta Chaudhuri and Ghosh (2015a, 2015b) explored the behavior pattern of Indian stock market volatility using clustering method and tried forecasting in Neural Network framework. The basic contention was that before forecasting, it is important to understand the volatility patterns. If there exits too many clusters in time series data, then the data is random and hence forecasting may not be very dependable. The paper also looked at the explanatory variables of volatility and arrived at an optimum number of inputs. There exits two measures of volatility namely Historic Volatility (HV) and Implied Volatility (IV). The former is derived from past data of stock market returns and is backward looking. The latter is derived from the Black and Scholes options pricing model and is the volatility that is expected to prevail in the near future as implied by the option price. We use both these variables in our analysis, along with Implied Volatility measure in the US Market (CBOE VIX) to incorporate financial integration. In this paper, the three volatility indicators namely HV, INDIA VIX (IV) and CBOE VIX are initially decomposed into approximate and detail components that pick up the low and high frequency components of the respective time series. These components are also referred as trend and cyclic components. These components reflect different frequency bands corresponding to short run, medium run and long run time periods. Finally the low and high frequency components are reconstructed via inverse Discrete Wavelet Transformation (DWT).

Rest of this article is structured as follows. Section 2 highlights the overall objective of the research and methodologies adopted for accomplishment. Survey of related literature has been summarized in Section 3. Wavelet decomposition and subsequent analysis of volatility indicators are narrated in Section 4. Results of test of associations among INDIA VIX, HV and CBOE VIX on decomposed components are presented in Section 5. Predictive modelling framework and the results are explained in details in Section 6. Subsequently in Section 7 causal interactions among volatility indicators have been critically examined and an alternative prediction framework is presented. Section 8 concludes paper.

2. OBJECTIVE OF THE STUDY

In this paper we apply wavelet analysis to understand stock market volatility. If we look at the overall pattern of implied volatility in the Indian stock market as given by INDIA VIX in Figure 1, we can observe that, over time, there are some steady periods with occasional spikes in the data.

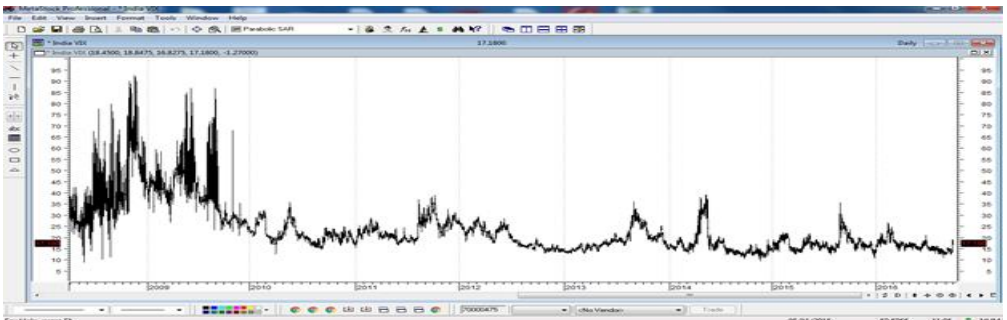

Figure 1: Movement of INDIA VIX

Wavelet analysis allows us to decompose the overall series into various components to understand the dominant components in the time series. For forecasting efficiency, we decompose the time series data, forecast component wise, and then aggregate the forecast values to arrive at the overall figure. As discussed, assessment of degree of association among the volatility measures and constructing forecasting frameworks for predicting future figures of them are the key objectives of this research. Observations for the three indicators, CBOE VIX, INDIA VIX and HV for the time period March 3, 2008 to June 16, 2016 have been collected for analysis. Initially all three indicators have been decomposed via DWT using Haar wavelets into different frequency components (high and low) to understand the dynamics of movements. All subsequent analysis are carried out on these components. Pearson correlation coefficient measures and cross correlation values are computed to quantify and understand the degree of association among the indicators of volatility. Next, using Random Forest and Auto Regressive Integrated Moving Average models as statistical learning tools, attempt has been made to forecast future values of volatility indicators. Granger causality tests in Vector Auto Regression environment are conducted among the high frequency components (high volatile parts) of CBOE VIX, INDIA VIX and HV to analyze the causal nexus among them and a predictive model, composed of Random Forest and Multi-Layer Feedforward Neural Network, is presented.

3. RELATED WORK

Research on stock market volatility has used traditional econometric methods like ARCH (Autoregressive Conditional Heteroskedastic) and GARCH (Generalized Autoregressive Conditional Heteroskedastic) models for analysis. However, advanced machine learning techniques like wavelet analysis has also been used as reported in literature. Panda and Deo (2014) studied the volatility spillover effect between rupee-dollar exchange rate and CNX Nifty returns during the 2008 financial crisis using GARCH and EGARCH (Exponential GARCH) models. Study of Birau et al. (2015) observed volatility shocks and clustering in S&P Bombay Stock Exchange BANKEX index over the period January 2002 to June 2014 using standard GARCH model. Erdemir (2012) applied Symlet-8 wavelet for decomposition and forecasting S&P Index Options Implied Volatility. Ramsey and Zhang (1997) applied Wavelet decomposition to examine the foreign exchange rate dynamics. Study made by Gençay et al. (2001) showed the effectiveness of Wavelet filtering to deal with nonstationary and time varying features of time series. Khandelwal et al. (2015) applied ARIMA and Artificial Neural Networks (ANN) models on decomposed time series data using DWT for efficient forecasting. Barragan et al. (2013) attempted to study spillover effect of shocks in oil market to stock market indexes of Germany, Japan, UK and USA during the period 1990-2011 which includes Iraqi invasion of Kuwait, OPEC outbreak using Wavelet decomposition and test for contagion. Lu et al. (2016) examined the volatility of Chinese energy market using hybrid ANN and GARCH type models and Exponential GARCH (EGARCH) was found to be the most prominent one among all models. Efimova and Serletis (2014) deployed univariate and multivariate GARCH models to study the spillovers and interactions among oil, natural gas and electricity price in USA for the period from 2001 to 2013. Vegendla and Enke (2013) carried out a comparative study of Multilayer Feedforward Neural Network, Recurrent Neural Network and GARCH models for forecasting historic volatility, implied volatility and model based volatility of NASDAQ, DJIA, NYSE and S & P 500. Jothimani et al. (2016) presented a novel framework combining maximal overlap Discrete Wavelet Transformation and Artificial Neural Network and Support Vector Regression to forecast the National Stock Exchange fifty index from September 2007 to July 2015. Lahmiri (2014) used DWT and Back propagation neural network for predictive modelling of S&P 500 price index and stock prices of Apple, Dell, Hewlett–Packard, IBM, Microsoft and Oracle from February 28th, 2011 to March 11th, 2011. The proposed methodology has been found outperforming traditional Auto Regressive Moving Average (ARMA) and Random Walk model. Study of Gençay et al. (2005) emphasized the non-stationarity and asymmetry of volatility using wavelet transformation methods. Lee (2004) observed and measured volatility spillover effects of developed stock market (USA) to emerging stock market (Korea). Liu et al. (2017) explore the volatility spillover between oil and stock market using wavelet based GARCH-BEKK model. Their research investigated the spillovers effect of WTI crude oil prices on S&P 500 index and MICEX index during the period Jan, 2003 to December 2014 which was further subdivided into pre-crisis, crisis and post-crisis sub periods. Results suggest that linkage between oil and S&P 500 index weakens in long term whereas exact reverse phenomenon is observed between MICEX index and oil market i.e. strong linkage during entire time period.

