**Mid Semester Exam. PHY657. 22/2/2025 Time : 120 Minutes.**

Consider a two dimensional metric given by

$$ds^2 = \frac{9t}{4}dx^2 - \frac{9t}{2}dxdt + \frac{3}{4}(2+3t)dt^2$$

a) Compute the non-zero Christoffel symbols. The formula is $\Gamma^{\alpha}_{\beta\gamma} = \frac{1}{2}g^{\alpha\mu}\left(g_{\mu\beta,\gamma} + g_{\mu\gamma,\beta} - g_{\beta\gamma,\mu}\right)$.

b) Notice that $x$ is cyclic. Hence the Euler-Lagrange equation for $x$ from an appropriate Lagrangian will be of first order. Write this equation.

c) Define the tangent vector $u^{\mu}$. Use $u^{\mu}u_{\mu} = 1$. Solve the normalization condition in conjunction with the equation obtained in part (b) to obtain $\dot{x}$ and $\dot{t}$.

d) Compute $u^{\alpha}{}_{;\beta}$. The formula is $A^{\alpha}_{;\beta} = A^{\alpha}{}_{,\beta} + \Gamma^{\alpha}_{\mu\beta}A^{\mu}$.

e) Set the arbitrary constant in the expressions for $\dot{x}$ and $\dot{t}$ **to zero**. Hence compute the expansion scalar.