

**ASSIGNMENT-4**  
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**2019TT10953**

**Problem 8.8:**

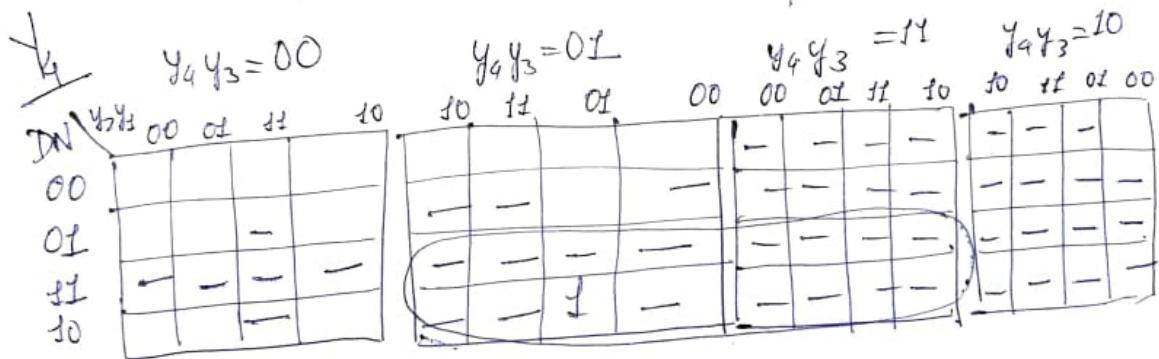
Derive the circuits that implement the state tables in Figures 8.55 and 8.56. Compare the costs of these circuits.

**ANSWER:**

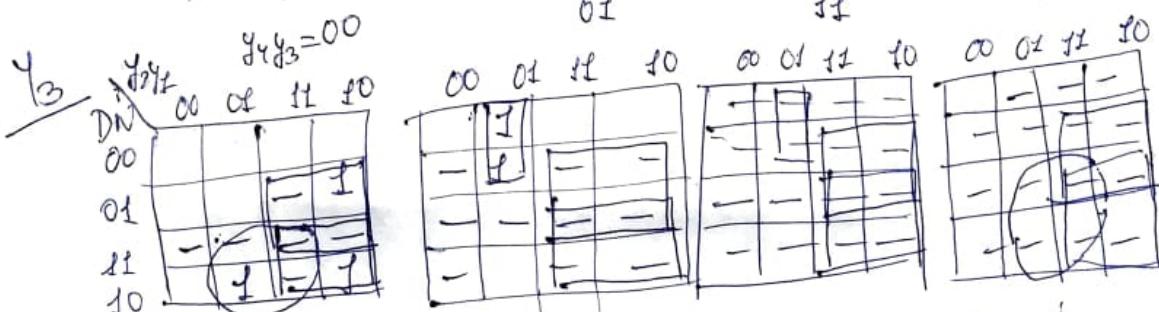
Figure 8.55:

Present state	Next state				Output $z$
	$DN = 00$	$01$	$10$	$11$	
S1	S1	S3	S2	—	0
S2	S2	S4	S5	—	0
S3	S3	S6	S7	—	0
S4	S1	—	—	—	1
S5	S3	—	—	—	1
S6	S6	S8	S9	—	0
S7	S1	—	—	—	1
S8	S1	—	—	—	1
S9	S3	—	—	—	1

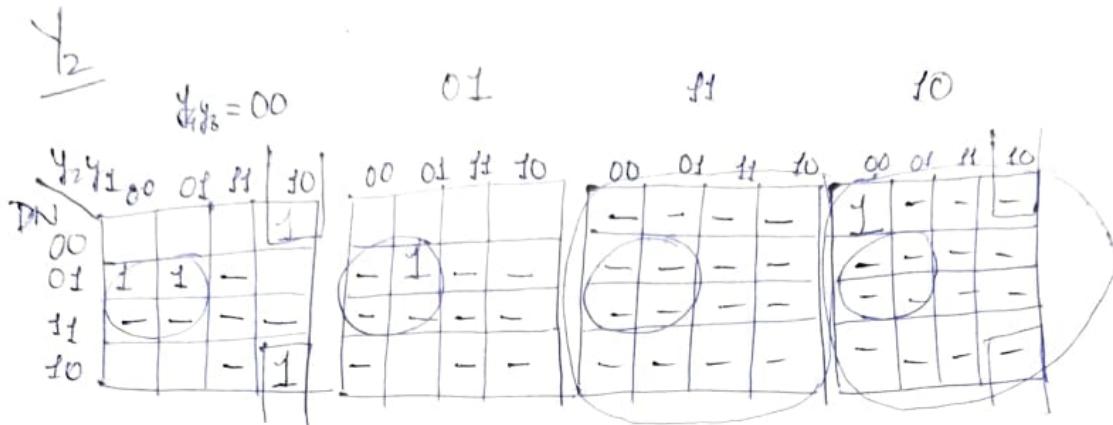
Present state $y_4y_3y_2y_1$	Next state $DN = Q_0 Q_1 10 11$	Output $Z$
0000	0000 0010 0001 -	0
0001	0001 0011 0100 -	0
0010	0010 0101 0110 -	0
0011	0000 - - -	1
0100	0010 - - -	1
0101	0101 0111 1000 -	0
0110	0000 - - -	1
0111	0000 - - -	1
1000	0010 - - -	1



