Particle Physics (PHY680A)

Assignment 1 (Submission date: 31.01.2024)
Indian Institute of Technology, Kanpur

Instructor: Sabyasachi Chakraborty
✉: sabyac@iitk.ac.in
☎: 0512-279-2349 (O)

1. See the conversion table here. Derive any one of the tabulated relations.

2. Show that for one particle states $\langle p|p\rangle = (2\pi)^3\, 2E_p\delta^3(0)$.

3. Prove the Lorentz invariance of the Lorentz invariant phase space element derived in the class.

4. While computing the scattering cross-section, we defined $F = |\vec{v}_1 - \vec{v}_2|\,(2E_1)\,(2E_2)$. Show that

$$F = 4\left[(p_1.p_2)^2 - m_1^2 m_2^2\right]^{1/2} \ . \tag{1}$$

Obtain the expression for F in both COM and FT frames.

5. **(a)** A useful variable often used in collider physics is rapidity, defined as

$$y = \frac{1}{2}\ln\left(\frac{E+p_z}{E-p_z}\right) \ , \tag{2}$$

where p_z is the momentum along the beam direction. For a highly relativistic particle of mass m, express y in terms of θ, i.e., the angle made by the particle trajectory with the beam direction.

(b) Starting from the definition of y, show that under Lorentz boosts parallel to the z axis, rapidity transforms as

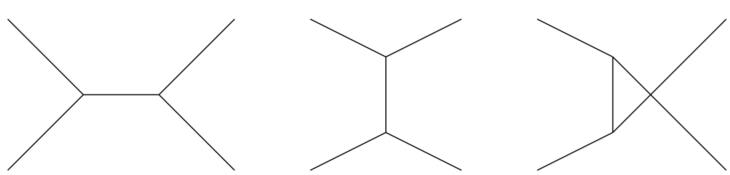
$$y' = y - \tanh^{-1}\beta \ . \tag{3}$$

You might need the following: $x' = \gamma\,(x - \beta y)$ *and* $\tanh(\ln x) = (1 - x^{-2})/(1 + x^{-2})$.

6. In the class, we introduced the Mandelstam variables s, t, u. Show that they satisfy

$$s + t + u = \sum_j m_j^2 \ , \tag{4}$$

where m_j are the invariant masses of the particles. We note in passing that in case of $2 \to 2$ scattering, s, t, u represent the following diagrams. The s channel is an annihilation process



where the intermediate state has $p_\mu^2 = s > 0$. The t and u channel are scattering diagrams and $t < 0$, $u < 0$.

7. In the class, we derived the expression of the differential cross section with respect to t (see Eq. (2.25)). Show that, this is nothing but

$$\frac{d\sigma}{dt} = \frac{1}{16\pi} \frac{1}{\left[s-(m_1+m_2)^2\right]\left[s-(m_1-m_2)^2\right]} |\mathcal{M}|^2 \ . \tag{5}$$

Would you say that this expression is completely Lorentz invariant?