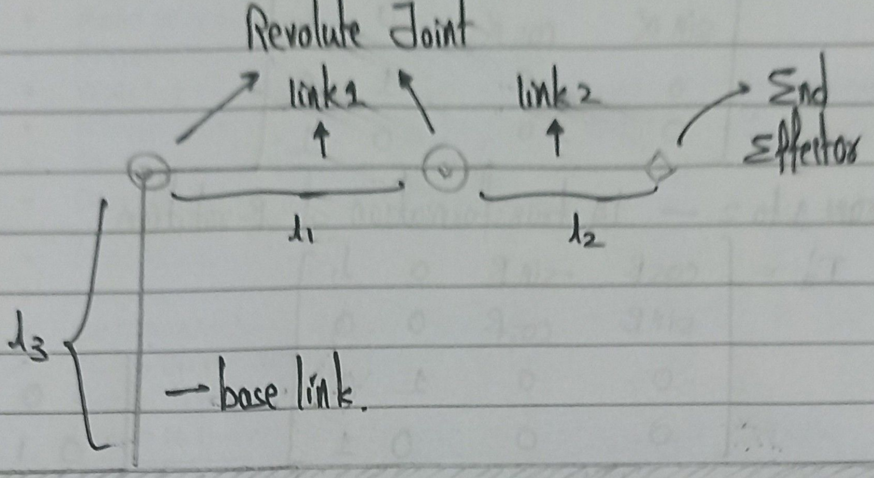
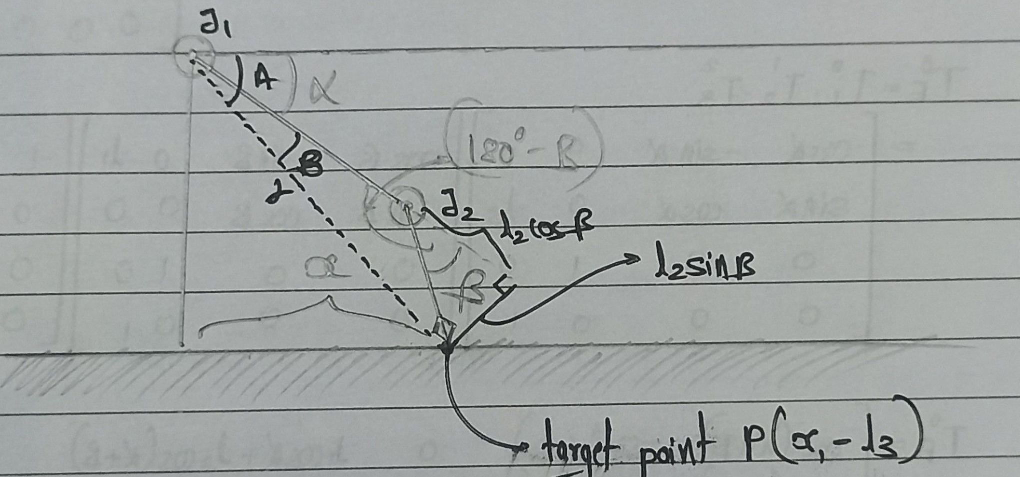
Kinematic Analysis of a 2-DOF Planar Robotic Arm





$r^2 = l_3^2 + x^2$

Applying cosines law for $(J_1 J_2 P)\Delta \rightarrow r^2 = l_1^2 + l_2^2 - 2l_1l_2\cos(180° - \beta)$

$$x^2 + l_3^2 = l_1^2 + l_2^2 + 2l_1l_2\cos\beta$$

$$\beta = \cos^{-1}\left\{\frac{x^2 + l_3^2 - l_1^2 - l_2^2}{2l_1l_2}\right\}$$

$$A - B = \alpha \Rightarrow \tan A = \frac{l_3}{x} \Rightarrow A = \tan^{-1}\left(\frac{l_3}{x}\right)$$

$$\tan B = \frac{l_2\sin\beta}{l_1 + l_2\cos\beta} \Rightarrow B = \tan^{-1}\left(\frac{l_2\sin\beta}{l_1 + l_2\cos\beta}\right)$$

$$\alpha = \tan^{-1}\left(\frac{l_3}{x}\right) - \tan^{-1}\left(\frac{l_2\sin\beta}{l_1 + l_2\cos\beta}\right)$$

2. Frame 0 to 1 → Base to joint 1 → l_3 translation, α rotation

$$T_1^0 = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 & 0 \\ \sin\alpha & \cos\alpha & 0 & l_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

From 1 to 2 → l_1 transformation of β rotation

$$T_2^1 = \begin{bmatrix} \cos\beta & -\sin\beta & 0 & l_1 \\ \sin\beta & \cos\beta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

From 2 to End-effector → l_2 translation → $\begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$$T_E^0 = T_1^0 \cdot T_2^1 \cdot T_3^2$$

$$= \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 & 0 \\ \sin\alpha & \cos\alpha & 0 & l_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\beta & -\sin\beta & 0 & l_1 \\ \sin\beta & \cos\beta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_E^0 = \begin{bmatrix} \cos(\alpha+\beta) & -\sin(\alpha+\beta) & 0 & l_1\cos\alpha + l_2\cos(\alpha+\beta) \\ \sin(\alpha+\beta) & \cos(\alpha+\beta) & 0 & l_3 + l_1\sin\alpha + l_2\sin(\alpha+\beta) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$