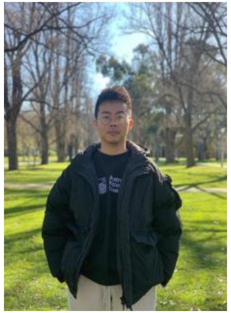
COMP2610/6261: INFORMATION THEORY

Week 1 Tutorial: Elementary Probability

Australian National University

TEACHING TEAM:



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TUTORIALS AND ASSIGNMENTS OUTLINE:

Week	Topic	Tutor
1	Elementary Probability	Manish
2	Probability and Bayesian Interface	Manish
3	Entropy and Information	Angela
4	Relative Entropy and Mutual Information	Naisheng
5	Fundamental and Probabilistic Inequalities	Naisheng
6	Joint and conditional Entropy, Markov Chain	Naisheng
7	AEP and Source Coding	Naisheng
8	Arithmetic Coding	Naisheng
9	Huffman and Shannon-Fano-Elias Code	Naisheng
10	Channel capacity, Block codes	Naisheng
11	Channel Coding Theorems	Zhifeng
12	Review and Exam Preparation	Manish

Week		Tutor
1		
2		
3	Assignment 1 Release : Friday 5:00 PM	Manish
4	Assignment 1 Drop-in sessions (Fri.)	Manish
5	Assignment 1 Due: Friday 5:00 PM	
6	Assignment 2 Release: Friday 5:00 PM	Angela
7	Assignment 1 Marks released & feedback (Tue.) Assignment 2 Drop-in sessions (Wed.)	Manish Angela
8	Assignment 2 Due: Monday 9:00 AM	
9		
10	Assignment 3 Release Assignment 2 Marks released & feedback	Zhifeng Angela
11	Assignment 3 Drop-in sessions	Zhifeng
12	Assignment 3 Due: Friday, 5:00 PM	

SOME KEY CONCEPTS:

Probability (study of uncertainty) quantifies the likelihood of a particular event to occur.

For example: What are the chances to get an even number when you roll a fair six-sided dice? 

- ❖ Experiment: Any procedure that can be infinitely repeated and has a well-defined set of outcomes. E.g., Throwing Dice
- ❖ Sample Space (S): Set of all possible outcomes of an experiment. S = {1, 2, 3, 4, 5, 6}
- ❖ Event (E): Set of favorable outcomes of an experiment to which a probability is assigned. E = {2, 4, 6}
  - *__Independent Event:__ When the outcome of one event does not affect the outcome of another. E.g., Flipping a coin*
  - *__Dependent Event:__ When the outcome of one event does affect the outcome of another. E.g., Drawing cards from a deck without replacement.*
- ❖ Probability (P): Likelihood of an event happening. *P(E)* = $\frac{No.of\ favourable\ outcomes\ (|E|)}{Total\ no.of\ possible\ outcomes\ (|S|)}$. E.g., P(E) = 3/6 = ½
  - *Probability is a number between 0 to 1 i.e., $0 \leq P(E) \leq 1$.*
  - *Sum of probabilities of all the elements in a sample space S is always 1.*
  - *Probability of an event (E) and its complement (E') is always 1 i.e., P(E) + P (E') = 1.*

Question 1: A spinner is divided into 5 equal sections, with sections labeled 1, 2, 3, 4, and 5. Compute the probability of:

a) Spinning a 4 on the spinner.

b) Spinning an even number on the spinner.

c) Spinning a prime number on the spinner.

Solution:

Step 1: Find the sample space (S). In this case, S = {1, 2, 3, 4, 5}.

Step 2: Check if equiprobable or not. As the spinner is divided into 5 equal sections, hence *P(1) = P(2) = P(3) = P(4) = P(5) = 1/5.*

Step 3: Find events corresponding to each probability assigned. For example, E1 (spinning a 4 on the spinner) = {4}, or E2 (spinning an even number on the spinner) = {2, 4}.

1. A spinner is divided into 5 equal sections, with sections labeled 1, 2, 3, 4 and 5. Compute the probability of:

Probability of event A [P(A)] = $\frac{Number\ of\ favourable\ outcomes\ (n(A))}{Total\ number\ of\ possible\ outcomes\ (n(S))}$

Sample space (S) = {1,2,3,4,5}

So, P(1) = P(2) = P(3) = P(4) = P(5) = $\frac{Number\ of\ favourable\ outcomes}{Total\ number\ of\ possible\ outcomes}$ = 1/5

a) spinning a 4 on the spinner.

