Complex Variables and Functions:

Our focus in the present chapter is on functions of a complex variable and on their analytical properties. We know that by defining a complex function $f(z)$ to have the same power series expansion (in z) as the expansion (in x) of the corresponding real function $f(x)$, the real and complex definitions coincide when z is real.

We also know that by use of the polar representation, $z = re^{i\theta}$, we can compute powers and roots of complex quantities. In particular, we noted that roots, viewed as fractional powers, become **multivalued** functions in the complex domain, due to the fact that $\exp(2n\pi i) = 1$ for all positive and negative integers n. We thus found $z^{1/2}$ to have two values (not a surprise, since for positive real x, we have $\pm\sqrt{x}$). But we also noted that $z^{1/m}$ will have m different complex values. We also noted that logarithm becomes multivalued when extended to complex values, with

$$\ln z = \ln\left(re^{i\theta}\right)\mathrm{i} = \ln r + i(\theta + 2n\pi),$$

with n any positive or negative integer (including 0).