ASSIGNMENT COVER SHEET

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This coversheet must be attached to the front of your assessment

The assessment is due on 28/08/2023, 9:05 AM unless otherwise specified in the course outline.

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Course Code	COMP6261
Course Name	Information Theory
Assignment Item	Assignment 1
Due Date	28/08/2023,
Date Submitted	26/08/2023

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Signature Nanthawat Anancharoenpakorn

Question 1 : section II

- Random variable $X = \{ \underset{\dots XXXXX}{XXX}, A_X = \{A, M, S\}, P_X = \frac{1}{3} \}$
- once win 3 rounds, game over, max rounds = 5
- random v. $Y = \{ y, A_y = \{3, 4, 5\}, P_y = \square \}$

→ Compute all relevant probability

3 teams play against each other until one of them win 3 rounds.
with 3 game | there are 3 teams. Hence 3 ways.
with $(Y = 3) = 3 \times \left(\frac{1}{3}\right)^3 = \frac{1}{9}$ ∦

with 4 game | let fix A to be the winner then
$\boxed{\_\ \_\ \_}$ A, ${}^3C_2$ since A must have won 2 out of 3 games
then ${}^3C_2 = \frac{3!}{1!\,2!} = 3$. since there are 3 teams,
there are $3 \times 3 = 9$ events. Since $\underline{A}\,\underline{A}\,\underline{M}A \neq AASA$
then we need to multiply by 2 which is $9 \times 2 = 18$
ways. Hence, 18 ways with $(Y = 4) = 18 \times \left(\frac{1}{3}\right)^4 = \frac{2}{9}$

summary	n. events	each prob	game prob.
$y = 3$	3	$(0.33)^3$	$\frac{1}{9}$
$y = 4$	18	$(0.33)^4$	$\frac{2}{9}$
$y = 5$	36	$(0.33)^5$	$\frac{4}{27}$

(a)

$$h(P = AMSSS) = \log \frac{1}{(1/3)^5} = 7.92 \text{ bits}$$

(b) $$H(P) = 3\left(\frac{1}{27}\right)\log(27) + 18\left(\frac{1}{81}\right)\log(81) + 36\left(\frac{1}{243}\right)\log(243) = 3.11 \text{ bit}$$

(c) $$H(Q) = \frac{1}{9}\log(9) + \frac{2}{9}\log\left(\frac{9}{2}\right) + \frac{4}{27}\log\left(\frac{27}{4}\right)$$

$$= 1.24 \text{ bit}$$

(d) $H(P \mid Q = 4) = 18 \times \left(\frac{1}{3}\right)^4 \times \log\left(\frac{1}{(1/3)^4}\right) = 1.4$ bit

e) $H(P \mid G) = H(P) + H(Q \mid P) - H(Q)$ since $H(Q \mid P) = 0$
because Q is a deterministic of P Hence $H(P \mid G) = 1.87$

f) $H(Q \mid P) = 0$ because Q is deterministic of P,
meaning that knowing P will allow to know Q with
certainty example. $P = A\,A\,A$, then $Q = 3$

g) if (civilization win first round ≠ civil win the last round)
then 33%. → N 66%. → normal

if (civil loss 4 consecutive) then 50%. → N
50%. → normal.

summary

$y = 3$	3	$(1/3)^3$	[same]
$y = 4$	12	$(1/3)^4$	
	6	$(1/3)^4 \times \frac{2}{3}$. 66%.	
$y = 5$			

$H(G) =$

h)

Question 2

- Value (v) $V: \{A, 2, 3, .., K\}$ $|V| = 13$
- Suit (s) $S: \{$ heart, diamond, .. $\}$ $|S| = 4$
- Color : c $C: \{$ black, red $\}$ $|C| = 2$
- Face f $F: \{0, 1\}$ $|F| = 2$

① (a) since face card are $\{J, Q, K\}$ which has 2 reds per value. Then prob $(c = red, f = 1) = \frac{6}{52}$

$h(c = red; f = 1) = \log_2 \frac{1}{(6/52)} = 3.1154$ bits ※

(b) $h(V = K \mid f = 1) = \log_2 \frac{1}{(4/12)} = 1.5849$ bits ※
since there are 4 kings and 12 face cards.

(c) since there are 4 suits $H(S) = -\sum p(s) \log_2 p(s)$
$= 4 \times \frac{1}{4} \log\left(\frac{1}{1/4}\right) = 2$ bits and $H(V, S) = 5.7$ bits ※

(d) $= H(V) + H(S) - H(V, S) = 0$ ※ there is independent.

