

COMP2610/COMP6261 – Information Theory

Tutorial 1: Elementary Probability (Soln.)

Week 1, Semester 2, 2023

1. A spinner is divided into 5 equal sections, with sections labeled 1, 2, 3, 4 and 5. Compute the probability of:



$$\text{Probability of event A } [P(A)] = \frac{\text{Number of favourable outcomes } (n(A))}{\text{Total number of possible outcomes } (n(S))}$$

Sample space (S) = {1,2,3,4,5}

$$\text{So, } P(1) = P(2) = P(3) = P(4) = P(5) = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}} = 1/5$$

a) spinning a 4 on the spinner.

$$P(4) = 1/5$$

b) spinning an even number on the spinner.

Event of spinning an even number on the spinner (E1) = {2,4}

$$P(\text{even number}) = P(2) + P(4) = 1/5 + 1/5 = 2/5$$

c) Spinning a prime number on the spinner.

Event of spinning a prime number on the spinner (E2) = {2,3,5}

$$P(\text{prime number}) = P(2) + P(3) + P(5) = 1/5 + 1/5 + 1/5 = 3/5$$

2. Let us assume that ACT number plates have three letters followed by three numbers (e.g., YOA077). What will be the probability that a randomly chosen number plate will have an ACT with the number ending in a 7 (ACT##7)?

$$\text{Probability of event A } [P(A)] = \frac{\text{Number of favourable outcomes } (n(A))}{\text{Total number of possible outcomes } (n(S))}$$

Sample space for integers (E1) = {0,1,2,3,4,5,6,7,8,9}

$$\Rightarrow P(0) = P(1) = P(2) = \dots = P(9) = 1/10$$

And Sample space for alphabets (E2) = {A,B,C,D,E,F,G,H,.....,X,Y,Z}

$$\Rightarrow P(A) = P(B) = P(C) = \dots = P(Z) = 1/26$$

So basically, we have 6 positions (L₅, L₄, L₃, L₂, L₁, L₀) out of which four is fixed as ACT__7.

=> L_2 and L_1 can take any integer between 0 to 9.

Hence probability that a randomly chosen number plate will have an ACT with the number ending in a 7 will

$$\text{be } P(E) = \left(\frac{1}{26}\right) * \left(\frac{1}{26}\right) * \left(\frac{1}{26}\right) * \left(\frac{10}{10}\right) * \left(\frac{10}{10}\right) * \left(\frac{1}{10}\right) = \frac{100}{(26)^3 * (10)^3} = \frac{1}{175760}$$

3. ACT Govt. plan to enforce speed limits during the morning rush hour on four different routes into the city. The traps on routes A, B, C, and D are operated 40% , 30%, 20%, and 30% of the time, respectively. Arya always speeds to work, and she has probability 0.2, 0.1, 0.5, and 0.2 of using those routes. Compute the probability of:

According to question,

$$P(A) = 0.2$$

$$P(B) = 0.1$$

$$P(C) = 0.5$$

$$P(D) = 0.2$$

$$P(\text{traps_A}) = 0.4$$

$$P(\text{traps_B}) = 0.3$$

$$P(\text{traps_C}) = 0.2$$

$$P(\text{traps_D}) = 0.3$$

a) Arya getting a ticket on any one morning.

To calculate Arya getting tickets on any one morning we need to sum the probabilities of getting a ticket by the frequencies with which she travels each route ($P(\text{event})$);

$$\Rightarrow P(A) * P(\text{traps_A}) + P(B) * P(\text{traps_B}) + P(C) * P(\text{traps_C}) + P(D) * P(\text{traps_D})$$

$$\Rightarrow 0.2 * 0.4 + 0.1 * 0.3 + 0.5 * 0.2 + 0.2 * 0.3$$

$$\Rightarrow 0.08 + 0.03 + 0.10 + 0.06$$

$$\Rightarrow \mathbf{0.27}$$

b) Arya will go five mornings without the tickets.

Since sum of probabilities of an event and its complementary event is 1 i.e., $P(\text{event}) + P(\text{event})^C = 1$

$$\Rightarrow \text{Probability that Arya will go without the tickets in any morning } (P(\text{event})^C) = 1 - P(\text{event})$$

$$\Rightarrow 1 - 0.27$$

$$\Rightarrow \mathbf{0.73}$$

$$\Rightarrow \text{Probability that Arya will go Five mornings without the ticket will be } = (0.73)^5 = \mathbf{0.2073}$$

4. In an urn there are 5 blue, 3 red, and 2 yellow marbles. If you draw 3 marbles, what is the probability that less than 2 will be red if:

Blue	Red	Yellow	Total
5	3	2	10

$$\text{Probability of blue marble } (P(B)) = 5/10 = 0.5$$

Probability of red marble ($P(R) = 3/10 = 0.3$)

Probability of yellow marble ($P(Y) = 2/10 = 0.2$)

If three marbles are drawn simultaneously, then probability of drawing less than 2 red marbles when

a) the marbles are drawn with replacement.

