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School of Computing
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Semester 2, 2023
Self-assessment

COMP3670: Introduction to Machine Learning

Problem 1: Matrix addition and Multiplication
(1pt) We have three matrices: $\boldsymbol{A} \in R^{3\times 2}$, *i.e.,* real-valued 3 by 2 matrix; $\boldsymbol{B} \in R^{2\times 1}$; $\boldsymbol{C} \in R^{3\times 1}$.
$\boldsymbol{A} = \begin{bmatrix} 2 & -3 \\ 5 & 6 \\ 1 & -3 \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}, \boldsymbol{C} = \begin{bmatrix} -5 \\ 1 \\ 0 \end{bmatrix}$. Calculate $\boldsymbol{AB}+\boldsymbol{C}$.

Problem 2: Gaussian Elimination for System of Linear Equations
(2 pts) Solve the following system of linear equations. You can use any method you know of, such as intuitively solving it, or using the constructive Gaussian Elimination method.

$$\begin{cases} x_1 + x_2 + x_3 = 8 \\ x_2 + 2x_3 = 2 \end{cases}$$

Problem 3: Group
(1pt) Consider the set $\{1, -1\}$ together with the operation multiplication (*i.e.,* $\times$). Is this set a Group? Please explain.

Problem 4: Abelian Group
(2pt) Determine if the set of matrices of the form

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix},$$

where θ is a real number, forms an Abelian Group under matrix multiplication.

Problem 5: properties of matrix transpose
(1pt) For $\boldsymbol{A} \in R^{m \times n}$, $\boldsymbol{B} \in R^{m \times n}$, prove that $(\boldsymbol{A}+\boldsymbol{B})^T = \boldsymbol{A}^T + \boldsymbol{B}^T$

Problem 6: Matrix Inverse
(1pt) Find the inverse of

$$\mathbf{A} = \begin{bmatrix} 4 & 1 \\ 1 & 5 \end{bmatrix}.$$