COMP 2610/6261: Information Theory

Week 12 Tutorial Questions

Question 1: Suppose X and Y are random variables with the following joint distribution p(x, y):

X	0	1	2
0	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{6}$
1	$\frac{1}{4}$	$\frac{1}{3}$	0

Answer the following questions.

- (i) Are X and Y independent? Explain your answer.
- (ii) Compute the expected value of *X* and *Y*, i.e., $\mathbb{E}[X]$ and $\mathbb{E}[Y]$.
- (iii) Compute the expected value of XY, i.e., $\mathbb{E}[XY]$.

Question 2: Three random variables X, Y, Z are given,

- (i) Comparing the term H(Y) + H(X, Y, Z) with the term H(X, Y) + H(Y, Z) in general and determine which terms is greater than or equal to the other.
- (ii) What is the condition for random variables X, Y, Z that these two terms are equal to each other, i.e., H(Y) + H(X,Y,Z) = H(X,Y) + H(Y,Z)

Question 3: Consider the Random Variable X and answer the following questions.

X	x_1	x_2	<i>x</i> ₃	x_4	<i>x</i> ₅
$p(x_i)$	0.4	0.25	0.15	0.1	0.1

- (a) What is the entropy H(X)?
- (b) Find the binary Huffman code for X.
- (c) Find the Shannon-Fano-Elias code for X.