Exercise 1

John Bag = { 15 red , 5 , blue } 20 total

Asheley { 10 red , 5 blue } 15 total.

1) $\frac{15}{20} = \frac{3}{4}$ #

2) John Bag

- red $15/20$
  - red $14/19$
  - blue $5/19$
- blue $5/20$
  - red $15/19$
  - blue $4/19$

second pick is Red

$$= \frac{15}{20}\left(\frac{14}{19}\right) + \left(\frac{5}{20}\right)\left(\frac{15}{19}\right)$$

$$= \frac{3}{4} \text{ or } 0.75 \ \#$$

3) mix bag = { 25 red , 10 blue } let S = (first pick, second pick) S = (red , blue)

two eggs from A. : $\frac{10}{25} \times \frac{5}{10} = \frac{1}{5}$ #

Exercise 2

1) $P(X=k) = (1-p)^{k-1} \times p = (1-p)^4 p$ #

2) prob. of Tao getting free drink is p. Since the event is independent to each other or meaning that the first free drink or any previous drink doesn't change the prob of Tao getting free drink. #

3) $L(p) = (1-p)^9 \times p$ we take derivative with respect to p and set it to 0, we get

$$\frac{dL(p)}{dp} = (1-p)^9 \times p$$

$$= 9(1-p)^8 \times p \times -1 + (1-p)^9$$

$$0 = -9p(1-p)^8 + (1-p)^9$$

$$9p(1-p)^8 = (1-p)^9 \quad \text{then solve for } p$$

Hence, $p_{ml} = \frac{1}{10}$ #

4) α posterior = 2 + 1 = 3 , β posterior = 2 + 9 = 11 Hence, mode

$$= \frac{3-1}{3+11-2} = \frac{1}{6} \ \#$$

Bayes approach give higher value of the estimate, slightly more chance to get a free drink. This is due to consideration of prior belief given Beta distribution with observed data

Exercise 3

1) $p(x|y_1, y_2) = \underbrace{p(y_1|x)\, p(y_2|x)}_{\text{likelihood}} \times \underbrace{p(x)}_{\text{prior}}$ since

$$p(x) = N(x; 0, \sigma_0^2) = \frac{1}{\sqrt{2\pi\sigma_0^2}} \exp\left(-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}\right)$$

$$p(y_1|x) = N(y_1; x, \sigma_1^2) = \frac{1}{\sqrt{2\pi\sigma_1^2}} \exp\left(-\frac{1}{2}\frac{(y_1-x)^2}{\sigma_1^2}\right)$$

$$p(y_2|x) = N(y_2; x, \sigma_2^2) = \frac{1}{\sqrt{2\pi\sigma_2^2}} \exp\left(-\frac{1}{2}\frac{(y_2-x)^2}{\sigma_2^2}\right)$$

Hence,

$$p(x|y_1, y_2) = \frac{1}{2\sqrt{2}\,\pi^{3/2}\sqrt{\sigma_0^2}\sqrt{\sigma_1^2}\sqrt{\sigma_2^2}} \exp\left(-\frac{1}{2}\left(\frac{x^2}{\sigma_0^2} + \frac{(x-y_1)^2}{\sigma_1^2} + \frac{(x-y_2)^2}{\sigma_2^2}\right)\right)$$

$$\text{Mean}_{posterior} = \frac{\left(\frac{y_1}{\sigma_1^2} + \frac{y_2}{\sigma_2^2}\right)}{\left(\frac{1}{\sigma_0^2} + \frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}\right)}$$

$$\text{Variance}_{posterior} = \left(\frac{1}{\sigma_0^2} + \frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}\right)^{-1}$$

2) (a) when measurement noise close to infinity, posterior distribution will be dominated by prior, measurement is less informative

(b) when prior variance is large as infinity, posterior is dominated by the measurement, as prior is less informative.

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Exercise 4

1) posterior = likelihood x prior where

likelihood = $p(y|B) = \text{Gamma}(y; \alpha, B) = \frac{B^{\alpha}}{\Gamma(\alpha)} y^{\alpha-1} e^{-By}$

prior gamma = $\text{Gamma}(B; \alpha_0, B_0) = p(B)$

$\hookrightarrow p(B) = \frac{B_0^{\alpha_0}}{\Gamma(\alpha_0)} B^{\alpha_0 - 1} e^{-BB_0}$

since posterior $p(B|y) = p(y|B) \times p(B)$ we can substitute

$$p(B|y) \propto (B^{\alpha} y^{\alpha-1} e^{-By}) \times (B^{\alpha_0 - 1} e^{-BB_0})$$

$$p(B|y) \propto B^{\alpha + \alpha_0 - 1} e^{-B(y+B_0)}$$ which is the

form of $\text{Gamma}(B; \alpha_0 + \alpha, B_0 + y)$

2) posterior = likelihood x prior where

likelihood $p(y|\mu, T) = \sqrt{\frac{T}{2\pi}} e^{-\frac{(y-\mu)^2 T}{2}} = N(y; \mu, T^{-1})$

prior $p(\mu, T) = \frac{B^{\alpha_0} \sqrt{T_0}}{\Gamma(\alpha_0) \sqrt{2\pi}} T^{\alpha_0 - \frac{1}{2}} e^{-B_0 T} e^{-\frac{TT_0(\mu - \mu_0)^2}{2}}$

posterior $p(\mu, T|y) = p(y|\mu, T) \times p(\mu, T)$ Then

$$p(\mu, T|y) \propto \sqrt{T} e^{-\frac{(y-\mu)^2}{2}} \times T^{\alpha_0 - 1} e^{-B_0 T} e^{-\frac{T(\mu-\mu_0)^2}{2}}$$

$$p(\mu, T|y) \propto T^{\alpha_0 + \frac{1}{2} - 1} e^{-T\left(B_0 + \frac{(y-\mu)^2}{2} + \frac{(\mu-\mu_0)^2}{2}\right)}$$

which is the form of Normal gamma.