4. WAVELET DECOMPOSITION

Recently, there has been significant use of Wavelets (Discrete Wavelet Transformation (DWT), Wavelet Dictionary, Wavelet Filtering) in understanding time series data in finance where the original series can be segregated into linear (detail part) and nonlinear (approximation part) components, while preserving orthogonality to carry out multiresolution analysis. It is a novel filtering technique that separates nonlinearity and presence of other erratic features in financial time series without affecting inherent features such as spillovers, clustering and heteroscedasticity. Gençay et al. (2001) showed the effectiveness of Wavelet filtering to deal with nonstationary and time varying features of time series. To decompose a financial time series into orthogonal components, Wavelet transformation represents the original series as a set of superimposed wavelets. To meet the goal, the original function ($f(t)$) is translated and dilated onto father ($\varphi(t)$) and mother ($\psi(t)$) wavelets at different scales. The mother ($\psi(t)$) wavelets represent the details having unit energy and zero mean whereas the father ($\varphi(t)$) defines the approximations with mean value of one. Mathematically it can be expressed as:

$\int \varphi(t)dt = 1$ ………………….. (1)

$\int \psi(t)dt = 0$ ………………….. (2)

If r_{it} denotes the value of time series *i* at time *t*, then the time series is approximated using the detail coefficients $\psi_{J,k}^{r_i}$ and approximation coefficients $\varphi_{J,k}^{r_i}$ as given by equation 3.

$r_{it} = a_J^{r_i}(t) + d_J^{r_i}(t) + d_{J-1}^{r_i}(t) + \cdots + d_1^{r_i}(t)$ ………….. (3)

Where $a_J^{r_i}(t) = \sum_k \varphi_{J,k}^{r_i} \phi_{J,k}(t)$ ………………… (4)

$d_j^{r_i}(t) = \sum_k \psi_{J,k}^{r_i} \omega_{j,k}(t)$ ………………… (5)

$\phi_{J,k}(t) = 2^{-J/2}\phi\left(\frac{t-2^J k}{2^J}\right)$ ……………... (6)

$\omega_{j,k}(t) = 2^{-j/2}\omega\left(\frac{t-2^{j}k}{2^{j}}\right)$ ……………… (7)
For j=1, 2, ….., J.
As discussed $a_J^{r_i}(t)$ and $D_j^{r_i}(t)$ are known as approximation and detail levels respectively. The discrete wavelet transform of any function *f(t)* is expressed as
$\widetilde{W_{l,k}} = \int_{-\infty}^{\infty} f(t)\psi_{l,k}(t)\,dt$ ……………………… (8)
The inverse wavelet transform is defined as:
$f(t) = \sum_{l=-\infty}^{\infty}\sum_{k=-\infty}^{\infty}\widetilde{W_{l,k}} \times \psi_{l,k}(t)$ ……………. (9)
In this study, five detail levels $(d_5(t),\ d_4(t),\ d_3(t), d_2(t),\ d_1(t))$ and one approximation level $(a_5(t))$ have been considered for decomposition and subsequent multilevel resolution analyses. Figure 2, 3 and 4 represent the visualization of DWT process for decomposing the respective time series into appropriate scales. Components of lower frequency range prevail for longer periods while higher frequency components prevail for shorter durations. Mapping of the scales to their corresponding time domain characterizations are shown in the following table.

Table 1: Time Scales

Scale	Frequency
Approximation (A5)	3rd March, 2008 to 16th June, 2016
Detail Scale 1 (D1)	Up to 15 months
Detail Scale 2 (D2)	Up to 30 months
Detail Scale 3 (D3)	Up to 45 months
Detail Scale 4 (D4)	Up to 60 months
Detail Scale 5 (D5)	Up to 75 months

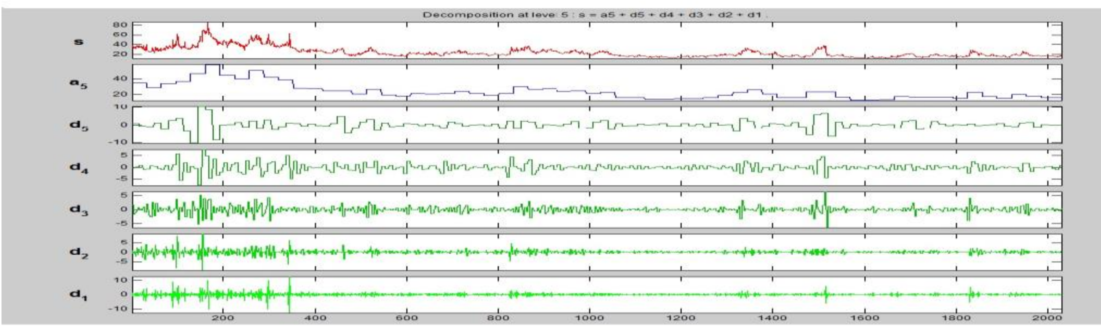

Figure 2: Wavelet decomposition of INDIA VIX
Approximation scale accounts for the low frequency component i.e. relatively stable part of the volatility of entire time span considered in this study. Clear presence of detail scales during the entire time periods can be observed in INDIA VIX.

