Particle Physics (PHY680A)

Assignment 4 (Submission date: 16.03.2024)
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1. Consider the case of spontaneous symmetry breaking of a global $U(1)$ symmetry. From there, consider the elastic scattering of the Goldstone bosons against the radial mode, i.e.,

$$h(p_1) + \xi(p_2) \to h(p3) + \xi(p4) \ . \tag{1}$$

How many diagrams contribute to this process at the tree level? Calculate the amplitude in both the representations and show that they are equal.

2. Consider the Lagrangian

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \Phi\right)^T \left(\partial^\mu \Phi\right) - \frac{\mu^2}{2}\Phi^T\Phi - \frac{\lambda}{4}\left(\Phi^T\Phi\right)^2 \ , \tag{2}$$

here $\Phi = (\phi_1, \phi_2, \phi_3)^T$ transforms like a triplet of an $SU(2)$ symmetry and ϕ_i's are real. Assume that the system settles down at the minimum $\langle\phi_3\rangle = v$ and $\langle\phi_1\rangle = \langle\phi_2\rangle = 0$.

- Obtain the Lagrangian in terms of the quantum fields. Find out the remnant symmetry of the Lagrangian after spontaneous symmetry breaking,
- Find the masses of the fields in the broken theory.

3. Despite the success of the SM, the prospect of finding other Higgs particles is not yet ruled out. In this example, we consider higher scalar $SU(2)$ multiplets and the task would be to deduce the modification of the mass relationship between the vector bosons W and Z. For a general value of weak isospin T, we have a multiplet of $2T+1$ complex components, corresponding to $T_3 = -T, ..., T$ which can be written as

$$\Phi = \begin{pmatrix}\phi_{T,T} & \phi_{T,T-1} & . & . & \phi_{T,-T}\end{pmatrix}^T \ . \tag{3}$$

We assume the weak hypercharge of this multiplet to be Y. The neutral component of the scalar field then has the weak isospin projection $T_3^0 = -Y$. The vacuum value of the multiplet can be written as

$$\Phi_0 = \frac{v}{\sqrt{2}}\begin{pmatrix}0 & . & . & 1 & . & . & 0\end{pmatrix}^T \ . \tag{4}$$

Here Φ_0 is the eigenvector of the $SU(2)$ generator T_3 corresponding to the eigenvalue $T_3^0 (= -Y)$. The vector boson masses can be obtained from the following Lagrangian

$$\mathcal{L} = \Phi_0^\dagger \left(g A_\mu^a T_a + g' Y B_\mu\right)\left(g A^{b\mu} T_b + g' Y B^\mu\right)\Phi_0 \ , \tag{5}$$

where T_a, $a = 1, 2, 3$ are $(2T + 1) \times (2T + 1)$ matrices representing the SU(2) generators, with T_3 as the diagonal and Y is a multiple of unit matrix. Introduce the isospin raising and lowering operators $T_\pm = T_1 \pm iT_2$ and the $W^\pm$ fields. Now, show that

$$\rho = \frac{T(T+1) - Y^2}{2Y^2} \ . \tag{6}$$

We have mentioned in class that having $\rho = 1$ at the classical level is desirable. This implies $T(T + 1) - 3Y^2 = 0$, which can be satisfied by $(T, Y) = (1/2, 1/2), (3, 2)$ etc. This is a serious constraint on the different types of Higgs multiplets you want to introduce in your model. Also, show that if there exist several Higgs scalar multiplets with different vacuum values, then one obtains

$$\rho = \frac{\sum_{T,Y} |v_{T,Y}|^2 \, T(T+1) - Y^2}{2 \sum_{T,Y} |v_{T,Y}|^2 \, Y^2} \ . \tag{7}$$

This implies that if you have only Higgs doublets, you would always get $\rho = 1$, independent of the values of $v_{T,Y}$.

4. Go to my notes on Coleman-Weinberg potential. The final form has been obtained using cut-off regularization method. Follow all the steps and reproduce Eq. (23). If you have too much coffee, then you can also try to reproduce the final form using dimensional regularization.