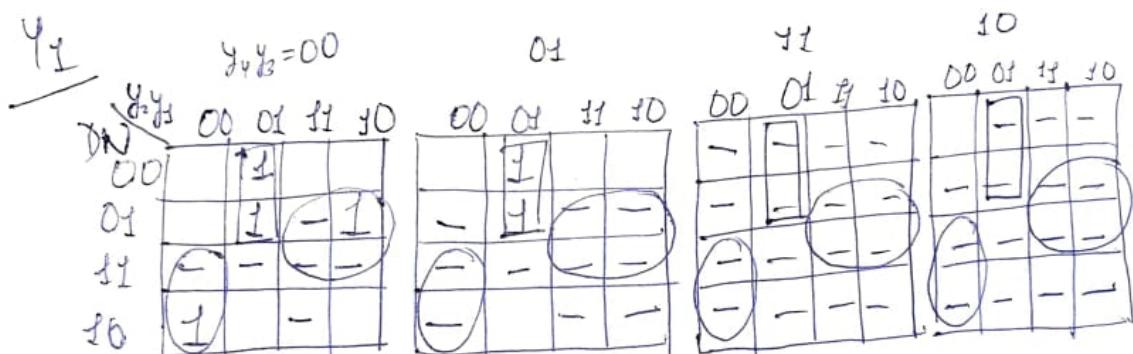
$$\therefore y_4 = D y_3$$



$$y_3 = \bar{D} y_3 \bar{y}_2 y_1 + N y_3 + D y_2 + \bar{D} \bar{y}_3 y_1$$

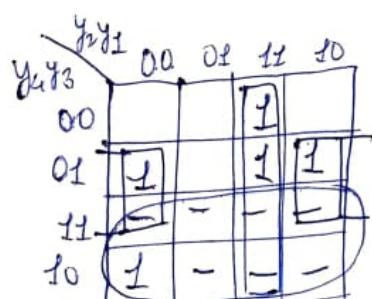


$$y_2 = N \bar{y}_2 + \bar{N} \bar{y}_3 y_2 \bar{y}_1 + y_4$$



$$y_1 = \bar{D} \bar{y}_2 y_3 + N y_2 + \bar{D} \bar{y}_2 \bar{y}_1$$

Output Z



$$Z = y_3 \bar{y}_1 + y_1 y_2 + y_4$$

Figure 8.56:

Present state	Next state				Output $z$
	$DN = 00$	$01$	$10$	$11$	
S1	S1	S3	S2	—	0
S2	S2	S4	S5	—	0
S3	S3	S2	S4	—	0
S4	S1	—	—	—	1
S5	S3	—	—	—	1

Present state	Next state	Output $z$
$y_3 y_2 y_1$	$y_3 y_2 y_1$	
000	000	0
001	001	0
010	010	0
011	000	1
100	010	1

$y_3$

$y_3$	$y_2 y_1$	$y_3 = 0$	$y_3 = 1$
DN	$y_2 y_1$	00 01 11 10	00 01 11 10
00		- - - -	- - - -
01		- - - -	- - - -
11	- (1 1) -	- - - -	- - - -
10	- (1 1) -	- - - -	- - - -

$$y_3 = D y_1$$

$y_2$

$y_2$	$y_2 = 0$	$y_2 = 1$
DN	$y_2 y_1$	$y_2 y_1$
00	00 01 11 10	00 01 11 10
01	(1 1) - -	- - - -
11	- - - -	- - - -
10	- - (1 1) -	- - - -

$$y_2 = N \bar{y}_2 + \bar{N} y_2 \bar{y}_1 + y_3$$

$y_1$

$y_1$	$y_1 = 0$	$y_1 = 1$
DN	$y_2 y_1$	$y_2 y_1$
00	00 01 11 10	00 01 11 10
01	(1 1) - -	- - - -
11	- - - -	- - - -
10	- - (1 1) -	- - - -

