

Considering the given set B along with binary operations +,.,', if it forms a Boolean algebra, it satisfies the following properties:

Identity Laws

$$x + 0 = x$$

$$x \cdot 1 = x$$

Complements Laws

$$x + x' = 1$$

$$x \cdot x' = 0$$

Associative Laws

$$(x + y) + z = x + (y + z)$$

$$(x \cdot y) \cdot z = x \cdot (y \cdot z)$$

Commutative Laws

$$x + y = y + x$$

$$x \cdot y = y \cdot x$$

Distributive Laws

$$x + (y \cdot z) = (x + y) \cdot (x + z)$$

$$x \cdot (y + z) = (x \cdot y) + (x \cdot z)$$

The consequences of the above axioms of Boolean algebra are:

(1) Idempotent Laws: $x + x = x$ and $x \cdot x = x$

(2) Domination Laws: $x + 1 = 1$ and $x \cdot 0 = 0$

(3) Absorption Laws: $(x \cdot y) + x = x$ and $(x + y) \cdot x = x$

(4) $x + y = 1$ and $x \cdot y = 0$ if and only if $y = x'$

(5) Double Complements Law: $(x')' = x$

(6) DeMorgan's Laws: $(x \cdot y)' = x' + y'$ and $(x + y)' = x' \cdot y'$

For a set(along with +,.) to be Boolean algebra, it should satisfy the following conditions:

a) If a belongs to the set B, so should a'.

b) If a and b belong to the set B, so should a+b and a.b.

We will use this definition to prove that the given tuple $\langle B, ., +, ', I+, I. \rangle$ forms a Boolean algebra.

In this set, “zero” is defined as that element of set B which satisfies the identity laws, for any element a of set B, $a \cdot a' = \text{“zero”}$ and $a + \text{“zero”} = a$.

Likewise, “one” is defined as $a \cdot \text{“one”} = a$ and $a + a' = \text{“one”}$.

In the question, identities of $+$ and \cdot operations are defined as $I + = 1$ and $I \cdot = N$. So, considering the above definitions, we can say that “zero” is 1 and “one” is N.