End Sem PHY690M. 30 April, 2024. Time : 3 Hours.

Please show all workings.

1a) In flat spacetime, consider an event that occurs at the origin of a Cartesian system.
Write the defining relation for the corresponding null hypersurface and hence construct the normal vector k_α. Write a set of parametric equations defining the hypersurface, clearly identifying the intrinsic coordinates.
From the parametric equations compute the tangent vectors.
Finally, construct the two metrics on the null hypersurface. [2+2+2+2]

1b) On a three-dimensional timelike hypersurface, define the intrinsic covariant derivative of a three vector A_a as a suitable projection of the covariant derivative $A_{\alpha;\beta}$. Now show that this is indeed equal to the covariant derivative constructed out of the three-dimensional induced metric. [2]

2) Set up the problem of Oppenheimer-Snyder collapse where the internal and external metrics are given by $ds_-^2 = -d\tau^2 + a^2(\tau)(d\chi^2 + \sin^2\chi\, d\Omega^2)$ and $ds_+^2 = -f dt^2 + f^{-1}dr^2 + r^2 d\Omega^2$. Calculate a) the induced metric and b) the extrinsic curvature on both sides. Clearly state any assumptions that you make. [2+2]

Useful info: If n_α is the unit normal to a hypersurface and is defined by $n_\alpha = \frac{\epsilon\Phi_{,\alpha}}{|g^{\mu\nu}\Phi_{,\mu}\Phi_{,\nu}|^{1/2}}$ with Φ being the defining relation for the hypersurface and $\epsilon = \pm 1$ for timelike/spacelike hypersurface, then the extrinsic curvature is defined as $K_{ab} = n_{\alpha;\beta}e_a^\alpha e_b^\beta$.

3) In the Lagrangian formulation of GR, set up the Einstein-Hilbert action. Vary this action and indicate how it necessitates the Gibbons-Hawking boundary term. Now include the matter part and varying this, define the energy-momentum tensor. [4+2]

4) Motivate and draw the Penrose diagram for Minkowski space. [5]