

COMP2610/COMP6261 – Information Theory

Tutorial 7

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Question 1. Inequality

Suppose a coin is tossed n times. The coin is known to land “heads” with probability p . The number of observed “heads” is recorded as a random variable X .

- (a) What is the exact probability of X being $n - 1$ or more?
- (b) Using Markov’s inequality, compute a bound on the same probability as the previous part.
- (c) Suppose $n = 2$. For what values of p will the bound from Markov’s inequality be within 1% of the exact probability?

Question 2. AEP and Source Coding

A sequence of bits is generated by i.i.d. draws from an ensemble with probabilities $p_0 = 0.995$ and $p_1 = 0.005$. Sequences are coded in 100-bit blocks. Every 100-bit block with at most three 1s is assigned a codeword. Those blocks with more than three 1s are not assigned codewords.

- (a) What is the minimum required length of the assigned codewords if they are all to be of the same length?
- (b) Calculate the probability of observing a 100-bit block that has no associated codeword
- (c) (Harder) Use Chebyshev’s inequality to bound the probability of observing a 100-bit block for which no codeword has been assigned. Compare the bound to the probability just calculated.

Question 3. Typical Set

Let X_N be an extended ensemble for X with $\mathcal{A}_X = \{0, 1\}$ and $P_X = \{0.4, 0.6\}$.

- (a) Calculate the entropy $H(X)$.
- (b) Let $N = 25$ and $\beta = 0.1$.
 - i. Which sequences in X_N fall in the typical set $T_{N\beta}$?
 - ii. Compute $P(x \in T_{N\beta})$, the probability of a sequence from X_N falling in the typical set.
 - iii. How many elements are there in $T_{N\beta}$?
 - iv. How many elements are in the smallest δ -sufficient subset S_δ for $\delta = 0.9$?
 - v. What is the essential bit content $H_\delta(X_N)$ for $\delta = 0.9$?