COL 751 : Assignment - 3

Total marks: $25 + 25 + 10 = 60$

1 Global *k*-Min-cut

A k-cut of a graph $G = (V, E)$ is a partition of V into k non-empty sets $A_1, \ldots, A_k$. The size of the cut is the number of edges connecting vertices from different sets.

Show that a simple variant of the random edge contraction algorithm of Karger and Stein can be used to efficiently compute a minimum 3-cut of an undirected and unweighted graph $G = (V, E)$ on n vertices. Analyze the success probability and running time of your algorithm.

Also extend your solution to the case of minimum k-cuts.

2 Edmonds' Matching algorithm

Provide an efficient implementation of Edmonds' cycle contraction algorithm for computing maximum matching that for an n vertex and m edges graph takes $O(mn \log n)$ time.

Also argue that your solution can be extended to obtain a Tutte-Berge maximizer in same time complexity.

Hint: Your algorithm should not explicitly contract odd length cycles encountered, but instead use union-find data structure to maintain for each vertex v the base of the cycle to which it currently belongs.

3 Cycle cover and Disjoint Perfect Matching

A *disjoint cycle cover* for a graph $G = (V, E)$ is a collection $C_1, C_2, \ldots, C_k$, $k \geqslant 1$ of vertex disjoint cycles such that $V = V(C_1) \cup \cdots \cup V(C_k)$.

Prove that a bipartite graph $G = (X, Y, E)$ has two edge disjoint perfect-matching if and only if G contains a disjoint cycle cover.

Describe in words how can you obtain a polynomial time algorithm for deciding whether a given bipartite graph contains two edge disjoint perfect-matching.

Hint: The problem can be solved by a single max-flow computation.