(e) according to data processing inequality and as color is determined by S $I(V; C) \leq I(V; S)$ hence $I(V : C) = 0$ since mutual information can't be negative ※

② Remove 13 ♣ and 7 hearts of 2 to 5 and, A♢ and 2♢

- Value $= \{A, .., K\}$, Face $= \{0, 1\}$, suit $= \{$ heart, diamond, clubs $\}$ $|S| = 3$, color $= \{$ black, red $\}$.

(a) $H(S) = \frac{8}{32} \log\left(\frac{1}{8/32}\right) + \frac{11}{32} \log\left(\frac{1}{11/32}\right) + \frac{13}{32} \log\left(\frac{1}{13/32}\right)$

$H(S) = 1.5575$ bits, $H(V, S) = 5$ bits ※

(b) $I(V; S) = H(V) + H(S) - H(V, S) = 5 + 1.5575 - 5 = 1.5575$. Then reason it different because we have remove some of the card out which lead to the result that suits are not uniform anymore. Then knowing S provide information about V. ※

c) $I(V; S \mid C) = 0.207$, because we reduce the card a lot which lead to less possibility for black.

eg. if we know the card is black, then we know for sure that it must be club. Knowing color cheng the probability make it more certain when we know color of the card.

Question 3

a) $I(X; Y) = \frac{1}{2} \log(2) + \frac{1}{2} \log(2) = 2$ bits Because it dependent to each other completely.

b) $I(X; Y) = H(X) - H(X \mid Y) = 0.58$

c) since the first and last element are the same, we can compute by log sum inequality. $H_2 - H_1 =$

$-2 \frac{(P_i + P_j)}{2} \log\left(\frac{p_i + P_j}{2}\right) + P_i \log p_i + p_j \log p_j \geq 0$.

$H_2 \geq H_1$ imply that entropy will be larger for most uniform distribution.

(d) $X : \{x, x \in X : [1, 2, .., m]\}$, $P_X = \{q_1, .., q_m \mid q_i$ in decreasing order$\}$ • new r.v. Y with 2 more outcome that each prob is divided by 3 and the last term is $\frac{2}{3}$.

$H(X) = -\sum_i^m q_i \log q_i$, $H(Y) = -\sum_i^m \frac{q_i}{3} \log \frac{q_i}{3} + \frac{2}{3} \log \frac{2}{3}$

write $H(Y)$ in term of $H(x)$:

$$H(Y) = -\sum_i^m \frac{q_i}{3} \log \frac{q_i}{3} + \frac{2}{3} \log \frac{2}{3}$$

$$= -\frac{1}{3} \sum_{i=1}^m q_i \log q_i + \frac{2}{3} \log \frac{2}{3}$$

$$= -\frac{1}{3} H(x) + \frac{2}{3} \log \frac{2}{3}$$ ※

Question 4

$Y : (y, \{0,1\}, \{0.5, 0.5\})$

$X : (x, \{a, b, c\}, P_X :$

$y =$ $x =$	a	b	c
0	$1/5$	$3/5$	$1/5$
1	$\frac{3}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

(a) $I(X;Y) = H(X) + H(Y) - H(X,Y)$

$H(X) = 1.58$ bit

$H(Y) = 1$ bit

$H(X|y) = 0.5(1.37) + 0.5(1.37) = 1.37$ bit

$H(X,Y) = 1 + 1.37 = 2.37$

hence, $I(X;Y) = 1.58 + 1 - 2.37 = 0.21$

(b) $D_{KL}(p||q) = \sum_x p(x) \log \frac{p(x)}{q(x)}$

$I(X;Y) = \sum_{x,y} p(x,y) \log_2 \frac{p(x,y)}{p(x)p(y)}$

let y be a binary random v. $(y, y \in \{0,1\}$ $P_Y = \{\frac{1}{2}, \frac{1}{2}\})$

$I(x;y) = \frac{1}{2} \sum_x p(x|Y=1) \log \frac{p(x|Y=1)}{m(x)} + \frac{1}{2} \sum p(x|Y=0) \log \frac{p(x|Y=0)}{m(x)}$

$m(x) = \frac{p(x|Y=1) + p(x|Y=0)}{2}$

$I(x;y) = \frac{1}{2} D_{KL}(p||m) + \frac{1}{2} D_{KL}(q||m)$

(c) As $p(Z=x|Y=y) = p(X=x|Y=1-y)$, it means that conditional of $Z|Y$ is equal to $X|1-y$ then $I(Z;Y) = \frac{1}{2} D_{KL}(q||m) + \frac{1}{2} D_{KL}(p||m)$. Therefore $I(Z;Y) = I(X;Y)$ according to the formula above. since Z and X have same conditional distribution, if we know Y, it will also reduce uncertainty about Z in the same as reduce uncertainty in X.

(d)

Question 5