P (4) = **1/5**

b) spinning an even number on the spinner.

Event of spinning an even number on the spinner (E1) = {2,4}

P (even number) = P(2) + P(4) = 1/5 + 1/5 = **2/5**

c) Spinning a prime number on the spinner.

Event of spinning a prime number on the spinner (E2) = {2,3,5}

P (prime number) = P(2) + P(3) + P(5) = 1/5 + 1/5 + 1/5 = **3/5**

Question 2: Let us assume that ACT number plates have three letters followed by three numbers (e.g., YOA077). What will be the probability that a randomly chosen number plate will have an ACT with the number ending in a 7 (ACT##7)?

Solution: Probability of event A [P(A)] = $\frac{Number\ of\ favourable\ outcomes\ (n(A))}{Total\ number\ of\ possible\ outcomes\ (n(S))}$

Sample space for integers (E1) = {0,1,2,3,4,5,6,7,8,9}

⇨ P(0) = P(1) = P(2)………… = P(9) = 1 / 10

And Sample space for alphabets (E2) = {A,B,C,D,E,F,G,H……………………………,X,Y,Z}

⇨ P(A) = P(B) = P(C) = ……………………………… = P(Z) = 1/26

So basically, we have 6 positions (L_5, L_4, L_3, L_2, L_1, L_0) out of which four is fixed as ACT_ _7.

Question 2: Let us assume that ACT number plates have three letters followed by three numbers (e.g., YOA077). What will be the probability that a randomly chosen number plate will have an ACT with the number ending in a 7 (ACT##7)?

Solution (Cont.):

=> L_2 and L_1 can take any integer between 0 to 9.

Hence probability that a randomly chosen number plate will have an ACT with the number ending in a 7 will be $P(E) = (\frac{1}{26}) * (\frac{1}{26}) * (\frac{1}{26}) * (\frac{10}{10}) * \frac{10}{10}) * (\frac{1}{10}) = \frac{100}{(26)^3 * (10)^3} = \frac{1}{175760}$

Question 3: ACT Govt. plan to enforce speed limits during the morning rush hour on four different routes into the city. The traps on routes A, B, C, and D are operated 40% , 30%, 20%, and 30% of the time, respectively. Arya always speeds to work, and she has probability 0.2, 0.1, 0.5, and 0.2 of using those routes. Compute the probability of:

a) Arya getting a ticket on any one morning.

b) Arya will go five mornings without the tickets.

Solution:

According to question,

P(A) = 0.2	P(B) = 0.1	P(C) = 0.5	P(D) = 0.2
P(traps_A) = 0.4	P(traps_B) = 0.3	P(traps_C) = 0.2	P(traps_D) = 0.3

a) Arya getting a ticket on any one morning.

To calculate Arya getting tickets on any one morning we need to sum the probabilities of getting a ticket by the frequencies with which she travels each route (P(event));

⇨ P(A) * P(traps_A) + P(B) * P(traps_B) + P(C) * P(traps_C) + P(D) * P(traps_D)

⇨ 0.2 * 0.4 + 0.1 * 0.3 + 0.5 * 0.2 + 0.2 * 0.3

⇨ 0.08 + 0.03 + 0.10 + 0.06

⇨ **0.27**

b) Arya will go five mornings without the tickets.

Since sum of probabilities of an event and its complementary event is 1 i.e., $P(\text{event}) + P(\text{event})^C = 1$

⇨ Probability that Arya will go without the tickets in any morning $(P(\text{event})^C) = 1 - P(\text{event})$

⇨ 1 - 0.27

⇨ **0.73**

⇨ **Probability that Arya will go Five mornings without the ticket will be = $(0.73)^5$ = 0.2073**

Question 4: In an urn there are 5 blue, 3 red, and 2 yellow marbles. If you draw 3 marbles, what is the probability that less than 2 will be red if:

a) marbles are drawn with replacement.

b) marbles are drawn without replacement.

Solution:

Blue	Red	Yellow	Total
5	3	2	10

Probability of blue marble (P (B)) = 5/10 = 0.5

Probability of red marble (P (R)) = 3/10 = 0.3

Probability of yellow marble (P (Y)) = 2/10 = 0.2

If three marbles are drawn simultaneously , then probability of drawing less than 2 red marbles when

a) the marbles are drawn with replacement.

