

COMP2610 / COMP6261 - Information Theory

Tutorial 3: Entropy and Information

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1. Let X be a random variable with possible outcomes $\{1,2,3\}$. Let the probabilities of the outcomes be

$$\begin{aligned}p(X = 1) &= \frac{\theta}{2} \\p(X = 2) &= \frac{\theta}{2} \\p(X = 3) &= 1 - \theta\end{aligned}$$

for some parameter $\theta \in [0,1]$.

Suppose we see N observations of the random variable, $\{x_1, \dots, x_N\}$. Let n_i denote the number of times that we observe the outcome $X = i$, i.e.

$$n_i = \sum_{k=1}^N \begin{cases} 1 & \text{if } x_k = i \\ 0 & \text{else.} \end{cases}$$

- (a) Write down the likelihood function of θ given the observations $\{x_1, \dots, x_N\}$ in terms of n_1, n_2, n_3 .

- (b) Suppose the observations are

$\{3, 3, 1, 2, 3, 2, 2, 1, 3, 1\}$.

Compute the maximum likelihood estimate of θ . (*Hint*: Compute the log-likelihood function, and check when the derivative is zero.)

2. Consider the following joint distribution over X, Y :

$p(X,Y)$		X			
		1	2	3	4
Y	1	0	0	1/8	1/8
	2	1/8	1/16	1/16	0
	3	1/8	1/8	0	0
	4	0	1/16	1/16	1/8

- (a) Show that X and Y are not statistically independent. (*Hint*: You need only show that for at least one specific x,y pair, $p(X = x, Y = y)$ not equal to $p(X = x)p(Y = y)$.)

- (b) Compute the following quantities:

- (i) $H(X)$
- (ii) $H(Y)$
- (iii) $H(X|Y)$
- (iv) $H(Y|X)$
- (v) $H(X,Y)$

3. A standard deck of cards contains 4 *suits* — $\heartsuit, \diamondsuit, \clubsuit, \spadesuit$ (“hearts”, “diamonds”, “clubs”, “spades”) — each with 13 *values* — A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K (The A, J, Q, K are called “Ace”, “Jack”, “Queen”, “King”). Each card has a *colour*: hearts and diamonds are coloured red; clubs and spades are black. Cards with values J, Q, K are called *face cards*.

Each of the 52 cards in a deck is identified by its value v and suit s and denoted vs . For example, $2\heartsuit, J\clubsuit$, and $7\spadesuit$ are the “two of hearts”, “Jack of clubs”, and “7 of spades”, respectively. The variable c will be used to denote a card’s colour. Let $f = 1$ if a card is a face card and $f = 0$ otherwise.

A card is drawn at random from a thoroughly shuffled deck. Calculate:

- (a) The information in observing a red King, i.e., $h(c = \text{red}, v = K)$
 - (b) The conditional information in observing a King given a face card was drawn, i.e., $h(v = K | f = 1)$
 - (c) The entropies $H(S)$ and $H(V, S)$.
4. Let X be a random variable taking on a finite number of values. What is the (general) inequality relationship of $H(X)$ and $H(Y)$ if
- a. $Y = 2^X$?
 - b. $Y = \cos X$?