$$y_1 = \bar{D} \bar{y}_2 y_1 + N y_2 \bar{y}_1 + D \bar{y}_1$$

Output Z

$y_3$	$y_2 y_1$	00	01	11	10
0		-	-	-	-
1	1	-	-	1	-

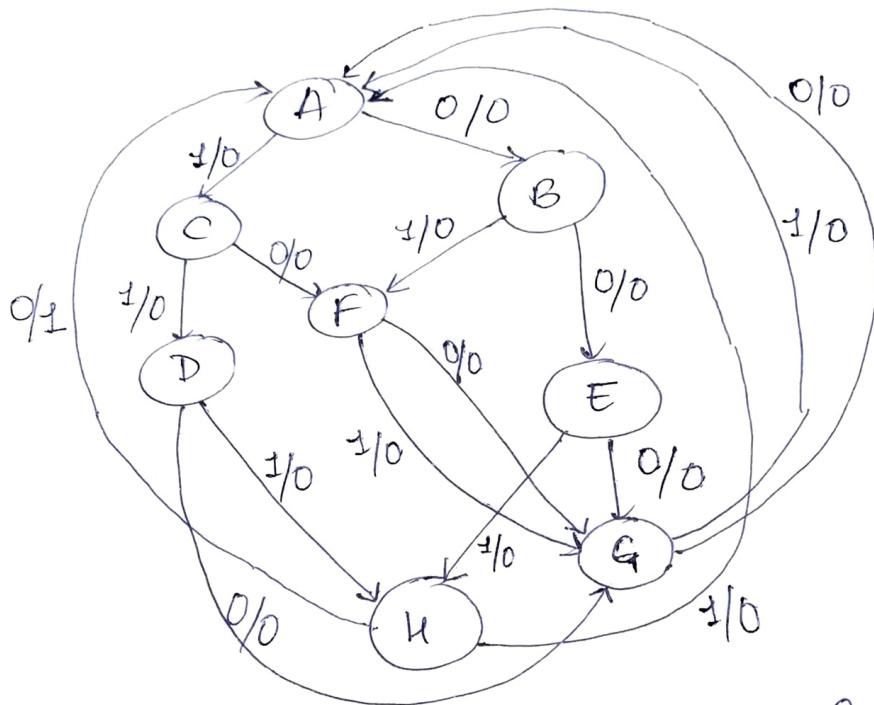
$$Z = y_3 \neq + y_2 y_1$$

Hence, these expressions define a much cheaper circuit than the Figure 8.55.

**PROBLEM 8.11:**

State A represents reset.

States F and G represent incorrect sequences.



Reset state	Next state		Output =	
	$w=0$	$w=1$	$w=0$	$w=1$
A	B	C	0	0
B	E	F	0	0
C	F	D	0	0
D	G	H	0	0
E	G	H	0	0
F	G	G	0	0
G	A	A	0	0
H	A	A	1	0

### STATE PARTITIONING :

$(A B C D E F G H)$   
 $(A B C D E F G) (H)$   
 $(A B C F G) (D E) (H)$   
 $(A C F G) (B) (D E) (H)$   
 $(A F G) (C) (B) (D E) (H)$   
 $(F G) (A) (C) (B) (D E) (H)$   
 $(F) (G) ((A)) (C) (B) (D E) (H)$   
 i) D and E are equivalent.

Min state stable for Mealy-type FSM :

Present state	Next state		Output	
	$w=0$	$w=1$	$w=0$	$w=1$
A	B	C	0	0
B	D	F	0	0
C	E	D	0	0
D	G	H	0	0
F	G	G	0	0
G	A	A	0	0
H	A	A	1	0

Problem 8.18:

Present state	Next state		Output $z$	
	$w=0$	$w=1$	$w=0$	$w=1$
A	B	C	0	0
B	D	-	0	1
C	F	E	0	1
D	B	G	0	0
E	F	C	0	1
F	E	D	0	1
G	F	-	0	0

