The Fundamental Fields Theorem

A Structural Classification of Classical Interaction Fields Under Closure

Simon F. Gates†

Abstract

Which classical interaction fields can be fundamental in principle? This work addresses that upstream classification question by introducing a conservative, deductive notion of *closure*: a fundamental classical interaction field must admit self-contained conserved quantities in regimes where conservation is well defined, without external bookkeeping or regime-dependent prescriptions.

Imposing only classical covariance and self-contained conservation, together with derived admissibility criteria such as exhaustion under symmetry, we classify candidate classical fields eliminatively. We prove that any admissible fundamental interaction must be universal, self-coupled, and geometrically mediated. In the classical domain, this uniquely selects gravity as the only closing interaction field. All non-gravitational interaction fields are necessarily open in the stated sense, relying on independent charge sectors or external prescriptions.

The result does not modify gravitational dynamics or assume a specific action. It explains, at a structural level, why gravity alone exhibits rigidity and exactness in its clean regimes and provides the logical foundation for the downstream Gravitational Closure (Ananke) Theorem.

1. Introduction

1.1 The upstream classification problem

Modern theoretical physics possesses powerful frameworks for constructing and analysing field theories, yet it lacks a clear answer to a more basic question:

Which classical fields can be fundamental in principle?

Most existing results address how to build theories once the field content is assumed. Representation theory classifies particles; gauge principles organise interactions; variational methods constrain admissible actions. None of these frameworks ask whether a given classical field is structurally capable of standing alone as a fundamental interaction.

As a result, gravity, gauge fields, and effective descriptions are often discussed on equal footing, despite exhibiting radically different structural behaviour in their clean symmetry regimes.

This paper addresses that upstream gap.

† Correspondence: simonfgates@gmail.com

1.2 What "classification" means here

The classification undertaken here is structural, not phenomenological.

We do not ask:

- which theories fit data best,
- which models are most economical,
- or which interactions quantise most cleanly.

Instead, we ask a logically prior question:

If a classical interaction field is claimed to be fundamental, what minimal structural requirements must it satisfy, and which fields meet those requirements?

This is a question about admissibility, not about empirical adequacy.

1.3 Why this question has been avoided

There are three historical reasons why such a classification has not been made explicit:

1. **Success of effective field theory**: modern physics tolerates open, regulator-dependent frameworks because they work. This has blurred the distinction between *effective* and *fundamental*.
2. **Special treatment of gravity**: gravity is often treated geometrically rather than as a field to be compared structurally with others, allowing its exceptional rigidity to pass without explanation.
3. **Reluctance to formalise "fundamental"**: the term is widely used but rarely defined, as definitions invite philosophical scrutiny.

The result is an implicit hierarchy that is relied upon in practice but not stated as a theorem.

1.4 The central idea

The central idea of this work is that closure provides the missing classificatory criterion.

In regimes where conserved quantities are well-defined, a fundamental interaction field should:

- regulate its own conservation laws,
- exhaust its degrees of freedom under symmetry,
- and require no external bookkeeping to remain consistent.

These requirements are already tacitly enforced when physicists decide which theories are "exact" and which are "phenomenological." This paper makes them explicit and uses them deductively.

1.5 What this paper does

This paper:

1. Defines fundamental classical interaction fields in a precise, conditional sense;
2. States minimal upstream axioms consistent with established practice;
3. Uses these axioms to classify candidate field structures;
4. Proves that gravity is uniquely admissible as a closing classical interaction field;
5. Establishes a clean logical foundation for the downstream Gravitational Closure (Ananke) Theorem.

No assumptions are made about the detailed dynamics of gravity, the form of the action, or the existence of additional degrees of freedom.

1.6 Relation to existing results

The present work is not a replacement for existing classification theorems; it is upstream of them.

- Representation-theoretic results classify particle states once a field content is assumed.
- Gauge-theoretic frameworks organise interactions given a charge structure.
- Lovelock-type theorems classify admissible actions once metric gravity is assumed.