Method 1 : Conventional approach

The probabilities are fixed. Hence the probability of no red at all is $P(\text{not } R) = 1 - P(R) = 1 - 0.3 = 0.7$.

⇒ Probability that no red will be drawn in all three drawing of marble, $P(\text{event : } R = 0) = P(\text{not } R)^3$

⇒ **$P(\text{event : } R = 0) = (0.7)^3 = 0.343$**

Since there are three ways to get one red - in the first draw, second draw or third draw.

In all three cases, the probability will be the same. Hence,

⇒ **$P(\text{event : } R = 1) = 3 * (P(R) * P(\text{not } R) * P(\text{not } R)) = 3 * (0.3 * 0.7 * 0.7) = 3 * (0.147) = 0.441$**

⇒ **$P(\text{event : } R < 2) = P(\text{event : } R = 0) + P(\text{event : } R = 1) = 0.343 + 0.441 = 0.784$**

Method 2 : Binomial distribution

$$P_X = {}^n C_x p^x q^{n-x}$$

Where, **n** = number of trials

p = probability of success on a single trial

q = probability of failure on a single trial = $1 - p$

$P(\text{event : } R < 2) = P(\text{event : } R = 0) + P(\text{event : } R = 1) = {}^3 C_0 (P(R))^0 (1-P(R))^{3-0} + {}^3 C_1 (P(R))^1 (1-P(R))^{3-1}$

⇒ **$P(\text{event : } R < 2) = {}^3 C_0 (0.3)^0 (0.7)^3 + {}^3 C_1 (0.3)^1 (0.7)^2 = (0.7)^3 + 3 * (0.3 * 0.7 * 0.7) = 0.343 + 0.441$**

⇒ **$P(\text{event : } R < 2) = 0.784$**

b) the marbles are drawn without replacement.

So, 3 of the 10 marbles are red. The probability of drawing less than two is the sum of the probabilities of drawing either 1 or none:

$$\begin{aligned} P(\text{Red} < 2) &= \frac{(\text{No. of ways to select (0 red and 3 other marbles)} + \text{No. of ways to select (1 red and 2 other marbles)})}{\text{No. of ways to select 3 marbles out of 10}} \\ &= \frac{{}^3 C_0 ({}^7 C_3) + {}^3 C_1 ({}^7 C_2)}{{}^{10} C_3} = \frac{1 * (35) + 3 * 21}{120} = \frac{98}{120} = \frac{49}{60} \end{aligned}$$

5. Nick will miss an important Cricket match while taking his Information theory exam, so he sets both his VCRs to record it. The first VCR has 70% chances to successfully record the match and the second VCR has 60% chances to successfully record the match. What is the probability that he gets home after the exam and finds? (Note: Here we assume that events A and B are independent, so with $P(A) = 0.7$ and $P(B) = 0.6$ and their set complements A^c and B^c occurring with probabilities 0.3 and 0.4 respectively).

According to question,

Probability of VCR 1 recording successfully ($P(A)$) = 0.7

⇒ Probability of VCR 1 not recording successfully ($P(A^c)$) = $1 - P(A) = 1 - 0.7 = 0.3$

Similarly, Probability of VCR 2 recording successfully ($P(B)$) = 0.6

⇒ Probability of VCR 2 not recording successfully ($P(B^c)$) = $1 - P(B) = 1 - 0.6 = 0.4$

Let E_1 be the event when no copies of the Cricket match will be available,

E_2 be the event when 1 copy of the Cricket match will be available, and

E_3 be the event when two copies of the Cricket match will be available.

a) No copies of the Cricket match?

$$P(E_1) = P(A^c \text{ and } B^c) = P(A^c)P(B^c) = (0.3) * (0.4) = \mathbf{0.12}$$

b) One copy of the Cricket match?

Here we need to account that any one VCR (out of the two) needs to record. So,

$$P(E_2) = P(A \text{ and } B^c) + P(A^c \text{ and } B) = P(A) * P(B^c) + P(A^c) * P(B) = 0.7 * 0.4 + 0.3 * 0.6$$

$$\Rightarrow P(E_2) = 0.28 + 0.18$$

$$\Rightarrow P(E_2) = \mathbf{0.46}$$

c) Two copies of the Cricket match?

Here we need to account that both VCRs are recording simultaneously. So,

$$P(E_3) = P(A \text{ and } B) = P(A) * P(B)$$

$$\Rightarrow P(E_3) = 0.7 * 0.6 = \mathbf{0.42}$$