$$P_1 = (ABCDEF)$$

$$P_2 = (ADG)(BCEF)$$

$$P_3 = (ADG)(B)(CCEF)$$

$$P_4 = (AD)(G)(B)(CCEF)$$

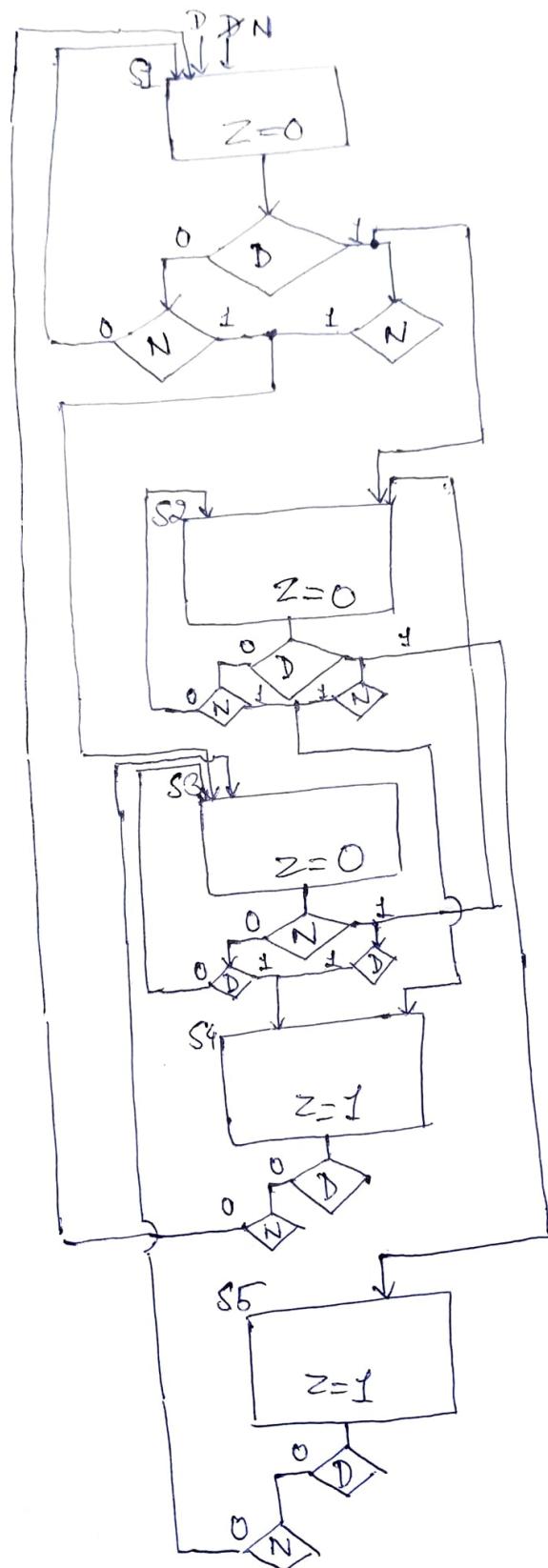
$$P_5 = (A)(D)(G)(B)(CCE)(F)$$

in 6 states in all.

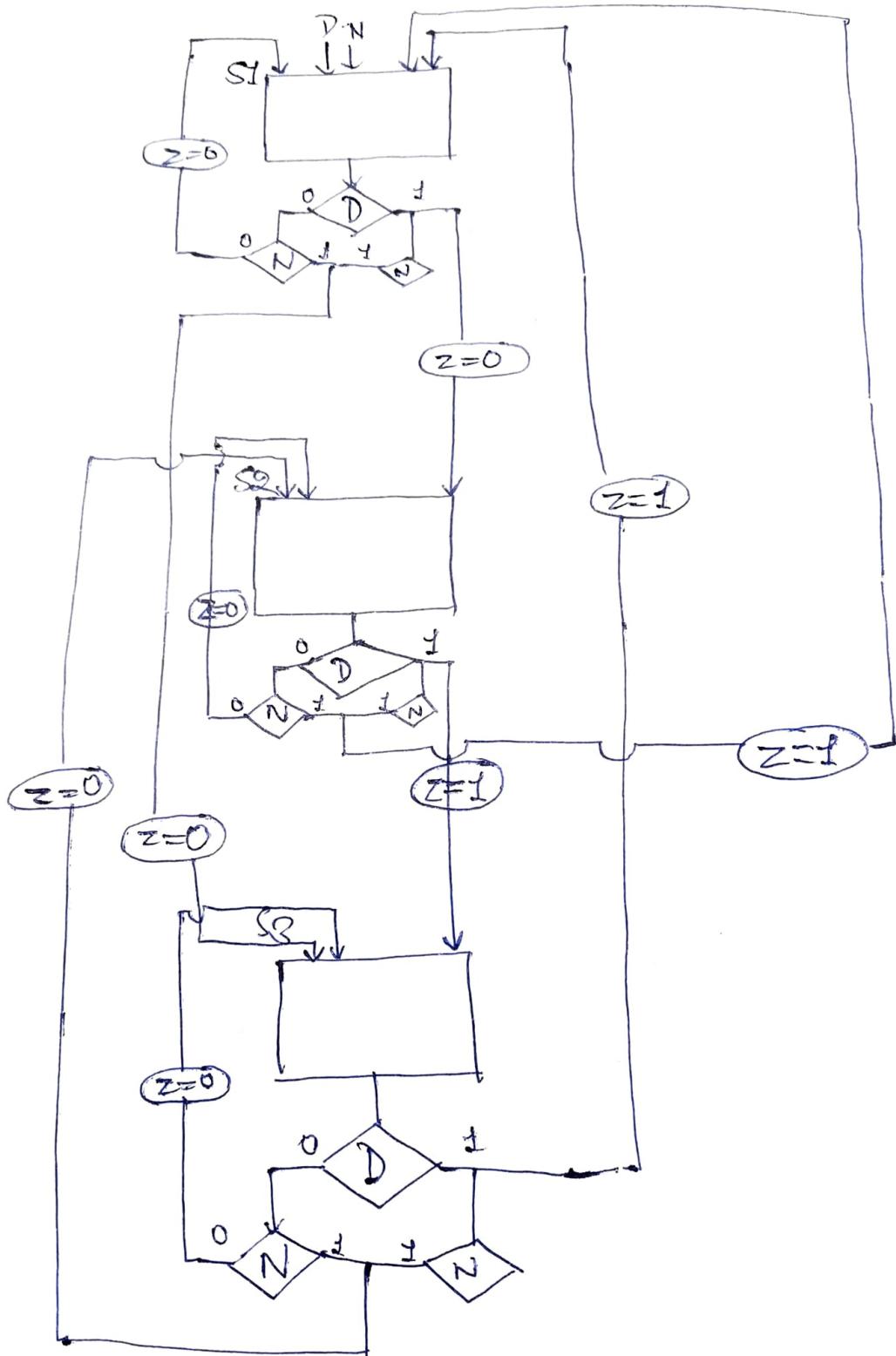
CE are equivalent.