By contrast, this work asks a logically prior question: which classical interaction fields are admissible in principle, before specifying field content or action.

In this sense, the result complements, rather than competes with, existing theorems.

1.7 Why gravity emerges as exceptional

Gravity's exceptional status in classical physics is well known empirically, but poorly explained structurally. Its exactness in vacuum, rigidity under symmetry, and geometric mediation are typically treated as special facts rather than consequences.

The Fundamental Fields Theorem shows that these features are not coincidental. They follow necessarily once closure and law-completeness are imposed.

Gravity is not assumed to be special; it is forced to be.

1.8 What the theorem does *not* claim

For clarity, the theorem does not claim that:

- gravity is the only interaction in nature,
- non-gravitational fields are non-fundamental in all senses,
- quantum gravity is resolved or constrained,
- dark matter or dark energy are illusory.

The theorem addresses only classical interaction fields and only the question of structural closure.

1.9 Structure of the paper

The remainder of the paper is organised as follows:

- **Section 2** defines fundamental fields, closure regimes, and admissibility criteria.
- **Section 3** states the upstream axioms and derived diagnostics.
- **Section 4** applies these criteria to candidate classes of classical fields.
- **Section 5** states and proves the Fundamental Fields Theorem.
- **Section 6** discusses implications and relation to the Gravitational Closure (Ananke) Theorem.

1.10 Summary

By making explicit a set of conservative structural requirements already implicit in physics practice, this work provides the first deductive classification of fundamental classical interaction fields.

The result is simple but consequential: only gravity can close.

2. Definitions and Scope

This section fixes terminology and scope precisely. All subsequent results are conditional on these definitions. Any objection to later conclusions must therefore reject one or more definitions explicitly.

2.1 Classical field and theory

Definition 2.1 (Classical field).
A *classical field* is a set of tensorial or geometric quantities Ψ defined on a smooth spacetime manifold M, governed by covariant field equations derivable from a variational principle or equivalent local dynamical law.

No assumptions are made regarding quantisation, ultraviolet completion, or microscopic ontology.

2.2 Fundamental classical field

Definition 2.2 (Fundamental classical field).
A classical field Ψ is said to be *fundamental* if, within its declared classical domain of validity:

1. its field equations are law-complete, requiring no regime-dependent prescriptions, interpolations, or external supplements;
2. it is not defined as an effective or emergent description of more primitive degrees of freedom;
3. it applies universally within its scope, without modification of its defining equations across physical regimes.

This definition is conditional: the present work classifies fields claiming fundamentality under these criteria.

2.3 Classical covariance

Definition 2.3 (Classical covariance).
A field theory is *classically covariant* if its equations of motion, constraints, and conserved quantities are invariant under arbitrary smooth diffeomorphisms of the spacetime manifold (or the appropriate symmetry group for the domain considered), with no fixed background structures.

This excludes preferred frames, background metrics, or coordinate-dependent definitions of physical quantities.

2.4 Closure regimes

Definition 2.4 (Closure regime).
A *closure regime* is a class of spacetime domains admitting covariantly defined conserved quantities associated with the field, including but not limited to:

- stationary or static spacetimes (Komar-type charges),
- asymptotically flat spacetimes (ADM or Bondi charges),
- symmetry-reduced sectors admitting Noether charges.

No claim is made that such quantities exist in all conceivable spacetimes. Closure is evaluated only in regimes where conservation is standardly defined in established physics.

2.5 Self-contained conserved functional (closure)

Definition 2.5 (Self-contained conserved functional).
A classical field Ψ admits *closure* if, in every closure regime, there exists a covariantly defined conserved functional

$$Q[\Psi],$$

constructed solely from Ψ and the geometric structures it dynamically determines, such that:

1. Q is finite on all admissible configurations;
2. conservation of Q follows from the field equations alone;
3. no external sectors, regulators, hidden variables, or regime-dependent rules are required to define or preserve Q.

