Recitation 2: Discrete Probability

Brown University CS145: Probability & Computing

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Example 1:

In a World Series, teams A and B play until one team has won four games. Assume that each game played is won by team A with probability p, independently of all previous games.

a) *For $g = 4$ through 7, find a formula in terms of p and $q = 1 - p$ for the probability that team A wins in g games.*

b) *What is the probability that team A wins the World Series, in terms of p and q?*

c) *Let X be a binomial$(7, p)$ random variable. Explain why $P(A\ wins) = P(X \geq 4)$*

d) *Let G represent the number of games played. What is the distribution of G? For what value of p is G independent of the winner of the series?*

Solution:

a) For $g = 4$ games, team A wins with probability p^4. For $g > 4$, team A has to win 4 games, but not the first 4. So they need to win any 4 games out of the first $g - 1$ games, and win the last game. Thus the probability equals:

$$\binom{g-1}{3} p^4 q^{g-4} \tag{1}$$

b) Following equation 1

$$P(team\ A\ wins) = \sum_{g=4}^{7} \binom{g-1}{3} p^4 q^{g-4} \tag{2}$$

c) The question can also be stated as: Play 7 games in total, and not terminate until 7 games are played. A has to win more than 4 games – that's basically equal to $Binomial(7, p)$. Or if you want to use a mathematical view

$$\begin{aligned} P(X \geq 4) &= p^7 + 7p^6q + 21p^5q^2 + 35p^4q^3 \\ &= p^7 + p^6q + 6p^6q + 6p^5q^2 + 15p^5q^2 + 35p^4q^3 \\ &= p^6 + 6p^5q + 15p^4q^2 + 20p^4q^3 \\ &= p^6 + p^5q + 5p^5q + 15p^4q^2 + 20p^4q^3 \\ &= p^5 + 5p^4q + 10p^4q^2 + 20p^4q^3 \\ &= p^4 + 4p^4q + 10p^4q^2 + 20p^4q^3 \end{aligned} \tag{3}$$

d) Let G be the number of games played until the first wins. Then according to equation 1

$$\begin{aligned} P(G = g) &= P(A\ wins\ in\ g\ games) + P(B\ wins\ in\ g\ games) \\ &= \binom{g-1}{3} p^4 q^{g-4} + \binom{g-1}{3} q^4 p^{g-4} \end{aligned} \tag{4}$$

The expression won't depend on p only if $p = \frac{1}{2}$