Present state	Next state		Output	
	$w=0$	$w=1$	$w=0$	$w=1$
A	B	C	0	0
B	D	-	0	1
C	F	C	0	1
D	B	G	0	0
E	C	D	0	1
F	F	-	0	0

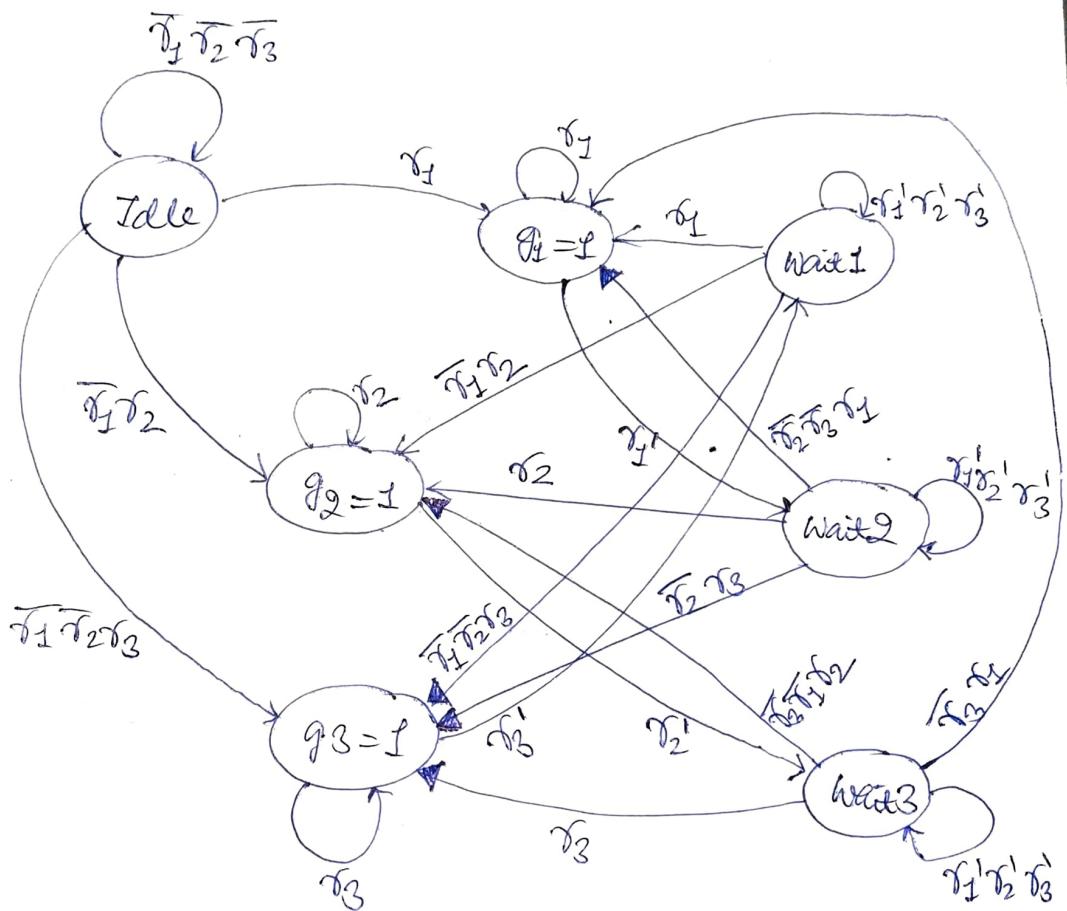
Problem 8.36:



Problem 8.37:



### Problem 8.38



I have made use of rotating priorities.

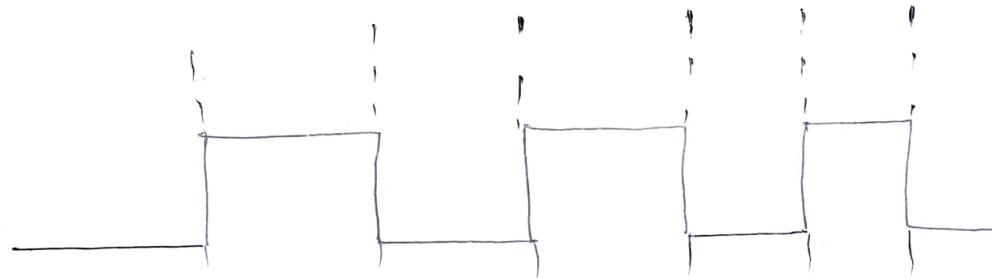
PROBLEM 9.2:

$$z_2 = \overline{\overline{c} + z_1}$$

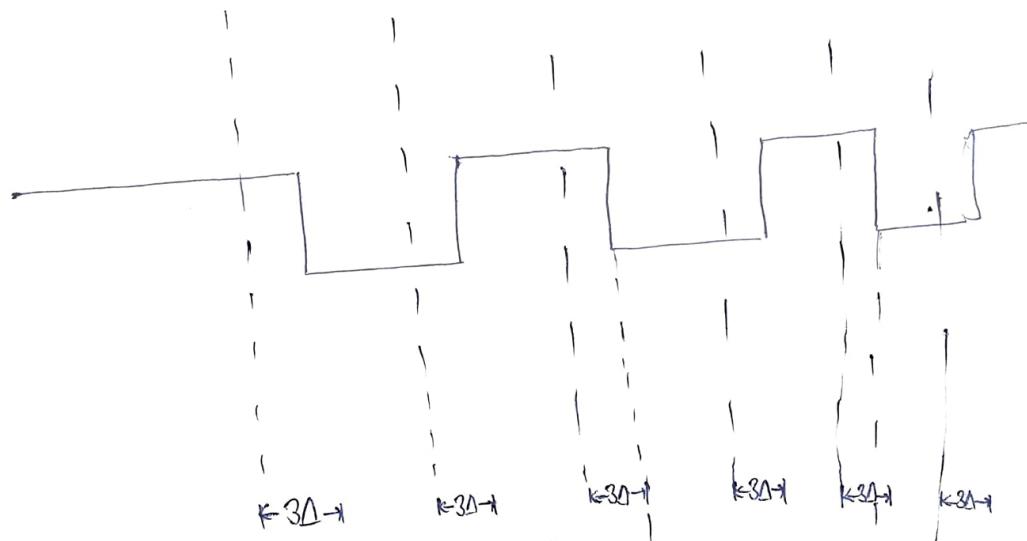
$$= c \cdot \overline{z_1}$$

$$z_1 = \frac{c \cdot \overline{z_1} + c}{\overline{c}}$$

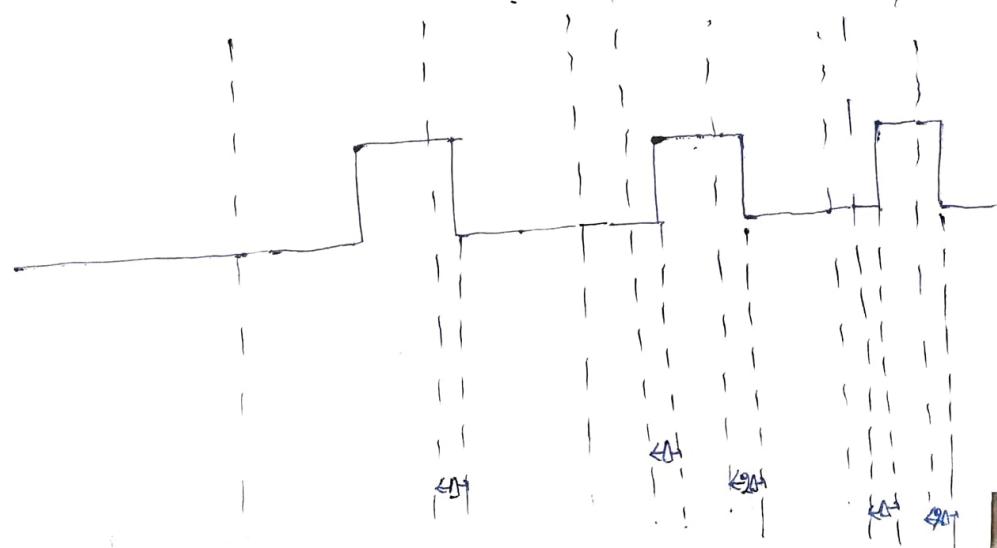
c



$z_1$



$z_2$



Present state $z_2 z_1$	Next state $C=0$	Next state $C=1$	Output $z_2 z_1$
0 0	01	10	01 10
0 1	01	00	01 00
1 0	01	10	01 10
1 1	01	00	01 00

Present state	Next state $C=0$	Next state $C=1$	Output $C=0$ $C=1$
A	B	C	01 10
B	(B)	A	01 00
C	B	(C)	01 10
D	B	A	01 00

### PROBLEM 9.3

$$P_1 = (ABCDEFGHIJKLMNOPRST)(JKLNOSUV)$$

$$P_2 = (ADGIMPT)(BEGHR)(JLOSV)(KNU)(CF)$$

$$P_3 = (ADGIP)(MT)(BEGH)(R)(JLOSV)(KNU)(CF)$$

$$P_4 = (A)(GIP)(D)(MT)(B)(EH)(R)(JLOSV)(KNU)(CF)$$

$$P_5 = (A)(G)(J)(P)(D)(MT)(B)(E)(H)(R)(JLOSV)(KNU)(CF)$$

Present state

Present state $Q_2Q_1Q_0 = 000$	Next state				Output
	01	10	11	12	
A	(A)	B	C	-	0
B	D	(B)	-	-	0
C	P	-	(C)	-	0
D	(D)	E	F	-	0
E	G	(E)	-	-	0
F	M	-	(F)	-	0
G	(G)	H	I	-	0
H	J	(H)	-	-	0
I	A	-	(I)	-	1
J	(J)	K	I	-	0
K	A	(K)	-	-	1
M	(M)	K	I	-	0
P	(P)	R	I	-	0
R	M	(R)	-	-	0