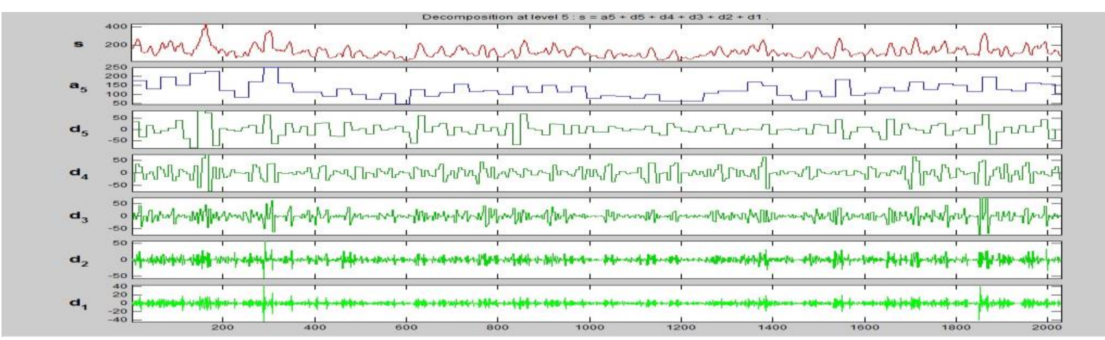

Figure 3: Wavelet decomposition of INDIA VIX
In case of HV, visually it is quite apparent that the magnitude of detail scales in compared to detail scales of INDIA VIX is high. Distinct presence of detail components throughout the entire duration may infer the presence of volatility clustering.

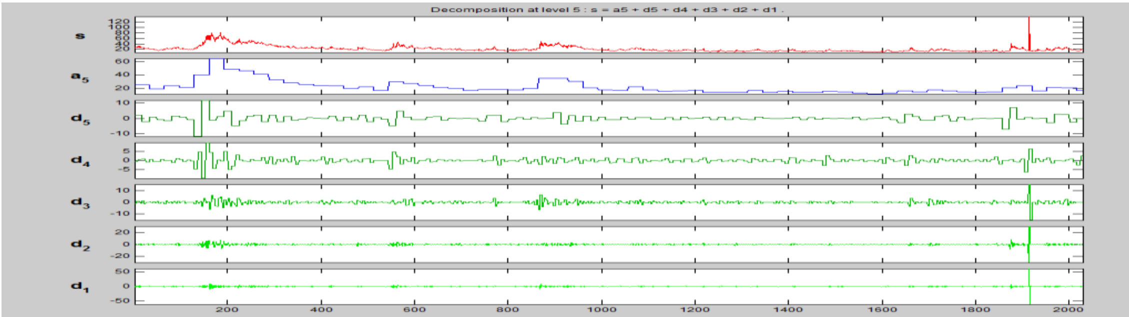

Figure 4: Wavelet decomposition of Volatility of CBOE VIX

Approximation part of CBOE VIX is found to be more dominant in compared to detail levels as they are almost faded out. So inference may be drawn that the movement CBOE VIX is not as highly volatile as of INDIA VIX and HV. This paper studies Scale-by-Scale interrelationship among CBOE VIX, INDIA VIX and HV. To gain deeper insights of decomposition, energy levels of the respective time series and their detail and approximation levels have been computed. It basically measures how much of variance original series is explained by the respective decomposed levels. Table 2 shows the results.

Table 2: Energy Levels

Energy (%)	Approximation Level (A5)	Detail Level 5 (D5)	Detail Level 4 (D5)	Detail Level 3 (D5)	Detail Level 2 (D5)	Detail Level 1 (D5)
INDIA VIX	98.03%	0.19%	0.19%	0.23%	0.43%	0.92%
HV	90.78%	0.15%	0.49%	1.52%	2.85%	4.21%
CBOE VIX	96.83%	0.82%	0.55%	0.46%	0.48%	0.86%

Energy levels further conform to the visual pattern of wavelet decomposition of respective volatility indicators. Detail scales of HV have been found to be very prominent in Figure 4, which according to energy level analysis are responsible for around 9.22% variance of HV.

5. Study of Association

To critically examine the nexus among three volatility indicators, we have computed Pearson's Correlation Coefficient and Cross Correlation Function (CCF) for checking correlations at different lags to carry out systematic test of association. Both the computations are made on each scale obtained through DWT since they are orthogonal to each other implying the nonexistence of overlapping information in them.

5.1 Association Test: Correlation test has been performed on decomposed components of different scales of three volatility indicators involved in this study in pairwise manner to assess point-to-point degree of association. Cross correlation test to measure the degree of association at different time lags has been performed too. Here, we have outlined the pair of scales and their correlation coefficient values which are statistically significant.

Table 3: Significant Correlations between pairs

Pairs	Correlation Coefficient	Significance
Approximation Levels of CBOE VIX and INDIA VIX	0.817	**
Approximation Levels of CBOE VIX and HV	0.537	**
Detail Level 5 of INDIA VIX and HV	0.455	**

** Significant at 5% level

However mere study of association through simple point wise correlation test cannot provide presence and nature of correlations at different lags. To overcome the shortcomings, Cross Correlation plots, generated by applying *ccf()* function in R programming language as adopted in the study of Sen and Datta Chaudhuri (2016), for the pairs having significant associations are shown below. Interestingly it has been noticed that some of the pairs of indicators exhibit weak negative association at different lags which further justify its usage in assessing association.

