Mid Semester Examination

Particle Physics (PHY680A), IIT Kanpur

Time: 2hrs (23.02.2024) Marks: 30

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1. Starting from the expression of the differential cross section for a $2 \to 2$ process in the center of mass frame
$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{CM}} = \frac{1}{64\pi^2 s}\frac{|\vec{p}_f|}{|\vec{p}_i|}\left|\overline{\mathcal{M}}\right|^2 \ ,$$
obtain the differential cross-section in terms of Lorentz invariant quantities such as Mandelstam variables and masses. Here, $|\overline{\mathcal{M}}|^2$ is the spin averaged amplitude squared, and $\vec{p}_f$ and $\vec{p}_i$ represent the initial and final state three momenta respectively. Assume m_1, m_2 to be the masses of the initial state and m_3, m_4 to be the masses for the final state particles. (Hint: Try to use the energy conservation.)

(10)

2. Consider the Lagrangian
$$\mathcal{L} \supset \lambda\, \phi\overline{\psi}\psi + m_\psi\, \overline{\psi}\psi \ ,$$
where λ is a real coupling constant between the fermion field ψ and the scalar field ϕ, of mass m_ϕ.

(a) Deduce the mass dimension for the coupling λ.

(a) Draw the Feynman diagram for the decay process $\phi\,(p_1) \to \psi(p_2)\overline{\psi}(p_3)$,

(b) Compute the decay width (Γ) for $\phi \to \psi\overline{\psi}$.

$(2 + 2 + 6 = 10)$

3. Consider the elastic neutrino-electron scattering process $\nu(k) + e(p) \to \nu(k') + e(p')$ in the four Fermi theory .

(a) Draw the Feynman diagrams depicting this scattering.

(b) Assume the following current
$$\begin{aligned} J^\mu_{(\nu)} &= \bar{\psi}_\nu \gamma^\mu \left(1 - \gamma_5\right) \psi_\nu \ , \\ J_{(e)\mu} &= \bar{\psi}_e \gamma_\mu \left(C_V - C_A\gamma_5\right) \psi_e \ , \end{aligned}$$
to compute the spin-averaged amplitude square ($|\overline{\mathcal{M}}|^2$) for this process. Write the result in terms of Lorentz invariant quantities. You can ignore the masses for the electrons.

(c) Consider the potential of a real scalar field:
$$V\left(\phi\right) = -\frac{\mu^2}{2}\phi^2 + \frac{2\xi}{3}\phi^3 + \frac{\lambda}{4}\phi^4 \ .$$
Plot the potential and find out the degeneracy in the minima when $\xi = 0$, What happens to the old minima if we switch on the cubic term?

$(2 + 5 + (1 + 2) = 10)$

Useful expression: You may need the following

$$\begin{aligned} &\text{Tr}\left[\not{k}_1(1+\gamma_5)\gamma_\nu\not{k}_2\gamma_\mu(1-\gamma_5)\right]\text{Tr}\left[\not{p}_1\gamma^\mu(C_V - C_A\gamma_5)\not{p}_2(C_V + C_A\gamma_5)\gamma^\nu\right] \\ &= 64\left[(C_A + C_V)^2(k_1.p_2)(k_2.p_1) + (C_A - C_V)^2(k_1.p_1)(k_2.p_2)\right] \ . \end{aligned}$$