PHY614

Classical Electrodynamics 2

Quiz – 1

Date: 04-10-2024] [Time: 30 minutes

Name:

Roll No.:

Question:

Show that the one-dimensional Schrodinger equation ($\hbar = m = 1$)

$$-\frac{1}{2}\frac{\partial^2\psi}{\partial x^2} = i\frac{\partial\psi}{\partial t}$$

is invariant under the Galilean transformation, when

$$\psi \to \psi' = \psi \exp\left(-ivx + \frac{iv^2 t}{2}\right).$$

The one-dimensional Galilean transformation is given by

$$x' = x - vt \,, \quad t' = t$$

$$\Rightarrow \quad x = x' + vt' \,, \quad t = t'$$

$$\Rightarrow \quad \frac{\partial}{\partial x'} = \frac{\partial x}{\partial x'}\frac{\partial}{\partial x} + \frac{\partial t}{\partial x'}\frac{\partial}{\partial t} = \frac{\partial}{\partial x} \,; \quad \frac{\partial}{\partial t'} = \frac{\partial x}{\partial t'}\frac{\partial}{\partial x} + \frac{\partial t}{\partial t'}\frac{\partial}{\partial t} = \frac{\partial}{\partial t} + v\frac{\partial}{\partial x}$$

We have the Schrödinger equation:

$$-\frac{1}{2}\frac{\partial^2\psi'}{\partial x'^2} = i\frac{\partial\psi'}{\partial t'}$$

$$\Rightarrow -\frac{1}{2}\frac{\partial^2}{\partial x^2}\left(\psi e^{-ivx + iv^2t/2}\right) = i\frac{\partial}{\partial t}\left(\psi e^{-ivx + iv^2t/2}\right) + iv\frac{\partial}{\partial x}\left(\psi e^{-ivx + iv^2t/2}\right)$$

$$\Rightarrow -\frac{1}{2}e^{iv^2t/2}e^{-ivx}\left(\frac{\partial^2\psi}{\partial x^2} - 2iv\frac{\partial\psi}{\partial x} - v^2\psi\right) = ie^{-ivx + iv^2t/2}\left(\frac{\partial\psi}{\partial t} + \frac{iv^2}{2}\psi\right) + iv e^{iv^2t/2}e^{-ivx}\left(\frac{\partial\psi}{\partial x} - iv\psi\right)$$

$$\Rightarrow -\frac{1}{2}\frac{\partial^2\psi}{\partial x^2} + iv\frac{\partial\psi}{\partial x} + \frac{1}{2}v^2\psi = i\frac{\partial\psi}{\partial x} - \frac{v^2}{2}\psi + iv\frac{\partial\psi}{\partial x} + v^2\psi$$

$$\Rightarrow -\frac{1}{2}\frac{\partial^2\psi}{\partial x} = i\frac{\partial\psi}{\partial t}$$

Thus the 1D Schrodinger equation is indeed invariant under the GT.