Closure is an internal property. Declaring "the universe as a whole" to be closed does not constitute closure of an interaction field.

2.6 Open vs closed fields

Definition 2.6 (Closed field).
A field is *closed* if it admits a self-contained conserved functional in all closure regimes.

Definition 2.7 (Open field).
A field is *open* if its conserved quantities require independent external sectors, phenomenological prescriptions, or residual functional freedom not fixed by the field equations.

2.7 External bookkeeping

Definition 2.8 (External bookkeeping).
External bookkeeping denotes any prescription required to maintain consistency or conservation that is not dynamically determined by the field equations themselves, including:

- independent conserved charge sectors,
- interpolation or screening functions,
- regulator-dependent counterterms,
- environment- or scale-dependent rule changes.

Any theory requiring such bookkeeping cannot qualify as fundamental under Definition 2.2.

2.8 Exhaustion under symmetry

Definition 2.9 (Exhaustion under symmetry).
A closed field Ψ is said to *exhaust under symmetry* if, when sufficient spacetime symmetry is imposed within a closure regime, all functional freedom in admissible solutions is eliminated, leaving at most a finite set of integration constants fixed by conservation laws and boundary conditions.

Failure of exhaustion indicates the persistence of hidden functional freedom and therefore implicit external bookkeeping. Exhaustion under symmetry is treated as an admissibility criterion derived from closure, not as an independent axiom.

2.9 Interaction field versus matter field

Definition 2.10 (Interaction field).
An *interaction field* is a field whose role is to mediate relations between other degrees of freedom and to organise their dynamics universally.

Definition 2.11 (Matter field).
A *matter field* is a field whose role is to supply content or sources but which does not, by itself, mediate universal interaction.

The present classification concerns interaction fields. Matter fields may be fundamental in other senses, but they cannot serve as closing interaction fields under the present criteria.

2.10 Response modes

Definition 2.12 (Response mode).
A *response mode* of a field is an independent way in which the field reacts to sources or boundary conditions, contributing additively to the conserved functional Q.

Response modes are counted by independent contributions to Q, not by field components or gauge artefacts.

2.11 Orthogonality of response modes (for later use)

Definition 2.13 (Orthogonal response modes).
Response modes are *orthogonal* if the conserved quadratic functional decomposes as a direct sum,

$$Q = Q_1 \oplus Q_2 \oplus \cdots ,$$

with no cross-terms between modes.

Orthogonality ensures that energy accounting is unambiguous and that no hidden coupling or gauge-dependent redistribution occurs between modes.

This definition is stated here for completeness; orthogonality is not imposed at the upstream level but enters explicitly in the Ananke theorem.

2.12 Scope statement

Definition 2.14 (Scope).
All results in this work apply to classical fields in closure regimes admitting covariantly defined conserved quantities. No claims are made regarding quantum completion, ultraviolet behaviour, or regimes in which conservation laws are ill-defined.

2.13 Summary of definitions

For clarity, the classification relies on the following minimal commitments:

- classical covariance;
- self-contained conserved quantities in closure regimes;
- exclusion of external bookkeeping;
- exhaustion under symmetry as an admissibility criterion;
- restriction to interaction fields.

Any objection to subsequent theorems must therefore reject one or more of these commitments explicitly.

3. Upstream Axioms and Admissibility Criteria

This section states the minimal axioms required for the upstream classification and clarifies which conditions are axiomatic and which are derived admissibility criteria. The aim is to minimise assumptions while preserving deductive force.

3.1 Axiom A — Classical covariance

Axiom A (Classical covariance).
Any candidate fundamental interaction field admits a covariant formulation: its field equations, constraints, and conserved quantities are invariant under the relevant spacetime symmetry group (diffeomorphisms in the relativistic domain), with no fixed background structures.

Remarks.

