PHY 614 Mid Semester II / Time 1.5 hrs. / 40 Marks

1. Given the vector form of the Lorentz transformations as

$$\overrightarrow{r'} = \vec{r} + \frac{\gamma - 1}{\beta^2}(\vec{\beta} \cdot \vec{r})\vec{\beta} - \vec{\beta}\gamma ct$$

and

$$t' = \gamma t - \frac{\vec{\beta} \cdot \vec{r}}{c}\gamma,$$

show that $r'^2 - c^2 t'^2 = r^2 - c^2 t^2$. **(15)**

2. Show that $S = -\frac{mc}{\gamma}\int \sqrt{u_i u^i}\, dt$ gives the correct equation of motion: $m\frac{d^2 x^i}{d\tau^2} = 0$, where u^i is the four velocity and τ is the proper time. (Hint: The trajectory is $x^i = x^i(\tau)$.) **(10)**

3. Explicitly evaluate $F^{ij}F_{ij}$. (Hint: This is a tensor contraction and not a matrix multiplication. So, expand over the dummy indices and then use definition and antisymmetry.) **(15)**