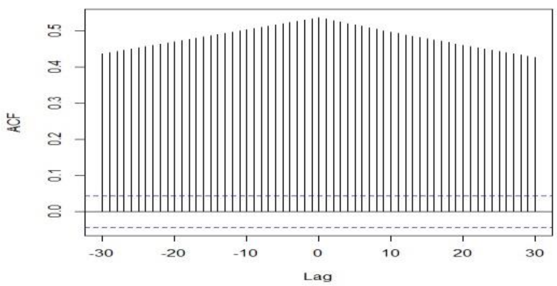

Figure 5: Cross Correlation between INDIAVIX A5 & HV A5

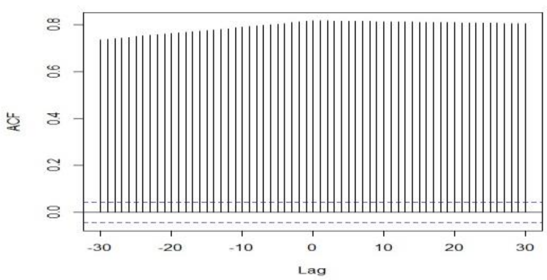

Figure 6: Cross Correlation between CBOEVIX A5 & INDIAVIX A5

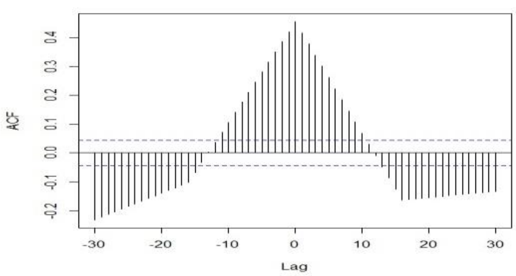

Figure 7: Cross Correlation between INDIA VIX (A5) & HV (A5)

6. PREDICTIVE MODELLING

Next effort has been made to develop univariate predictive modelling framework to forecast the respective individual volatility indicators. Similar to the work of Zhang et al. (2003) and Khandelwal et al. (2015) predictions have been made on segregated components of volatility indicators obtained through DWT. The original dataset is initially partitioned into training (80%) and test (20%) data set. Subsequently models are learned on training data set and evaluated on test data set. In this study Autoregressive Integrated Moving Average (ARIMA) as conventional econometric technique and Random Forest (RF) as advanced statistical learning method have been deployed to forecast CBOE VIX, INDIA VIX and HV. ARIMA is applied to be fitted on detail scales or linear components while RF is used for predictive modelling of approximation scales or nonlinear parts of respective time series. ARIMA model is implemented through *'auto.arima()'* function of R platform. Unlike ARIMA, RF needs explicit specifications of predictors beforehand in order to construct forecasting model. As univariate framework is chosen for modelling, lags of the dependent variable have been selected as predictors. To be precise, four predictors i.e., lagged values of dependent variable in one day, two days, three days and four days have been considered for RF based modelling of approximation scales. Mathematically functional forms of them are shown in the following equations.

$$CBOE\ VIX\_A5_t = f(CBOE\ VIX\_A5_{t-1}, CBOE\ VIX\_A5_{t-2}, CBOE\ VIX\_A5_{t-3}, CBOE\ VIX\_A5_{t-4}) \quad \text{………..} (10)$$

$$INDIA\ VIX\_A5_t = f(INDIA\ VIX\_A5_{t-1}, INDIA\ VIX\_A5_{t-2}, INDIA\ VIX\_A5_{t-3}, INDIA\ VIX\_A5_{t-4}) \quad \text{………} (11)$$

$HV\_A5_t = f(CBOE\ VIX\_A5_{t-1}, CBOE\ VIX\_A5_{t-2}, CBOE\ VIX\_A5_{t-3}, CBOE\ VIX\_A5_{t-4})$ ………………… (12)

Final forecast is made by summing the component wise obtained forecasted values as shown in the following equation.

$Forecast_{Final} = Forecast_{RF}^{A5} + Forecast_{ARIMA}^{D5} + Forecast_{ARIMA}^{D4} + Forecast_{ARIMA}^{D3} + Forecast_{ARIMA}^{D2} + Forecast_{ARIMA}^{D1}$ ………………… (13)

On respective components of three volatility indicators, predictions are made by RF and ARIMA models. Accuracy of the proposed forecasting framework has been evaluated applying some standard performance evaluation metrics, discussed later. RF and ARIMA models are briefly described below.

6.1 Random Forest (RF): Due to high accuracy level, robustness to outliers and efficacy in modelling nonlinear interrelationships to build efficient predictive models, Random Forest (RF) has garnered a lot of attention in academic research and resulted in many successful applications for general purpose machine learning problems (Lariviere and Van den Poel, 2005; and Liu et al., 2013). Originally proposed by Breiman (2001), RF is an ensemble based approach for classifications or regressions tasks, frequently utilized in predictive modeling problems. In general, decision trees used for classification and regression tasks is deployed as individual classifier in RF. Decision trees are created and updated based on randomly selected subset of training data. Each tree in RF is chosen randomly. At each node of that selected tree, the best variables/ features to split are chosen randomly too from the training set. The assignment of class label of an unknown instance (for classification) or prediction of continuous outcome (regression) is performed using majority voting or averaging strategy.

