Particle Physics (PHY680A)

Assignment 5 (Submission date: 31.03.2023)
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1. W **boson decay:** Show that the partial decay width for $W \to e\bar{\nu}$ is

$$\Gamma_W^0 \equiv \Gamma\left(W \to e\bar{\nu}\right) = \frac{g^2}{48\pi} M_W \ . \tag{1}$$

   To do this problem, use the vertex factor already derived in the class and the expression for the two body decay width which we did long time back. Remember, that g is related to G_F by matching SM with Fermi's theory. Use the value of G_F which we have discussed in the class and show that $\Gamma_W^0 = 0.225$ GeV.

   We have neglected electron mass in the above expression. In this limit

$$\Gamma_W^0 = \Gamma\left(W \to e\bar{\nu}_e\right) = \Gamma\left(W \to \mu\bar{\nu}_\mu\right) = \Gamma\left(W \to \tau\bar{\nu}_\tau\right) \ . \tag{2}$$

   Similarly, show that if you ignore quark masses then

$$\Gamma\left(W \to q'\bar{q}\right) = 3|V_{qq'}|^2 \Gamma_W^0 \ . \tag{3}$$

   Here V is the CKM matrix and 3 is the color factor (go to my notes and try to figure this out). You might need to look at page 19-21 of my handwritten notes to obtin the vertex factor. Finally, compare the values for $(q, q') = (u, d)$ and (c, s).

2. Z **boson decay:** Similar to previous problem, show that

$$\Gamma\left(Z \to \nu_e\bar{\nu}_e\right) = \frac{g^2}{96\pi} \frac{m_Z}{\cos^2\theta_W} \ . \tag{4}$$

   Here, the symbols have their usual meaning.

3. **Fermion Masses**: Recall that a direct mass term for fermions $m\bar{\psi}\psi$ is forbidden by gauge invariance. An attractive feature of the SM is that the same Higgs doublet which generates masses for the gauge bosons, also give masses to leptons and quarks. Such mass terms are generated by the $SU(2)_L \times U(1)_Y$ gauge invariant interactions of the form

$$\mathcal{L} \supset -y_e \left[\left(\bar{\nu}_e, \bar{e}\right)_L \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} e_R + \bar{e}_R \left(\phi^-, \ \bar{\phi}^0\right) \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \right] \ . \tag{5}$$

   Now, consider spontaneous breaking of the electroweak symmetry. In the unitary gauge, we can write

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} \ . \tag{6}$$

Using this definition, show that the Lagrangian expressed in terms of the masses of the fermions and its interactions with the Higgs field takes the form

$$\mathcal{L} \supset -\frac{y_e}{\sqrt{2}} v\left(\bar{e_L} e_R + \bar{e}_R e_L + h.c.\right) - \frac{y_e}{\sqrt{2}} (\bar{e}_L e_R + h.c.) h \ . \tag{7}$$

Choose, $m_e = y_e v/\sqrt{2}$. From Eq. (7), obtain the following decay width $\Gamma_{h\to e\bar{e}}$.

4. **Top quark decay:** Recall the discussion on charged current interactions between the W boson and leptons. Now, instead of leptons, obtain the vertex factor for W interacting with top and bottom quarks (you can consult my notes). Then, compute the top quark decay width for the process $t \to W^+ b$. Would there be any color factor N_C, which you would find for $h \to b\bar{b}$ (page 26-27 of my hand written notes)? If not, why?

5. **Cross-section calculation:** Compute the cross section for the process $e^+e^- \to \mu^+\mu^-$. You can ignore the lepton masses as well as the QED contribution. Therefore, the mediator would only be the Z boson.

6. **SM addressing the bad high energy behavior:** The following examples illustrates the role of Higgs boson in the SM, about gauge boson scattering. Consider the process

$$W_L^+(p_1) Z_L(p_2) \to W_L^+(p_3) Z_L(p_4) \ .$$

Just as we did in the class, for a momentum $p^\mu = (E, 0, 0, p_z)$, there are two transverse polarizations $\epsilon^\mu_{T_1} = (0, 1, 0, 0)$ and $\epsilon^\mu_{T_2} = (0, 0, 1, 0)$. The longitudinal polarization is $\epsilon^\mu_L = \frac{1}{m}(p_z, 0, 0, E)$. These all satisfy $\epsilon_i.p = 0$ and $\epsilon_i.\epsilon^*_j = -\delta_{ij}$. At high energies $\epsilon^\mu_L = \frac{p^\mu}{m}$. which is the origin of bad high-energy behavior. Nevertheless, compute the amplitude for

$$\mathcal{M}\left(W_L Z_L \to W_L Z_L\right) \ , \tag{8}$$

and comment on the result.