ENTRY NUMBER = 2019TT10953

$N = 53$
$M = 20190953$

<u>PROBLEM 1 :</u> $B = \{1, 53\}$ = set of divisors of N.

Given definitions: $a \cdot b = \gcd(a, b)$
$a + b = \mathrm{lcm}(a, b)$
$a' = N/a$

$I_+ = 1$ (identity of + operation)
$I_. = N$ (identity of . operation)

For any $a \in B$, $a \cdot \text{"one"} = a$
$\therefore$ "one" is N.

$a + \text{"zero"} = a$
$\therefore$ "zero" is 1.

$a \cdot \text{"one"} = a \cdot N = \gcd(a, N) = a$
$a + \text{"zero"} = a + 1 = \mathrm{lcm}(a, 1) = a$

Satisfies the identity law.

Checking for compliment laws,

$a + a' = a + N/a = \mathrm{lcm}(a, N/a) = N = \text{"one"}$

$a \cdot a' = \gcd(a, N/a) = 1 = \text{"zero"}$

[Our set has numbers which are coprime to each other.]

Hence, this law is also satisfied.

$B = \{1, 53\}$ $\quad N = 53$

For element 1 in the set, $1' = 53/1$
$= 53 \in B$.

For element 53 in the set, $53' = 53/53$
$= 1 \in B$.

For $\{1, 53\} \notin$ which belongs to the set,

$1 \cdot 53 = \gcd(1, 53) = 1 \in B$.

$1 + 53 = \text{lcm}(1, 53) = 53 \in B$.

$\therefore$ The tuple $\langle B, \cdot, +, ', I+, I. \rangle$ forms a Boolean algebra.