Method 1 : Conventional approach

The probabilities are fixed. Hence the probability of no red at all is P(not R) = 1 – P(R) = 1 – 0.3 = 0.7.

⇨ Probability that no red will be drawn in all three drawing of marble, P (event : R = 0) = $P(\text{not } R)^3$

⇨ **P (event : R = 0)** = $(0.7)^3$ = **0.343**

Since there are three ways to get one red - in the first draw, second draw or third draw.

In all three cases, the probability will be the same. Hence,

⇨ **P (event : R = 1)** = 3 * (P(R)*P(not R)* P(not R)) = 3 * (0.3 * 0.7 * 0.7) = 3 * (0.147) = **0.441**

⇨ **P (event : R < 2) =** P (event : R = 0) + P (event : R = 1) **=** 0.343 + 0.441 **= 0.784**

Method 2 : Binomial distribution

$$P_X = {}^nC_x\, p^x\, q^{n-x}$$

Where, **n** = number of trials
p = probability of success on a single trial
q = probability of failure on a single trial = 1 – p

P (event : R < 2) = P (event : R = 0) + P (event : R = 1) = $^3C_0\,(P(R))^0\,(1-P(R))^{3-0} + {}^3C_1\,(P(R))^1\,(1-P(R))^{3-1}$

⇨ **P (event : R < 2) =** $^3C_0\,(0.3)^0\,(0.7)^3 + {}^3C_1\,(0.3)^1\,(0.7)^2 = (0.7)^3 + 3 * (0.3 * 0.7 * 0.7)$ = 0.343 + 0.441

⇨ **P (event : R < 2) = 0.784**

b) the marbles are drawn without replacement.

So, 3 of the 10 marbles are red. The probability of drawing less than two is the sum of the probabilities of drawing either 1 or none:

$$P\ (Red < 2) = \frac{(\text{No.of ways to select}(0 \text{ red and } 3 \text{ other marbles}) + \text{No.of ways to select}(1 \text{ red and } 2 \text{ other marbles}))}{\text{No.of ways to select } 3 \text{ marbles out of } 10}$$

$$= ({}^{3}C_{0})\ ({}^{7}C_{3}) + ({}^{3}C_{1})\ ({}^{7}C_{2}) \,/\, ({}^{10}C_{3}) = \frac{1*(35)+3*21}{120} = \frac{98}{120} = \frac{49}{60}$$

Question 4: Nick will miss an important Cricket match while taking his Information theory exam, so he sets both his VCRs to record it. The first VCR has 70% chance to successfully recording the match and the second VCR has 60% chance of successfully recording the match. What is the probability that he gets home after the exam and finds? (Note: Here we assume that events A and B are independent, so with P(A) = 0.7 and P(B) = 0.6 and their set complements A^C and B^C occurring with probabilities 0.3 and 0.4 respectively).

a) No copies of the match.

b) One copy of the match.

c) Two copy of the match

Solution:

According to question,

Probability of VCR 1 recording successfully (P(A)) = 0.7

⇨ Probability of VCR 1 not recording successfully ($P(A^c)$) = 1 - P(A) = 1 - 0.7 = 0.3

Similarly, Probability of VCR 2 recording successfully (P(B)) = 0.6

⇨ Probability of VCR 2 not recording successfully ($P(B^c)$) = 1 - P(B) = 1 - 0.6 = 0.4

Let E1 be the event when no copies of the Cricket match will be available,

E2 be the event when 1 copy of the Cricket match will be available, and

E3 be the event when two copies of the Cricket match will be available.

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a) No copies of the Cricket match?

$P(E1) = P(A^c \text{ and } B^c) = P(A^c)P(B^c) = (0.3) * (0.4) = \mathbf{0.12}$

b) One copy of the Cricket match?

Here we need to account that any one VCR (out of the two) needs to record. So,

$P(E2) = P(A \text{ and } B^c) + P(A^c \text{ and } B) = P(A) * P(B^c) + P(A^c) * P(B) = 0.7 * 0.4 + 0.3 * 0.6$

⇨ $P(E2) = 0.28 + 0.18$

⇨ $P(E2) = \mathbf{0.46}$

c) Two copies of the Cricket match?

Here we need to account that both VCRs are recording simultaneously. So,

$P(E3) = P(A \text{ and } B) = P(A) * P(B)$

⇨ $P(E3) = 0.7 * 0.6 = \mathbf{0.42}$

THANK YOU

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