1. This axiom is not specific to gravity. It is a prerequisite for any theory purporting to describe a universal interaction in classical physics.
2. Rejecting Axiom A amounts to endorsing preferred frames or background-dependent laws at the fundamental level, a position outside the scope of the present work.

3.2 Axiom B — Self-contained conservation (closure)

Axiom B (Self-contained conservation).
A candidate fundamental interaction field must admit a self-contained conserved functional in every closure regime, as defined in Section 2.

Equivalently: conservation must follow from the field equations alone and must not rely on independent external sectors, prescriptions, or regime-dependent rules.

Remarks.

1. This axiom encodes the requirement of law completeness for a fundamental interaction field.
2. The axiom is scoped: it does not demand conservation where no conserved quantities are defined in established physics.
3. Axiom B is violated by theories whose consistency depends on external charge sectors, interpolation functions, or phenomenological supplements.

3.3 Minimality of the axioms

Axioms A and B are irreducible in the following sense:

- Axiom A cannot be derived from Axiom B, since conservation presupposes transformation properties.
- Axiom B cannot be derived from Axiom A, since covariance alone does not enforce self-contained conservation.

No additional upstream axioms are imposed. In particular, no assumptions are made regarding:

- the number of field components,
- the form of the action,
- the order of the field equations,
- the presence or absence of gauge symmetry.

All further restrictions enter only as admissibility criteria derived from Axioms A and B.

3.4 Admissibility Criterion C — Exhaustion under symmetry

Criterion C (Exhaustion under symmetry).
A candidate fundamental interaction field satisfying Axioms A and B must exhaust its degrees of freedom under sufficient spacetime symmetry in any closure regime.

That is, when symmetry conditions are imposed (e.g. stationarity, homogeneity, isotropy, spherical symmetry), the admissible solution space must collapse to a finite-dimensional family parameterised solely by integration constants fixed by conservation laws and boundary conditions.

Logical status.
Criterion C is not an independent axiom. It follows from Axiom B: if residual functional freedom remains under symmetry, then the field equations alone do not determine admissible solutions, and additional external choices are required. Such freedom constitutes implicit external bookkeeping and violates self-contained conservation.

Remarks.

1. Criterion C reflects the empirical behaviour of established interaction fields in their clean symmetry limits (e.g. uniqueness of vacuum gravitational exteriors).
2. This criterion does not deny the existence of PDE systems with functional freedom under symmetry; it excludes them from claims of fundamentality under the present definition.

3.5 Admissibility Criterion D — No external sourcing of structure

Criterion D (No external sourcing of structure).
A candidate fundamental interaction field may not rely on independent external sectors to define or stabilise its conserved quantities or large-scale structure.

In particular:

- conserved charges must not be supplied by an independent matter sector,
- universal structural features must not depend on phenomenological functions or screening prescriptions.

Logical status.
Criterion D is a direct restatement of Axiom B. Any theory violating Criterion D fails self-contained conservation.

Remarks.

1. Criterion D does not prohibit coupling to matter. It prohibits structural dependence on matter or auxiliary sectors.
2. Declaring "the combined system of field plus matter is closed" trivialises the classification and is therefore excluded.

3.6 Consequences of the admissibility criteria

Together, Criteria C and D exclude:

- gauge fields whose conserved quantities require independent charge sectors,
- theories with regime-dependent interpolation or screening mechanisms,
- models retaining arbitrary functional freedom in highly symmetric settings.

Importantly, these exclusions do not assert that such theories are false or useless; they assert only that such theories cannot qualify as fundamental closing interaction fields under Definitions 2.2 and 2.5.

3.7 Summary of upstream logical structure

The upstream framework consists of:

- **Axiom A (Classical covariance)**
- **Axiom B (Self-contained conservation / closure)**

together with two derived admissibility criteria:

- **Criterion C (Exhaustion under symmetry)**
- **Criterion D (No external sourcing of structure)**

No additional assumptions are imposed at this stage.

Crucially:

- Axioms A–B determine eligibility for fundamentality.
- Criteria C–D function as diagnostics for whether eligibility is satisfied in practice.

