SUBJECT: NO: DATE: / /

Excercise 1

1) To show that $x \notin U$, we need to show that x is not a linear combination of U. We can set

$$a\begin{bmatrix}-1\\1\\1\end{bmatrix} + b\begin{bmatrix}2\\-1\\-2\end{bmatrix} = \begin{bmatrix}8\\4\\16\end{bmatrix}$$ where a and b are scalar is $\in$ of R

So $-a + 2b = 8$, it show the contradict in
$a - b = 4$ equation which mean that
$a - 2b = 16$ $x \notin U$.

2,3 Since generating set U is a basis, represent as

$B = \begin{bmatrix}-1 & 2\\1 & -1\\1 & -2\end{bmatrix}$ then $B^TB = \begin{bmatrix}3 & -5\\-5 & 9\end{bmatrix}$, $B^Tx = \begin{bmatrix}12\\-20\end{bmatrix}$

Then we solve normal equation $B^TB\lambda = B^Tx$ to find λ

$$\begin{bmatrix}3 & -5\\-5 & 9\end{bmatrix}\begin{bmatrix}\lambda_1\\\lambda_2\end{bmatrix} = \begin{bmatrix}12\\-20\end{bmatrix} \rightarrow \lambda = \begin{bmatrix}4\\0\end{bmatrix}$$

now we need to compute $\pi_{(U)}(x) = B\lambda = \begin{bmatrix}-4\\4\\4\end{bmatrix}$

Then we compute the distance U: $\|x - \pi_U(x)\|$

$= \|[12 \ 0 \ 12]^T\| = \sqrt{288}$

4. Let u_1 and u_2 be the column of matrix $A = \begin{bmatrix}-1 & 2\\1 & -1\\1 & -2\end{bmatrix}$

We want to convert u_1, u_2 to v_1, v_2 where $v_1, v_2 \in B$.

$v_1 = \frac{u_1}{\sqrt{3}} = \frac{1}{\sqrt{3}}\begin{bmatrix}-1\\1\\1\end{bmatrix}$ $\quad w_2 = u_2 - (u_2 \cdot v_1)v_1$

and normalize

w_2 to get $v_2 = \begin{bmatrix}\frac{2}{\sqrt{6}}\\-1/\sqrt{6}\\-1/\sqrt{6}\end{bmatrix}$ Hence $B = \begin{bmatrix}-1/\sqrt{3} & 2/\sqrt{6}\\1/\sqrt{3} & -\frac{1}{\sqrt{6}}\\1/\sqrt{3} & -\frac{1}{\sqrt{6}}\end{bmatrix}$

5. we want to minimize $\|x - Q\theta\|^2 + \lambda\|\theta\|^2$ given vector θ

we can set the objective function: $F(\theta) = \|x - Q\theta\|^2 + \lambda\|\theta\|^2$

$= x^Tx - 2\theta^TQ^Tx + \theta^TQ^TQ\theta + \lambda\theta^T\theta$

To find minimum, let compute gradient $F(\theta)$ with respect to θ and set to 0, we get $\nabla F(\theta) = -2Q^Tx + 2Q^TQ\theta + 2\lambda\theta = 0$

$Q^Tx = (Q^TQ + \lambda I)\theta$ then we simplify to get

$\theta = \frac{1}{\lambda+1}Q^Tx$

6.

$$\theta = \frac{1}{11}\underbrace{\begin{bmatrix}2/\sqrt{6} & -1/\sqrt{6} & -1/\sqrt{6}\\-1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3}\end{bmatrix}}_{2\times3}\underbrace{\begin{bmatrix}8\\4\\16\end{bmatrix}}_{3\times1} = \frac{1}{11}\underbrace{\begin{bmatrix}-4/\sqrt{6}\\12/\sqrt{3}\end{bmatrix}}_{2\times1}$$

Excercise 2

1) $\frac{dx^TABCx}{dx}$ with $\frac{dx^TBx}{dx} = x^T(B+B^T)$ we get

$x^T(ABC + (ABC)^T)$

2) $\frac{d(Bx+b)^TC(Dx+d)}{dx}$ given $\frac{d}{ds}(x-As)^TW(x-As)$

$= -2(x-As)^TWA$ rule where W is symmetric we get

$-2(Dx+d)^TC^TB^TB^TCD$

3. $\frac{d\,tr(X^2)}{dX}$ then $\frac{d\,tr(X^2)}{dX} = 2X$

Excercise 3