6.2 Autoregressive Integrated Moving Average (ARIMA): Pioneered by Box and Jenkins (1968), ARIMA is a popular and effective econometric model for forecasting that estimates the future observations of any time series based on linear combination of past observations and white noise terms. It is a generalized version of famous Autoregressive Moving Average (ARMA) model where nonstationary time series is effectively modeled through appropriate difference steps that correspond to 'Integrated' part of the model. So basically for a stationary time series, the ARIMA forecasting equation simply consists of a linear combination of lagged values of dependent variable and lags of forecasted errors. For a non-seasonal ARIMA(p, d, q) model, 'p' denotes the number of autoregressive terms, 'd' is the number of differences required for stationarity and lastly 'q' refers to number of lagged forecast errors considered in the model. The parameters 'p' and 'q' can be identified using auto correlation function (ACF) and partial auto correlation function (PACF) whereas unit root tests, mentioned earlier may be deployed to fix the value of the parameter 'd'. Mathematically an ARIMA(p, d, q) can be represented as follows:

$\widehat{y_t} = \mu + \Phi_1 y_{t-1} + \cdots + \Phi_p y_{t-p} - \theta_1 e_{t-1} - \cdots - \theta_q e_{t-q}$ ……………………… (14)

6.3 Performance Evaluation: To assess the performance of the predictive modelling framework, four different performance metrics namely, Mean Squared Error (MSE), Nash-Sutcliffe Efficiency (NSE), Index of Agreement (IA) and Theil Inequality (TI) measure. They are calculated as:

$\mathrm{MSE} = \frac{1}{N}\sum_{i=1}^{N}\{Y_{act}(i) - Y_{pred}(i)\}^2$ ………………….. (15)

$\mathrm{NSE} = 1 - \frac{\sum_{i=1}^{N}\{Y_{act}(i) - Y_{pred}(i)\}^2}{\sum_{i=1}^{N}\{Y_{act}(i) - \overline{Y_{act}}\}^2}$ …………………..... (16)

$\mathrm{IA} = 1 - \frac{\sum_{i=1}^{N} Y_{act}(i) - Y_{pred}(i)^2}{\sum_{i=1}^{N}\{|Y_{pred}(i) - \overline{Y_{act}}| + |Y_{act}(i) - \overline{Y_{act}}|\}^2}$ …………… (17)

$\mathrm{TI} = \frac{\left[\frac{1}{N}\sum_{i=1}^{N}\left(Y_{act}(i) - Y_{pred}(i)\right)^2\right]^{1/2}}{\left[\frac{1}{N}\sum_{i=1}^{N} Y_{act}(i)^2\right]^{1/2} + \left[\frac{1}{N}\sum_{i=1}^{N} Y_{pred}(i)^2\right]^{1/2}}$ …………………….. (18)

Where $Y_{act}(i)$ and $Y_{pred}(i)$ are actual observed and predicted value of i^{th} sample. The sample size is denoted by N. While $\overline{Y_{act}}$ and $\overline{Y_{pred}}$ denote the average of actual and predicted values of N samples. Values of MSE must be as low as possible to indicate efficient prediction; ideally a value of zero signifies no error. NSE can range between - ∞ to 1. NSE value of 1 corresponds to a perfect match between model and observations whereas value less than zero occurs when the observed mean is a better predictor than the modeler. The range of IA lies between 0 (no fit) and 1 (perfect fit). TI Values range between [0, 1]. They should be close to 0 for good prediction while 1 implies no prediction at all.

6.4 Results: Since predictions obtained through combined RF and ARIMA models are sensitive to several parameters such as number of trees, stopping criteria, etc. Twenty experimental trials have been conducted by varying those parameters and mean scores of MSE, NSC, IA and TI of twenty experimental trials have been reported here. Results of forecasting HV are narrated in the following tables.

Table 4: Forecasting Performance for CBOE VIX

Data Segment	Performance Evaluators			
	MSE	NSC	IA	TI
Training Data	0.0012	0.8457	0.9309	0.0257
Test Data	0.0015	0.8418	0.9286	0.0271

Table 5: Forecasting Performance for INDIA VIX

Data Segment	Performance Evaluators			
	MSE	NSC	IA	TI
Training Data	0.0017	0.8689	0.9218	0.0281
Test Data	0.0011	0.8722	0.9240	0.0275

Table 6: Forecasting Performance for HV

Data Segment	Performance Evaluators			
	MSE	NSC	IA	TI
Training Data	0.0004	0.9134	0.9688	0.0105
Test Data	0.0005	0.9086	0.9675	0.0132

As the values of MSE and TI are negligible whereas NSC and TI measures are close to 1 for both training and test data set, it can easily be inferred that the usage of the proposed forecasting framework in predictive modelling of volatility measured in terms of CBOE VIX, INDIA VIX and HV has been quite effective.

7. GRANGER CAUSALITY MODEL AND AN ALTERNATIVE PREDICTIVE MODELLING FRAMEWORK

Although we have carried association test to analyze the dynamics of interactions among CBOE VIX, INDIA VIX and HV, to comprehend causal interactions it is essential to conduct Granger Causality test in Vector Auto Regression (VAR) environment. Using this framework, causal interactions among the detail scales of the respective volatility indicators has been explored, as high frequency components that account for the dynamic parts of volatility measures, are of much importance than their counterparts i.e. approximation levels.

7.1 Causality Test in VAR environment: To proceed with the analysis, initially stationarity of the dataset is required to be tested to verify the presence of unit roots. Augmented Dickey Fuller (ADF) and Phillips-Peron (PP) tests have been performed to execute the task.

7.1.1 ADF Test: For any univariate time series, y_t the ADF test simply performs regression to the following model:

$$\Delta y_t = \alpha + \beta_t + \gamma y_{t-1} + \delta_1 \Delta y_{t-1} + \delta_2 \Delta y_{t-2} + \cdots + \delta_{p-1} \Delta y_{t-p+1} + \varepsilon_t \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

Where α, β_t and p are constant, coefficient on a time trend and lag order of autoregressive process respectively. The model corresponds to random walk if $\alpha = 0$ and $\beta = 0$. While only $\beta = 0$ refers to modelling random walk with a drift. The Unit Root Test is then conducted under null hypothesis (H_o) $\gamma = 0$ against alternative hypothesis (H_1) $\gamma < 0$. The acceptance of null hypothesis surmises nonstationary time series.