This separation prevents circularity: gravity is not assumed to be special; it is identified as special by elimination.

3.8 What the upstream framework does not assume

For clarity, the upstream framework does not assume:

- metric primacy,
- second-order field equations,
- specific Lagrangian form,
- number of degrees of freedom,
- existence of a vacuum sector,
- universality of coupling.

All of these emerge, if at all, only after classification.

3.9 Transition statement

Having fixed the axioms and admissibility criteria, we now address the classification problem itself:

Given a classical covariant interaction field satisfying Axioms A–B and admissibility Criteria C–D, which field structures are admissible in principle?

The next section applies this framework to the major classes of classical fields to determine which, if any, can serve as fundamental closing interaction fields.

4. Candidate Classes of Classical Fields

We now survey the principal classes of classical fields that might plausibly serve as fundamental interaction fields and evaluate them against the upstream axioms and admissibility criteria established in Sections 2–3.

The purpose is eliminative rather than constructive: to determine which classes cannot close, and under what structural reasons.

4.1 Taxonomy of classical fields

For the purposes of classification, classical fields are grouped into four broad classes:

1. **Gauge-type interaction field:** (e.g. electromagnetism, Yang–Mills theories)
2. **Matter-type fields**: (e.g. scalar, fermionic, fluid-like fields)
3. **Geometric or self-coupled interaction fields**: (fields whose conserved quantities act as their own sources)
4. **Composite or effective media**: (effective field theories, hydrodynamic descriptions)

This taxonomy is intentionally coarse. The classification result will not depend on detailed microstructure.

4.2 Gauge-type interaction fields

Gauge-type fields are characterised by:

- a connection-like structure,
- gauge symmetry,
- conserved charges defined independently of the field configuration.

Examples: classical electromagnetism, non-Abelian Yang–Mills fields.

4.3 Closure analysis for gauge fields

Gauge fields fail closure under Axiom B for a structural reason:

- their conserved quantities (electric charge, colour charge) are independent external invariants,
- the field equations alone do not determine these charges,
- conserved quantities are supplied by an external matter sector.

This dependence persists even in closure regimes (vacuum, stationary, symmetry-reduced domains).

Accordingly, gauge fields violate Criterion D (No external sourcing of structure).

4.4 On the role of charges

It is important to distinguish:

- **coupling to matter**, which is permitted, from
- **structural dependence on an independent conserved sector**, which is not.

In gauge theories, the charge sector is not generated by the field itself. Declaring “field + charges” to be closed trivialises closure and is excluded by Definition 2.8.

Thus, gauge fields cannot serve as fundamental closing interaction fields, regardless of their empirical success.

4.5 Matter-type fields

Matter-type fields include:

- scalar fields,
- fermionic fields,
- fluid-like continuum descriptions,
- classical media fields.

Such fields may be fundamental as constituents, but they are not interaction fields in the present sense.

4.6 Closure analysis for matter fields

Matter-type fields fail closure as interaction fields for two independent reasons:

1. **Lack of universality**: matter fields do not mediate relations between all other degrees of freedom. Their influence is contingent and selective.
2. **Absence of self-closing dynamics**: their conserved quantities (particle number, internal charges, stress-energy) are defined relative to background structure or other interactions. They do not regulate the geometry or causal structure through which all interactions occur.

As a result, matter fields cannot satisfy Axiom B as fundamental interaction fields.

This exclusion is not a criticism of matter fields as physical entities; it reflects only their inapplicability as universal closing interactions.

4.7 Composite and effective media

Composite or effective fields include:

- hydrodynamic descriptions,
- elastic continua,
- effective field theories with cutoffs,
- phenomenological coarse-grained models.

These theories explicitly acknowledge:

- truncation,
- regime dependence,
- emergent validity.

They therefore violate Definition 2.2 by construction and are excluded from the present classification.

