COL 751 : Assignment - 2

Total marks: 60

1 k-vertex-connectivity Preserver

Let G be a k-vertex-connected graph with n vertices and m edges.

Algorithm 1: k-vertex-connectivity preserver

1 Let $r =$ _____.
2 **for** $i = 1$ *to* r **do**
3 Let S_i be a uniformly random subset of V obtained by picking vertices w.p. $\frac{1}{k-1}$.
4 Let T_i be a spanning forest of induced graph $G[S_i]$.
5 Return $H = \big(V, \cup_{i=1}^{r} E(T_i)\big)$.

(a) Obtain a bound on r in Algorithm 1 so as to ensure that with probability at least $(1-1/n)$ the following claim holds [10 marks]:

> For any two vertices $a, b \in V$ and any $F \subseteq V \setminus \{a, b\}$ of size $k-1$, there exists an $i \in [1, r]$ that satisfy $a, b \in S_i$ and $F \cap S_i = \emptyset$.

(b) Provide **correctness analysis** of Algorithm 1 after substituting in value of r to argue that H is k-vertex-connectivity preserver of G with probability $1 - 1/n$, and the expected number of edges in H is $O(nk^2 \log n)$. [10 marks]

2 Gomory Hu Tree and Random graphs

A random graph $G(n, p)$ is a graph in which each edge occurs independently with probability p.

(a) Prove that $G(n, p)$ for $p = \frac{5 \log n}{n}$ is connected with probability at least $1 - \frac{1}{n}$. [10 marks]
Hint: Perform separate analysis for all partitions of size $(i, n-i),\ i \in [1, \frac{n}{2}]$.

(b) Argue that with probability at least $1 - \frac{1}{n}$ Gomory-Hu tree of $G(n, p)$ for $p = \frac{\Theta(k \log n)}{n}$ has all edges of weight k or more. [10 marks]

(c) Perform experimental analysis of the k-edge-connected components in $G(n, p)$ for a **large collection** of choices for k and p and an **appropriately large enough** integer n to estimate the following.

- Size of largest k-edge-connected-component,
- Number of k-edge-connected components,
- Inflation point(s) for probability p, around which the number of k-edge connected components seem to change by large value.

Also provide detailed justification for your observations. [20 marks]

Remark You must not publish your code in any public repository. You are free to use any programming language as well as any inbuilt implementation of Gomory-Hu tree. See, for example, here