7.1.2 PP Test: PP test estimates the following equation for a time series, y_t.

$y_t = \alpha + \sum_{i=1}^{T} y_{t-i} + \varepsilon_t$ ………………….. (20)

The bias in the error term results when the variance of the true population is governed by

$\sigma^2 = \lim_{T\to\infty} E(T^{-1}S_T^2)$ ………………………. (21)

The variance of the residuals in the regression equation is given by

$\sigma_u^2 = \lim_{T\to\infty} T^{-1}\sum_{t=1}^{T} E(U_t^2)$ ………………… (22)

Consistent estimators of σ^2 and σ_u^2 are shown in the following equations

$S_{TK}^2 = T^{-1}\sum_{t=1}^{T}(U_t^2) + 2T^{-1}\sum_{t=1}^{K}\sum_{t=j+1}^{T} u_t u_{t-j}$ ………….. (23)

$S_u^2 = T^{-1}\sum_{t=1}^{T}(U_t^2)$ ………………….. (24)

Where k is the parameter of lag truncation. PP test statistic under the null hypothesis of stationarity of original time series i.e. I(0) is

$Z(t_\mu) = \langle S_u | S_{tk} \rangle t_\mu - \frac{1}{2}(S_{tk}^2 - S_u^2)\left[S_{tk}\{T^2\sum_{t=2}^{T}(Y_t - Y_{t-k})^2\}^{\frac{1}{2}}\right]^{-1}$ …………….. (25)

7.2 Results of Stationarity Tests: Unit Root Tests have been performed on approximation and detail level of INDIA VIX, CBOE VIX and HV obtain through DWT. Results are shown in tables below.

Table 7: Results of ADF Test on components of INDIA VIX

Variable	t-Statistic	p-value	Significant
D1	-19.24451	0.000	***
D2	-15.09185	0.000	***
D3	-10.32975	0.000	***
D4	-10.92968	0.000	***
D5	-9.662461	0.000	***

*** Significant at 1% level

Table 8: Results of ADF Test on components of CBOE VIX

Variable	t-Statistic	p-value	Significant
D1	-20.84629	0.000	***
D2	-14.35667	0.000	***
D3	-12.77401	0.000	***
D4	-10.66996	0.000	***
D5	-8.837743	0.000	***

*** Significant at 1% level

Table 9: Results of ADF Test on components of HV

Variable	t-Statistic	p-value	Significant
D1	-18.89749	0.000	***
D2	-15.76987	0.000	***
D3	-12.47438	0.000	***
D4	-12.19529	0.000	***
D5	-9.083409	0.000	***

*** Significant at 1% level

Table 10: Results of PP Test on components of INDIA VIX

Variable	Adjusted t-Statistic	p-value	Significant
D1	-726.6028	0.000	***
D2	-89.42109	0.000	***
D3	-25.43686	0.000	***
D4	-15.20628	0.000	***
D5	-10.36442	0.000	***

*** Significant at 1% level

Table 11: Results of PP Test on components of CBOE VIX

Variable	t-Statistic	p-value	Significant
D1	-973.7532	0.000	***
D2	-380.2673	0.000	***
D3	-140.1417	0.000	***
D4	-16.03222	0.000	***
D5	-11.34662	0.000	***

*** Significant at 1% level

Table 12: Results of PP Test on components of HV

Variable	t-Statistic	p-value	Significant
D1	-402.7529	0.000	***
D2	-88.40056	0.000	***
D3	-29.95794	0.000	***
D4	-13.87197	0.000	***
D5	-10.97403	0.000	***

*** Significant at 1% level

It is clear that all the detail levels of INDIA VIX and CBOE VIX are I(0). Hence no transformations of original variables are needed to executing Granger Causality test in VAR model. It is also well known that carrying out causality analysis in VAR environment is highly sensitive to the lag order of the variables which is also referred to as structural instability. To thwart the issue, a particular lag order is chosen within which the outcomes of VAR causality model stays stable using lag order selection criterion.

7.3 Results of Granger Causality in VAR Environment: At first we investigate the causal interrelationships of three volatility indicators on D1 scales (high frequency scales that persist for shorter duration). The results of VAR Granger Causality model are summarized in table 13.

Table 13: VAR Granger Causality/Block Exogeneity Wald Tests on detail levels one

Dependent Variable: D1 (CBOE VIX)			
Excluded	**Chi Square**	**Degrees of Freedom**	**Probability**
D1 (INDIA VIX)	6.370815	8	0.606
D1 (HV)	6.821093	8	0.556
All	13.26616	16	0.653
Dependent Variable: D1 (INDIA VIX)			
D1 (CBOE VIX)	22.04839	8	0.005
D1 (HV)	26.70643	8	0.001
All	49.54256	16	0.000
Dependent Variable: D1 (HV)			
D1 (CBOE VIX)	9.887509	8	0.273
D1 (INDIA VIX)	5.585953	8	0.694
All	15.92977	16	0.458

Results suggest that on this particular scale that account for high fluctuations in volatility in short range, CBOE VIX and HV have significant causal impact on INDIA VIX. However the nature of the relationship is unidirectional. Both CBOE VIX and HV are not influenced by INDIA VIX. There is no significant interactions between CBOE VIX and HV as well. The IR plot of the nature of relationships is displayed in figure 9. The shortcoming of Granger Causality is its inability to identify the sign and duration of relationships between pair of variables. Impulse responses (IR) obtained through VAR modelling can cater to this problem. The following figure represents the IR of VAR model on details level one.

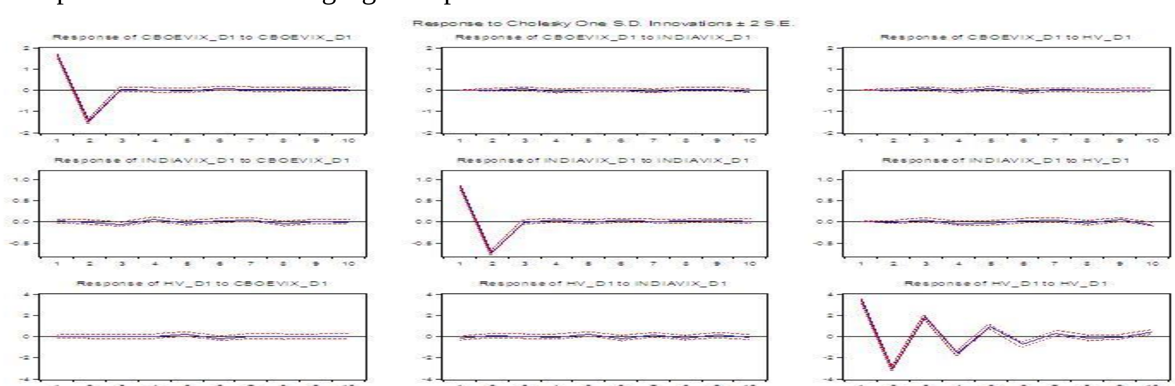

Figure 8: IR of VAR model for detail levels one

To validate the findings of causality model and IR techniques, variance decomposition has been computed as shown in the following tables.

