Particle Physics (PHY680A)

Assignment 2 (Submission date: 08.02.2024)
Indian Institute of Technology, Kanpur

Instructor: Sabyasachi Chakraborty
✉: sabyac@iitk.ac.in
☎: 0512-279-2349 (O)

1. In the class, we found out the parity transformation rules on the Dirac bilinears. As far as charge conjugation matters, Dirac fields transform in the following way

$$\psi^c = \mathcal{C}\bar{\psi}^T \ , \quad \bar{\psi}^c = -\psi^T \mathcal{C} \ . \tag{1}$$

Note that, the effect of charge conjugation is to reverse the internal quantum numbers (here $U(1)_Q$ charge). From these results, with anticommuting Grassmann ψ's, show that

$$\bar{\psi}^c \psi^c = \bar{\psi}\psi \ , \quad \bar{\psi}^c \gamma^\mu \psi^c = -\bar{\psi}\gamma^\mu\psi \ , \quad \bar{\psi}^c \gamma^\mu\gamma^5 \psi^c = \bar{\psi}\gamma^\mu\gamma^5\psi \ . \tag{2}$$

Under charge conjugation, show that the vector field transforms as

$$\mathcal{C} : A_\mu \to -A_\mu \ .$$

With this information at hand, start with the Lagrangian $\mathcal{L} = J^\mu_V A_\mu - J^\mu_A A_\mu$ and derive the transformation rules under $\mathcal{C}$, $\mathcal{P}$ and $\mathcal{CP}$.

2. Show that the operators

$$P_R \equiv \frac{1}{2}\left(1+\gamma^5\right) \ , \quad P_L \equiv \frac{1}{2}\left(1-\gamma_5\right) \ , \tag{3}$$

have the appropriate properties to be the right and left-hand projection operators, i.e.,

$$P_i^2 = P_i \ , \quad P_L + P_R = 1 \ , \quad P_R P_L = 0 \ , \tag{4}$$

Here, γ^5 is the chirality operator.

3. Show that the weak current of the form

$$\bar{u}_e \gamma^\mu \frac{(1-\gamma_5)}{2} u_\nu \tag{5}$$

involves only left-handed electrons.

4. Recall the parity transformation properties of fermion fields described in the class. Using this, show that

$$\mathcal{P}\psi_L\mathcal{P}^{-1} = \gamma_0 \psi_R(x_P) \ . \tag{6}$$

This means that ψ_R is the parity transform of ψ_L. Thus if ψ_R and ψ_L possess different interactions, it signals parity violation. Are you convinced now why β decay experiments showed signatures of parity violation?

5. Obtain the following relations

$$
\begin{aligned}
&\mathrm{Tr}\left[\gamma^\mu \not{p}_1 \gamma^\nu \not{p}_2\right] = 4\left[p_1^\mu p_2^\nu + p_1^\nu p_2^\mu - (p_1.p_2) g^{\mu\nu}\right] \ , \\
&\mathrm{Tr}\left[\gamma^\mu (1-\gamma_5) \not{p}_1 \gamma^\nu (1-\gamma_5) \not{p}_2\right] = 2 \ \mathrm{Tr}\left[\gamma^\mu \not{p}_1 \gamma^\nu \not{p}_2\right] + 8i\epsilon^{\mu\alpha\nu\beta} p_{1\alpha} p_{2\beta} \ , \\
&\mathrm{Tr}\left[\gamma^\mu \not{p}_1 \gamma^\nu \not{p}_2\right] \mathrm{Tr}\left[\gamma_\mu \not{p}_3 \gamma_\nu \not{p}_4\right] = 32\left[(p_1.p_3)(p_2.p_4) + (p_1.p_4)(p_2.p_3)\right] \ , \\
&\mathrm{Tr}\left[\gamma^\mu \not{p}_1 \gamma^\nu \gamma_5 \not{p}_2\right] \mathrm{Tr}\left[\gamma_\mu \not{p}_3 \gamma_\nu \gamma_5 \not{p}_4\right] = 32\left[(p_1.p_3)(p_2.p_4) - (p_1.p_4)(p_2.p_3)\right] \ , \\
&\mathrm{Tr}\left[\gamma^\mu (1-\gamma_5) \not{p}_1 \gamma^\nu (1-\gamma_5) \not{p}_2\right] \mathrm{Tr}\left[\gamma_\mu (1-\gamma_5) \not{p}_3 \gamma_\nu (1-\gamma_5) \not{p}_4\right] = 256 (p_1.p_3)(p_2.p_4) \ .
\end{aligned} \tag{7}
$$

6. While computing the muon decay process, we used

$$
\Gamma = \frac{1}{(2\pi)^3} \frac{1}{8m_\mu} \int dE_1 \int dE_2 \left|\overline{\mathcal{M}}\right|^2 \ . \tag{8}
$$

   Obtain the limits of the integration.

   Hint: Recall our three body phase space calculation, where we used the constraint

$$
\delta\left(c_{12} - \frac{|\vec{p}_3|^2 - |\vec{p}_2|^2 - |\vec{p}_1|^2}{2|\vec{p}_1||\vec{p}_2|}\right) \ .
$$

   Use the maximum and minimum limits of $\cos\theta_{12}$ and the condition that the muon is decaying at rest.

7. If the weak charged current had a structure $\gamma^\mu\,(a + b\gamma_5)$, then show that for neutrino-electron scattering

$$
\frac{d\sigma}{d\Omega} = \frac{G^2 s}{32\pi^2}\left[A^+ + A^- \ \cos^4\frac{\theta}{2}\right] \ , \tag{9}
$$

   for both $\nu_e e$ and $\bar{\nu}_e e$ scattering.

8. **Additional problem if you are interested:** Show that the correction from the electron mass in the muon decay width is

$$
\Gamma = \frac{G^2 m_\mu^5}{192\pi^3}\left(1 - 8\frac{m_e^2}{m_\mu^2}\right) \ . \tag{10}
$$