4.8 Interim conclusion

After application of Axioms A–B and Criteria C–D:

- **Gauge-type fields** are excluded due to dependence on external conserved charges.
- **Matter-type fields** are excluded as non-universal interactions.
- **Composite/effective fields** are excluded by explicit non-fundamentality.

The only remaining admissible class consists of self-coupled, universal interaction fields, to be examined next.

4.9 Self-coupled, universal interaction fields

A self-coupled universal interaction field is characterised by the following structural features:

1. **Universality**: the field couples to all forms of energy–momentum (or equivalent conserved quantities) without exception.
2. **Self-coupling**: the field's own conserved quantity contributes to its source. The interaction does not terminate in an external sector.
3. **Geometric mediation**: the field organises the spacetime or causal structure through which all other fields propagate.

These properties are not postulates; they arise as necessities once closure and universality are imposed.

4.10 Closure analysis for self-coupled universal fields

A self-coupled universal interaction field satisfies Axiom B in a way that other fields cannot:

- its conserved quantity is generated entirely by the field itself and the matter it universally couples to,
- no independent charge sector exists,
- conservation does not require regime-dependent prescriptions.

Moreover, under sufficient symmetry in closure regimes, such fields naturally exhibit exhaustion under symmetry: the solution space collapses to a finite set of parameters fixed by conservation laws and boundary conditions.

4.11 Universality and covariance imply geometric mediation

In a classically covariant theory, a universal interaction must act through the structure that defines:

- intervals,
- causal relations,
- the motion of all other fields.

Up to field redefinitions, this forces mediation through spacetime geometry (or an equivalent structure).

This step is crucial: it shows that universality + covariance leave no room for a non-geometric closing interaction.

4.12 Identification with gravity

The only classical interaction field known to satisfy all of the above is gravity:

- it couples universally,
- it is self-coupled,
- it mediates through geometry,
- it admits self-contained conserved quantities in closure regimes,
- it exhausts under symmetry in vacuum and stationary limits.

At this stage, no assumption has been made about the detailed form of the gravitational action or field equations.

Gravity is identified structurally, not phenomenologically.

4.13 Transition to the main theorem

The eliminative analysis of Sections 4.1–4.12 establishes that:

If a classical interaction field is fundamental in the sense of Definitions 2.2–2.5, then it must be a self-coupled, universal, geometrically mediated field.

The following section states and proves the Fundamental Fields Theorem, formalising this conclusion.

5. The Fundamental Fields Theorem

We now state and prove the main classification result implied by the upstream axioms and admissibility criteria.

5.1 Statement of the theorem

Theorem 5.1 (Fundamental Fields Theorem).
Consider a classical interaction field Ψ defined on a spacetime manifold and intended to be fundamental in the sense of Definition 2.2. Suppose that Ψ satisfies:

1. **Axiom A (Classical covariance)**;
2. **Axiom B (Self-contained conservation)** in all closure regimes;
3. **Admissibility Criteria C–D** (exhaustion under symmetry and absence of external bookkeeping).

Then Ψ must be a self-coupled, universal, geometrically mediated interaction field. In the classical domain, this uniquely identifies gravity as the only admissible fundamental closing interaction field.

All other classical interaction fields are necessarily open under the above definitions.

5.2 Logical structure of the proof

The proof proceeds by elimination and necessity, using the classification of Section 4.

- Step 1: Show that any admissible fundamental interaction must be universal.
- Step 2: Show that universality plus closure requires self-coupling.
- Step 3: Show that universality plus covariance forces geometric mediation.
- Step 4: Conclude uniqueness.

Each step excludes an entire class of alternatives without reference to phenomenology.

5.3 Proof — Step 1: Universality

Assume Ψ is a fundamental interaction field satisfying Axiom B.

If Ψ does not act universally, then there exist degrees of freedom whose dynamics are not regulated by Ψ. Conservation of the total system then requires an independent interaction to mediate those dynamics.

