COMP2610/COMP6261 – Information Theory

Tutorial 11

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Question 1.

A binary symmetric channel with input $x \in \{0,1\}$ and output $y \in \{0,1\}$ has transition matrix

$$Q = \begin{bmatrix} 1-q & q \\ q & 1-q \end{bmatrix}$$

Find the mutual information $I(X;Y)$ between the input and output for a general distribution $p_X = (p, 1-p)$ over inputs. Show that the capacity of this channel is $C = 1 - H(q)$ bits.

Question 2.

A binary erasure channel with input $x \in \{0,1\}$ and output $y \in \{0,e,1\}$ has transition matrix

$$Q = \begin{bmatrix} 1-q & 0 \\ q & q \\ 0 & 1-q \end{bmatrix}$$

Find the mutual information $I(X;Y)$ between the input and output for a general distribution $p_X = (p, 1-p)$ over inputs. Show that the capacity of this channel is $C = 1 - q$ bits.

Question 3.

Consider a channel with inputs $X = \{a,b,c\}$, outputs $Y = \{a,b,c,d\}$, and transition matrix

$$Q = \begin{bmatrix} 0.5 & 0 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 0.5 \end{bmatrix}$$

(a) Assuming $p_X = (0.25, 0.25, 0.5)$, what is the mutual information $I(X;Y)$ between the input and output ofthe channel?

(b) Assuming $p_X = (0.25, 0.25, 0.5)$, what is the average probability of the error of the channel if the maximum likelihood estimation is used?

(c) Calvin claims that he has constructed a block code for Q with rate 0.01 bits per transmission and maximal block error probability 1%. Is his claim possible? Justify your answer.

(d) Hobbes claims that he has constructed a block code for Q with rate 100 bits per transmission and maximal block error probability 1%. Is his claim possible? Justify your answer.

Question 4.

Find the channel capacity of the discrete memoryless channel $Y = X + Z$, where $Pr(Z = 0) = Pr(Z = a) = 1/2$. The alphabet for x is $X = 0, 1$. Assume that Z is independent of X. Observe that the channel capacity depends on the value of a.