Partially minimised flow table

I and K = compatible states

F and R = compatible states

Present state	Next state	Output
	$w_2 w_1 = 00$ of 10 11	
A	(A) B C -	0
B	D (B) - -	0
C	P - (C) -	0
D	(D) E F -	0
E	G (E) - -	0
F	M (F) - -	0
G	(G) H I -	0
H	J (H) - -	0
I	I (I) - -	0
J	T T I -	0
M	T T I -	0
P	(P) F I -	0

Minimal flow table

#### PROBLEM 9.4:

$(A B C E F G T L) (D H K M)$   
 $(A F) (B E G L) (C J) (D K) (H M)$   
 $(A F) (B G) (E L) (C J) (D K) (H M)$   
 $(A) (F) (B G) (E L) (C J) (D K) (H M)$

Present state	Next state	Output
	$w_2 w_1 = 00$ of 10 11	
A	(A) B C -	0
B	D (B) - H	0
C	F - (C) H	0
D	(D) E C -	1
E	A (E) - H	0
F	(F) E C -	0
H	- B C (K)	1

Partially minimised state  
~~(B, H)~~, ~~(A, H)~~, ~~(C, F)~~ is a compatible state.

Minimal flow table :

Present state	Next state	Output
	$w_2 w_1 = 00 \quad 01 \quad 10 \quad 11$	
A	(A)	0
B	(B)	0
C	(C)	0
D	(D)	1
E	(E)	0
H	(H)	1

PROBLEM 9.11:

$x_0 x_3$	$x_4 x_5$	00	01	11	10
00	00	1	1	3	2
01	01	1	1	7	6
11	11	12	13	35	34
10	10	8	9	34	30
	$x_0 x_3$				

$x_0 x_3$	$x_4 x_5$	00	01	11	10
00	00	1	1	3	2
01	01	20	21	23	22
11	11	20	21	31	30
10	10	24	25	27	26
	$x_0 x_3$				

$$\begin{aligned}
 &= \overline{x_4} \overline{x_2} \overline{x_4} \overline{x_5} + \overline{x_2} x_3 \overline{x_4} \overline{x_5} \\
 &+ x_3 \overline{x_4} x_5 + x_4 x_2 \overline{x_4} x_5 \\
 &+ \cancel{x_1 \overline{x_2} x_3} \cancel{x_4 x_2 \overline{x_3} \overline{x_4}} + \\
 &\overline{x_3} \overline{x_4} \overline{x_5}
 \end{aligned}$$

$$\begin{aligned}
 &= \overline{x_4} \overline{x_2} \overline{x_4} \overline{x_5} + \overline{x_2} x_3 \overline{x_4} \overline{x_5} + x_3 \overline{x_4} x_5 + \\
 &x_4 x_2 \overline{x_4} x_5 + x_4 x_2 \overline{x_3} \overline{x_4} + \overline{x_3} \overline{x_4} \overline{x_5}
 \end{aligned}$$

Problem 9.13:

$$\chi_2 \chi_3 \cancel{\chi_4 \chi_5}$$

	00	01	11	10
00	—	—	—	—
01	4	5	07	02
11	12	13	18	19
10	—	—	—	—
	1	2	3	4

$\chi_4 = 1$

	00	01	11	10
00	6	12	19	18
01	20	21	26	22
11	028	029	31	30
10	—	0	—	—
	24	25	27	26