This introduces either:

- an external interaction sector, or
- regime-dependent prescriptions.

Both violate self-contained conservation.

Therefore, any closed fundamental interaction field must act universally.

5.4 Proof — Step 2: Necessity of self-coupling

Let Ψ be a universal interaction field satisfying Axiom B.

If Ψ does not couple to its own conserved quantity, then the accounting of interaction energy requires an external rule to exclude or include the field's contribution.

This constitutes external bookkeeping and violates closure.

Therefore, a closed universal interaction must be self-coupled.

5.5 Proof — Step 3: Geometric mediation

Let Ψ be a universal, self-coupled interaction field satisfying Axiom A.

In a classically covariant theory, universality requires that all matter fields respond identically to Ψ in the absence of additional structure. This forces the interaction to act through the spacetime or causal structure common to all fields.

Up to field redefinitions, this implies mediation through spacetime geometry (or an equivalent geometric structure).

Non-geometric mediation would require preferred frames or background structures, violating Axiom A.

5.6 Proof — Step 4: Uniqueness

From Sections 5.3–5.5, any admissible fundamental interaction field must be:

- universal,
- self-coupled,
- geometrically mediated,
- classically covariant,
- self-contained in its conserved quantities.

Within the classical domain, this combination of properties uniquely characterises gravity.

No other known classical interaction field satisfies all of these requirements. Any deviation introduces either:

- external charge sectors,
- residual functional freedom under symmetry,
- background dependence,
- or regime-dependent prescriptions,

all of which violate the upstream axioms or admissibility criteria.

This establishes the classification result.

5.7 Corollary — Rigidity of vacuum gravity

Corollary 5.2.
In closure regimes devoid of non-gravitational sources, a fundamental gravitational interaction must exhibit rigidity: all admissible solutions under sufficient symmetry are fixed uniquely up to conserved integration constants.

This corollary explains, at a structural level, the empirical success of exact vacuum solutions in classical gravity.

5.8 Corollary — Openness of non-gravitational interaction fields

Corollary 5.3.
All non-gravitational classical interaction fields are necessarily open in the sense of Definition 2.7. Their conserved quantities depend on independent external sectors or residual functional freedom.

This openness does not undermine their empirical validity; it precludes only claims of self-closing fundamentality.

5.9 Relation to the Ananke (Gravitational Closure) Theorem

The Fundamental Fields Theorem establishes which classical interaction field can close. It does not determine how that field must organise its internal response.

The Ananke (Gravitational Closure) Theorem addresses this second question by imposing an additional structural requirement (orthogonality of response modes) on gravity and deriving its unique admissible response structure.

Logically:

- FFT isolates gravity.
- Ananke completes gravity.

5.10 Concluding remark

The classification achieved here does not privilege gravity by assumption. It isolates gravity as the only admissible closing interaction field once minimal, conservative criteria for fundamentality are imposed.

Any objection must therefore reject one or more of the definitions or axioms stated explicitly above.

6. Discussion and Implications

This section interprets the Fundamental Fields Theorem (FFT), clarifies what it does and does not claim, and situates it within established physics practice. The aim is to pre-empt misreadings and make explicit the methodological consequences.

6.1 What the theorem actually establishes

The FFT is a classification theorem, not a dynamical proposal. It establishes the following conditional statement:

If one demands that a classical interaction field be fundamental in the sense of law-completeness, covariance, and self-contained conservation in closure regimes, then gravity is uniquely admissible.

This result is eliminative and structural. It does not depend on:

- empirical curve fitting,
- assumptions about the detailed form of the gravitational action,
- or speculative extensions of known physics.

6.2 Why this is not a redefinition of gravity

The theorem does not redefine gravity. It does not assert new properties or modify known ones. Instead, it explains why gravity already behaves as it does in its clean regimes:

- rigidity in vacuum,
- absence of tunable vacuum degrees of freedom,
- geometric mediation,
- exactness of symmetry-reduced solutions.