Table 14: Variance Decomposition of Detail Level 1 (D1) of CBOE VIX

Period	S.E.	D1 (CBOE VIX)	D1 (INDIA VIX)	D1 (HV)
1	1.640255	100	0	0
2	2.192389	99.99932	0.000609	7.11E-05
3	2.194172	99.86718	0.072071	0.060753
4	2.195417	99.75522	0.13402	0.110759
5	2.197044	99.62824	0.133855	0.237904
6	2.199858	99.51212	0.135432	0.352451
7	2.20083	99.43983	0.194429	0.365736
8	2.201575	99.39375	0.2373	0.368948
9	2.202968	99.33757	0.290488	0.371945
10	2.203976	99.25898	0.368034	0.37299

Table 15: Variance Decomposition of Detail Level 1 (D1) of INDIA VIX

Period	S.E.	D1 (CBOE VIX)	D1 (INDIA VIX)	D1 (HV)
1	0.830645	0.035049	99.96495	0
2	1.10872	0.041937	99.93288	0.02518
3	1.111448	0.377127	99.45627	0.166599
4	1.114257	0.668185	99.04315	0.288661
5	1.115327	0.749361	98.86405	0.386588
6	1.116498	0.797032	98.75626	0.446708
7	1.118297	0.990547	98.45181	0.557644
8	1.119998	1.17906	98.17595	0.644989
9	1.121981	1.178023	97.92057	0.901409
10	1.124648	1.175355	97.45718	1.367468

Table 16: Variance Decomposition of Detail Level 1 (D1) of HV

Period	S.E.	D1 (CBOE VIX)	D1 (INDIA VIX)	D1 (HV)
1	3.457679	0.002108	0.229511	99.76838
2	4.60344	0.001742	0.154847	99.84341
3	4.959578	0.004382	0.133599	99.86202
4	5.204802	0.004799	0.130484	99.86472
5	5.293383	0.163328	0.271561	99.56511
6	5.35178	0.307118	0.381088	99.31179
7	5.362552	0.307163	0.442258	99.25058
8	5.367118	0.307948	0.502546	99.18951
9	5.368866	0.307838	0.543135	99.14903
10	5.389288	0.306019	0.571721	99.12226

As indicated by the VAR Granger Causality test, percentages of variance of INDIA VIX explained by CBOE VIX and HV are 1.18% and 1.36% respectively in short run. Whereas variance of both CBOE VIX and HV is largely explained by themselves. They are very marginally influenced by other indicators on this scale. Subsequently on D2 scale having lower frequency values but longer timespan than D1, causal dynamics of CBOE VIX, INDIA VIX and HV has been explored.

Table 17: VAR Granger Causality/Block Exogeneity Wald Tests on detail levels two

Dependent Variable: D2 (CBOE VIX)			
Excluded	**Chi Square**	**Degrees of Freedom**	**Probability**
D2 (INDIA VIX)	3.852919	8	0.870
D2 (HV)	9.033186	8	0.340
All	13.35383	16	0.647
Dependent Variable: D2 (INDIA VIX)			
D2 (CBOE VIX)	14.14614	8	0.078
D2 (HV)	14.48589	8	0.070
All	28.95927	16	0.024
Dependent Variable: D2 (HV)			
D2 (CBOE VIX)	2.865341	8	0.943
D2 (INDIA VIX)	7.489666	8	0.485

All	10.24034	16	0.854

It can be seen that at 5% significance level there is no statistically significant causal relationships among the indicators at this particular scale which was also verified by the IR and Variance decomposition. Similarly, VAR Granger Causality tests have been conducted on D3 scale and the findings have been mentioned in table 35.

Table 18: VAR Granger Causality/Block Exogeneity Wald Tests on detail levels three

Dependent Variable: D3 (CBOE VIX)			
Excluded	**Chi Square**	**Degrees of Freedom**	**Probability**
D3 (INDIA VIX)	1.952536	8	0.982
D3 (HV)	1.465148	8	0.993
All	3.610889	16	0.999
Dependent Variable: D3 (INDIA VIX)			
D3 (CBOE VIX)	2.664749	8	0.954
D3 (HV)	1.492213	8	0.993
All	4.050127	16	0.999
Dependent Variable: D3 (HV)			
D3 (CBOE VIX)	8.810282	8	0.3586
D3 (INDIA VIX)	15.13112	8	0.0566
All	24.17894	16	0.0857

Likewise D2 scale, there is no significant (at 5% level) causal interactions among the variables considered. Following the sequence, D4 levels of three indicators have been fed to VAR model to analyze their interrelationships.

Table 19: VAR Granger Causality/Block Exogeneity Wald Tests on detail levels four

Dependent Variable: D4 (CBOE VIX)			
Excluded	**Chi Square**	**Degrees of Freedom**	**Probability**
D4 (INDIA VIX)	4.699041	8	0.789
D4 (HV)	1.443701	8	0.994
All	6.543924	16	0.981
Dependent Variable: D4 (INDIA VIX)			
D4 (CBOE VIX)	2.248046	8	0.972
D4 (HV)	3.298988	8	0.914
All	4.686026	16	0.997
Dependent Variable: D4 (HV)			
D4 (CBOE VIX)	0.133239	8	1
D4 (INDIA VIX)	0.858378	8	0.999
All	1.075992	16	1

No causal interrelationships are prevailed on D4 scale in this exercise. Finally causality tests are conducted on D5 scales having moderately high frequency values that span for the entire observation periods of the sample to comprehend the interaction among the variables.