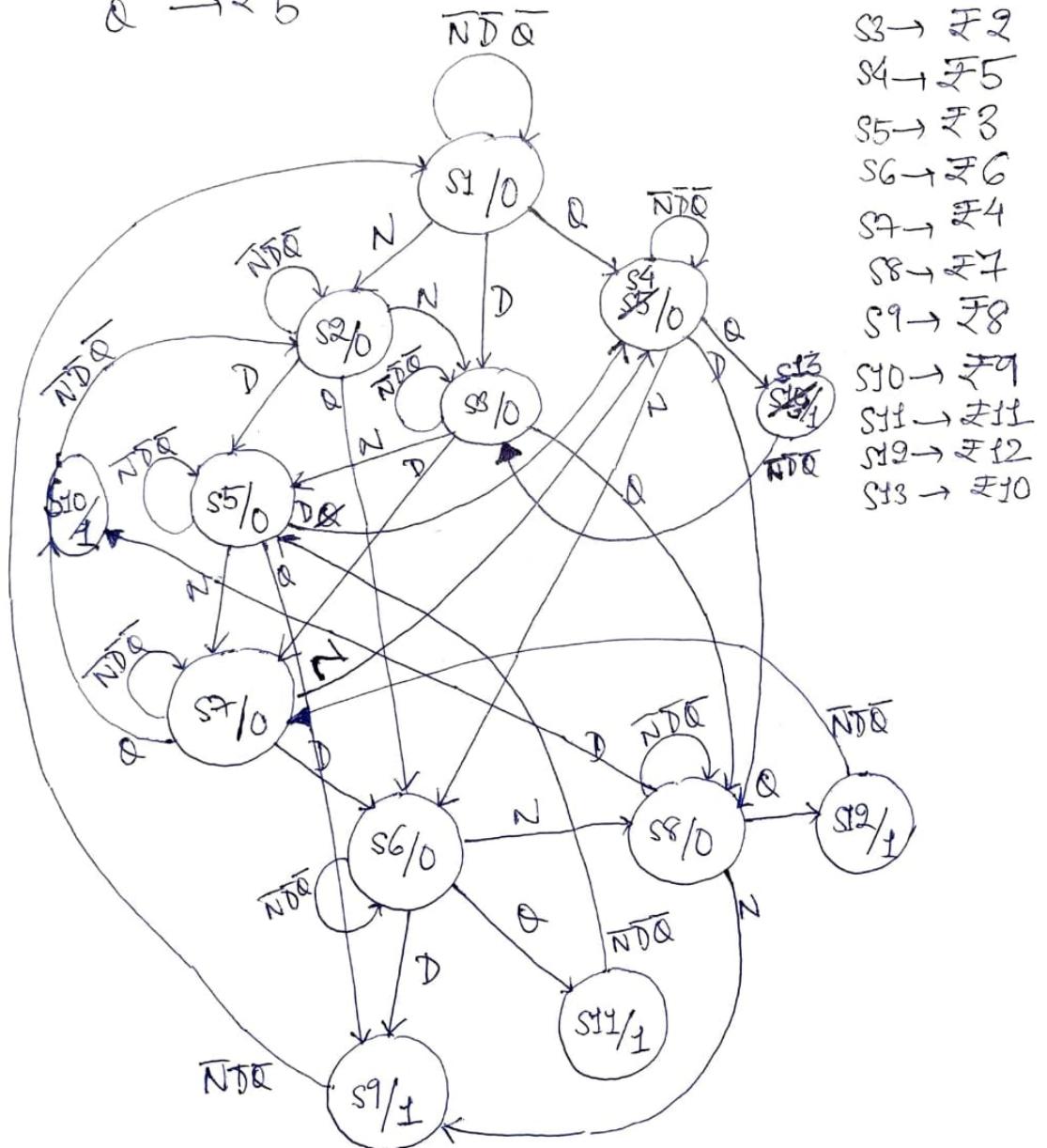
$$= \cancel{\chi_1 \chi_2 \chi_3 \chi_4} +$$

$$= (\overline{\chi_1} + \overline{\chi_2} + \chi_3 + \chi_4) \cdot (\overline{\chi_1} + \overline{\chi_2} + \chi_4 + \overline{\chi_5}) \cdot$$

$$(\overline{\chi_1} + \overline{\chi_3} + \overline{\chi_5}) \cdot (\chi_2 + \overline{\chi_3}) \cdot (\chi_3 + \chi_2 + \overline{\chi_4})$$

EXAMPLE 8.6:

$$\begin{aligned} N &\rightarrow \mathcal{E}_1 \\ D &\rightarrow \mathcal{E}_2 \\ Q &\rightarrow \mathcal{E}_5 \end{aligned}$$



Present state	NDA = .000	Next state						Output z
		001	010	011	100	101	110	
S1	S1	S4	S3	-	S2	-	-	0
S2	S2	S6	S5	-	S3	-	-	0
S3	S3	S8	S7	-	S5	-	-	0
S4	S4	S13	S8	-	S6	-	-	0
S5	S5	S9	S4	-	S7	-	-	0
S6	S6	S12	S9	-	S8	-	-	0
S7	S7	S10	S6	-	S4	-	-	0
S8	S8	S12	S10	-	S9	-	-	0
S9	S1	-	-	-	-	-	-	1
S10	<del>S10</del> S2	-	-	-	-	-	-	1
S11	S5	-	-	-	-	-	-	1
S12	S7	-	-	-	-	-	-	1
S13	S3	-	-	-	-	-	-	

$$P_1 = (S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8)(S_9, S_{10}, S_{11}, S_{12}, S_{13})$$

$$P_2 = (S_1, S_2, S_3)(S_4, S_5, S_6, S_7, S_8)(S_9, S_{10}, S_{13})(S_{11}, S_{12})$$

$$P_3 = (S_1)(S_2, S_3)(S_4, S_5, S_7)(S_6, S_8)(S_9)(S_{10}, S_{13})(S_{11}, S_{12})$$

$$P_4 = (S_1)(S_2)(S_3)(S_7, S_4)(S_5)(S_6)(S_8)(S_9)(S_{10})(S_{13})(S_{11}, S_{12})$$

$$P_5 = (S_1)(S_2)(S_3)(S_7)(S_4)(S_5)(S_6)(S_8)(S_9)(S_{10})(S_{13})(S_{11}, S_{12})$$

$\therefore$  No further reduction of states possible.