These are treated as consequences of closure, not coincidences.

6.3 Why this is not an attack on other interactions

The FFT does not claim that:

- electromagnetism is "non-fundamental" in all senses,
- Yang–Mills theories are incomplete or incorrect,
- matter fields are derivative or secondary in ontology.

It claims only that non-gravitational interaction fields are structurally open: their conserved quantities depend on independent external sectors or bookkeeping.

This is consistent with standard practice, where such fields are embedded in larger frameworks (matter + charges + interactions) and are not expected to close autonomously.

6.4 Relation to effective field theory

Effective field theory (EFT) does not contradict the FFT. On the contrary, EFT formalises openness:

- cutoffs,
- renormalisation prescriptions,
- scale-dependent couplings,

are explicit acknowledgements of non-closure.

The FFT simply makes explicit what EFT already assumes: openness is acceptable for effective theories, but incompatible with claims of fundamental closure.

6.5 Why closure is conservative, not radical

Closure is often perceived as a strong requirement only because it is rarely stated explicitly. In practice:

- GR is trusted precisely because its vacuum sector is rigid and closed.
- Deviations from GR are tolerated only when supplemented by phenomenology.
- Theories that require continuous patching are treated as provisional.

The FFT formalises this implicit hierarchy. It does not introduce new standards; it codifies existing ones.

6.6 What rejecting the theorem entails

To reject the FFT without contradiction, a critic must explicitly reject at least one of the following:

1. **Classical covariance** as a requirement for fundamental interactions;
2. **Law-completeness** as a criterion of fundamentality;
3. **Self-contained conservation** in regimes where conserved quantities are standard;
4. **Exhaustion under symmetry** as a diagnostic of closure.

Each rejection carries a clear methodological cost and must be defended on its own terms. The theorem itself is internally consistent.

6.7 Scope limitations and what is deliberately excluded

The FFT is intentionally limited in scope. It does not address:

- quantum gravity or UV completion,
- microscopic ontology of matter fields,
- cosmological initial conditions,
- numerical values of coupling constants,
- detailed dynamics of non-vacuum gravitational regimes.

These exclusions are not weaknesses. They preserve deductive clarity and prevent the theorem from depending on unsettled physics.

6.8 Anticipated objections and their status

The principal objections to the FFT fall into three classes:

1. **Semantic objections**: redefinitions of "fundamental" that abandon law-completeness or closure. These objections change the subject rather than refute the theorem.
2. **Philosophical objections**: anti-foundational or instrumentalist positions denying the relevance of structural classification. These are logically consistent but lie outside physics methodology.
3. **Technical scope objections**: concerns about conserved quantities in fully general spacetimes. These are addressed by explicit scoping to closure regimes consistent with established practice.

No known physical counterexample satisfies the axioms while violating the conclusion.

6.9 Why the theorem strengthens, rather than weakens, GR

The FFT explains why:

- vacuum GR is exact rather than approximate,
- Birkhoff-type rigidity is expected,
- higher-derivative or extra-field modifications require external justification,
- dark-sector supplements appear when closure is implicitly abandoned.

Rather than competing with GR, the FFT provides a structural justification for its privileged status in classical physics.

6.10 Bridge to the Ananke (Gravitational Closure) Theorem

With gravity isolated as the unique closing interaction field, a new question arises:

Given that gravity must close, what internal response structure is permitted?

This question cannot be answered at the upstream level. It requires an additional structural requirement: orthogonality of response modes.

The Ananke (Gravitational Closure) Theorem begins precisely at this point. It takes the output of the FFT as input and derives the unique admissible internal structure of gravity.

6.11 Final summary

The Fundamental Fields Theorem establishes that:

- closure is a meaningful and conservative criterion for fundamentality;
- gravity is uniquely admissible as a closing classical interaction field;
- openness of other interaction fields is structural, not accidental.

The theorem is conditional, deductive, and independent of phenomenology. Its conclusions follow directly from its definitions and axioms.