Particle Physics (PHY680A)

Assignment 3 (Submission date: 21.02.2024)
Indian Institute of Technology, Kanpur

Instructor: Sabyasachi Chakraborty
✉: sabyac@iitk.ac.in
☎: 0512-279-2349 (O)

1. In the class, we have introduced the CKM matrix, V as a 3×3 matrix with 9 real parameters. Show that these can be divided into 3 rotation angles and 6 phases. However, five of these 6 phases are unphysical. Show that they can be eliminated by redefining the fermion fields $f_n \to f_n e^{i\beta_n}$. This phase rotation does not affect anything else as in all other cases fermion fields enter in the form $\bar{f}_n \gamma_\mu f_n$.

   Hint: (If you choose to do this way, then explain all the results)
   Write the generic CKM matrix as

$$V = \begin{pmatrix} V_{11}e^{i\beta_{11}} & V_{12}e^{i\beta_{12}} & V_{13}e^{i\beta_{13}} \\ V_{21}e^{i\beta_{21}} & V_{22}e^{i\beta_{22}} & V_{23}e^{i\beta_{23}} \\ V_{31}e^{i\beta_{31}} & V_{32}e^{i\beta_{32}} & V_{33}e^{i\beta_{33}} \end{pmatrix} . \tag{1}$$

   Here all quantities are real and we also consider $V_{mn} \geq 0$ and $0 \leq \beta_{mn} \leq 2\pi$, for $m, n = 1, 2, 3$. Although there are 9 phase factors, they are not independent. Additional constraints can be obtained by imposing the unitarity condition on V:

$$\begin{aligned} V_{11}V_{21}e^{i(\beta_{11}-\beta_{21})} + V_{12}V_{22}e^{i(\beta_{12}-\beta_{22})} + V_{13}V_{23}e^{i(\beta_{13}-\beta_{23})} &= 0 \ , \\ V_{11}V_{31}e^{i(\beta_{11}-\beta_{31})} + V_{12}V_{32}e^{i(\beta_{12}-\beta_{32})} + V_{13}V_{33}e^{i(\beta_{13}-\beta_{33})} &= 0 \ , \\ V_{21}V_{31}e^{i(\beta_{21}-\beta_{31})} + V_{22}V_{32}e^{i(\beta_{22}-\beta_{32})} + V_{23}V_{33}e^{i(\beta_{23}-\beta_{33})} &= 0 \ . \end{aligned} \tag{2}$$

   As a result of the phase rotation of fermion fields

$$\begin{aligned} d'_{L_n} &= e^{-i\beta_{1n}} d'' \ , \ n = 1, 2, 3 \ , \\ \bar{u'}_{L_m} &= e^{-i(\beta_{m1}-\beta_{11})} \bar{u''} \ , \ m = 2, 3 \ . \end{aligned} \tag{3}$$

   five out of the nine phases of the mixing matrix can be set equal to zero without using Eq. (2). The remaining 4 phases are independent. Using the field rotation, Eq. (2) becomes

$$\begin{aligned} V_{11}V_{21} + V_{12}V_{22}e^{-i\beta_{22}} + V_{13}V_{23}e^{-i\beta_{23}} &= 0 \ , \\ V_{11}V_{31} + V_{12}V_{32}e^{-i\beta_{32}} + V_{13}V_{33}e^{-i\beta_{33}} &= 0 \ , \\ V_{21}V_{31} + V_{22}V_{32}e^{i(\beta_{22}-\beta_{32})} + V_{23}V_{33}e^{i(\beta_{23}-\beta_{33})} &= 0 \ . \end{aligned} \tag{4}$$

   The first condition helps us to express β_{22} in terms of β_{23} and elements V_{mn}. The second condition gives us the relation between β_{32} and β_{33}. After substituting these expressions into the third condition, β_{33} is expressed through the only remaining phase β_{23}.

2. We have discussed charged current processes of weak interaction as the neutral current was discovered much later. Surprisingly, experiments pointed out that neutral currents have a slightly different structure. **Compute the electron-neutrino cross section** for the neutral current process, using the following:

   The generic amplitude for the neutral current process can be written as

$$\mathcal{M} = \frac{4G}{\sqrt{2}}\, 2\rho\, J_\mu^{(e)} J^{\mu(\nu_e)} \ , \tag{5}$$

   where,

$$\begin{aligned} J_\mu^{(\nu_e)} &= \bar{u}_\nu \gamma^\mu (1-\gamma_5) u_\nu \ , \\ J_\mu^e &= \bar{u}_e \gamma_\mu \left(C_A V - C_A \gamma_5\right) u_\nu \ . \end{aligned} \tag{6}$$

3. Draw the relevant Feynman diagrams and explain why $\bar{K}$ meson (and not $K(d\bar{s})$ meson) should preferentially feature among the decay products of $D^+(c\bar{d})$ meson.

4. In the presence of CP violation, the physical states are not the same as the CP eigenstates. From experimental observations, we realized that the effects of CP violation were small. Assuming the following states,

$$\begin{aligned} |K_S\rangle &= \frac{1}{\sqrt{1+|\epsilon|^2}} \left(|K_1\rangle + \epsilon |K_2\rangle\right) \ , \\ |K_L\rangle &= \frac{1}{\sqrt{1+|\epsilon|^2}} \left(|K_2\rangle + \epsilon |K_1\rangle\right) \ . \end{aligned} \tag{7}$$

   Compute the $|K^0\rangle$-$|\overline{K^0}\rangle$ oscillation probability.

5. Consider the case of spontaneous symmetry breaking of a global $U(1)$ symmetry. From there, consider the elastic scattering of the Goldstone bosons against the radial mode, i.e.,

$$h(p_1) + \xi(p_2) \to h(p3) + \xi(p4) \ . \tag{8}$$

   How many diagrams contribute to this process at the tree level? Calculate the amplitude in both the representations and show that they are equal.