Table 20: VAR Granger Causality/Block Exogeneity Wald Tests on detail levels five

Dependent Variable: D5 (CBOE VIX)			
Excluded	**Chi Square**	**Degrees of Freedom**	**Probability**
D5 (INDIA VIX)	0.244002	2	0.885
D5 (HV)	0.043916	2	0.978
All	0.250925	4	0.993
Dependent Variable: D5 (INDIA VIX)			
D5 (CBOE VIX)	0.195365	2	0.907
D5 (HV)	0.001494	2	0.999

All	0.204158	4	0.995
Dependent Variable: D5 (HV)			
D5 (CBOE VIX)	0.386436	2	0.824
D5 (INDIA VIX)	0.005329	2	0.997
All	0.403401	4	0.982

It is quite evident that the movement of CBOE VIX, INDIA VIX and HV on D5 scales are independent of each other. Overall the Granger Causality Analysis in VAR environment attempts to extract interconnections among CBOE VIX, INDIA VIX and HV in component wise manner corresponding to different time scales corresponding to high frequency ranges. So only at detail level one, significant interactions have been noticed. Attempt has been made to leverage this finding in developing another predictive modelling framework which is mixture of univariate and multivariate models.

6.3 Alternative Predictive Modelling Framework: In Earlier section of the paper, architecture and performance of univariate forecasting model comprising RF and ARIMA have been elucidated in details. In this alternative model, predictors are selected on the basis of outcomes of Granger Causality tests in VAR framework. Predictive modelling of approximation parts of CBOE VIX, INDIA VIX and HV has been kept unchanged as it was not considered in causality tests. So forecasts obtained through RF model have been used for computing final forecasted values. Since INDIA VIX on detail level one is found to be influenced by CBOE VIX and HV hence the predictive model of INDIA VIX on this scale is must consider detail level one of CBOE VIX and HV apart from the lagged values of INDIA VIX. Original ARIMA model, on the other hand is not capable of developing multivariate forecasting model. As an alternative, Multilayer Feedforward Neural Network (MLFFNN) as standard Artificial Neural Network (ANN) model is deployed to carry out predictive modelling framework by learning the pattern of detail levels on five occasions for forecasting respective volatility indicators. Final forecast is made by applying equation 14.

Multilayer Feed-Forward Network (MLFFNN): It is a famous Artificial Neural Network (ANN) model that attempts to mimic the working nature of the human brain to discover the hidden nonlinear pattern between a set of inputs and outputs. Human brain is composed of approximately 10^{10} number of highly interconnected units called as neurons. In a typical layered architecture of ANN neurons are organized and interconnected in a hierarchical manner. There are three distinct interconnected layers in a standard MLFFNN model namely, an input layer, hidden layer(s) and an output layer. They are connected via neurons and strength of each connection is actually represented by numeric weight value. For prediction tasks, basically these weight values corresponding to decision boundary, are estimated using various optimization algorithms on training data set. Once the estimated values are stabilized after validation, trained MLFFNN is tested against a test data set to assess its predictive power. It has been successfully applied in various financial time series forecasting tasks as reported in literature (Vojinovic and Kecman (2001), Adhikari and Agrawal (2014), Zhang and Qi (2005), Andreou and Zombanakis (2006), etc.).

6.4 Forecasting Performance of Alternative Predictive Framework: On respective components of three volatility indicators predictions are made by RF and MLFFNN models. Final outcome is governed by equation 13 as explained earlier. Similar to original predictive framework, the alternative RF and MLFFNN based model is also sensitive to process parameters like number of trees, number of neurons in hidden layer, stopping criteria etc. Hence to keep uniformity twenty experimental trials are performed by changing the respective parameters and the overall forecasting efficiency and accuracy are measured in terms of mean scores of respective performance evaluation measures as discussed as summarized in the tables followed.

Table 21: Forecasting Performance for CBOE VIX

Data Segment	Performance Evaluators			
	MSE	NSC	IA	TI
Training Data	0.0010	0.8639	0.9432	0.0211
Test Data	0.0013	0.8508	0.9377	0.0227

Table 22: Forecasting Performance for INDIA VIX

Data Segment	Performance Evaluators			
	MSE	NSC	IA	TI
Training Data	0.0009	0.8894	0.9377	0.0257
Test Data	0.0010	0.8805	0.9391	0.0244

Table 23: Forecasting Performance for HV

Data Segment	Performance Evaluators			
	MSE	NSC	IA	TI
Training Data	0.0004	0.9189	0.9728	0.0088
Test Data	0.0004	0.9122	0.9691	0.0107

As the values of MSE and TI are negligible whereas NSC and TI measures are close to 1 for both training and test data set, it can easily be inferred that the usage of the proposed forecasting framework in predictive modelling of volatility measured in terms of CBOE VIX, INDIA VIX and HV has been quite effective.

8. CONCLUSION

This paper uses the wavelet decomposition approach to understand whether different measures of stock market volatility demonstrate primarily long term or short term fluctuations. The various experiments conducted was not on the aggregate time series, but on the components. Two different predictive modelling frameworks and Granger causality analysis in VAR framework have been applied on CBOE VIX, INDIA VIX and HV. Association and causal influence analysis on different scales of respective volatility measures has provided deep insights. The novelty of the second forecasting framework comprising RF and MLFFNN lies in integration of causality analysis for feature selection process. In future automatic extraction of key features through advanced Deep Learning Models need to be applied to augment the process of incorporating lagged values of endogenous constructs in building prediction models. Long run co-movements of CBOE VIX, INDIA VIX and HV have not been explored in this study which can be modeled through Johansen's Co-Integration tests. Scope of this research is restricted to volatility of stock markets of India and USA. Interactions, dynamics and predictions of volatility of stock markets of other developed and developing countries can also be understood using this research framework.

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