End Semester Examination

Particle Physics (PHY680A), IIT Kanpur

Time: 3hrs (23.04.2024) Marks: 50

Read the questions carefully and answer all of them. Assume $c = \hbar = 1$ for all the problems. Relevant formulas/hints are given at the back.

1. Consider the Lagrangian of a scalar field theory

$$\mathcal{L} = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \frac{1}{2}m^2\phi^2 - \lambda\phi^4 + \eta g_1\phi^6 + \eta g_2\phi^3\Box\phi + \mathcal{O}(\eta^2).$$

Here η is a constant with mass dimension $[-2]$ and g represents a dimensionless coupling constant. Use the following field redefinition: $\phi \to \phi + \eta g_2\phi^3$ and obtain the Lagrangian. Are there any redundant terms that can be dropped by making use of the above field redefinition? (7)

2. Under parity and charge conjugation, a fermion field transforms as $\hat{P}\psi(x)\hat{P}^{-1} = \gamma^0\psi(x_P)$ and $\hat{C}\psi(x)\hat{C} = C\bar{\psi}^T$. C is representation dependent and one can choose it as $C\gamma^0 = i\gamma^2$. Find out the transformation of the weak current under CP, i.e., CP: $\left(\bar{\psi}_1\gamma_\mu P_L\psi_2\right)$. (7)

3. Consider several Higgs multiplets, ϕ_i, with weak isospin T_i and hypercharge Y_i. These states' neutral component ($Q_i = 0$) acquires a vacuum expectation value v_i. Show that (7)

$$\rho = \frac{M_W^2}{M_Z^2\cos^2\theta_W} = \frac{\sum_i v_i^2[T_i(T_i+1) - Y_i^2]}{2\sum_i v_i^2 Y_i^2}.$$

4. Find the spin-averaged amplitude squared or $|\overline{\mathcal{M}}|^2$ for the process $Z(p) \to \nu(p_1)\bar{\nu}(p_2)$ in the Standard Model, in terms of Lorentz invariant quantities. You can derive or use the Lagrangian from your notes to compute. (7)

5. The amplitude squared for Higgs decays to fermion and anti-fermion pair is given by (do not show this)

$$|\overline{\mathcal{M}}|^2 = N_c\frac{2m_f^2}{v^2}m_j^2\left(1 - \frac{4m_f^2}{m_h^2}\right).$$

Compute the decay width $\Gamma\left(h \to f\bar{f}\right)$. (7)

6. Find $|\overline{\mathcal{M}}|^2$ for the inelastic neutrino-electron scattering $\nu_\mu(k) + e(p) \to \mu(p') + \nu_e(k')$ process. Draw the Feynman diagram(s), give proper arrows, and express the result in the rest frame of the electron. You can also use Fermi theory to compute the $|\overline{\mathcal{M}}|^2$. (7)

7. Start from the chiral Lagrangian defined as

$$\mathcal{L}_\chi = \frac{f^2}{8}\left\langle\partial^\mu\Sigma(x)\partial_\mu\Sigma(x)^\dagger\right\rangle, \text{ where } \Sigma(x) = \exp\left(\frac{2iM(x)}{f}\right), \text{ with } M(x) = \frac{\tau^a\pi^a(x)}{\sqrt{2}}.$$

Here $\langle\ldots\rangle$ denotes trace and $\pi(x)$ are the pion fields. Deduce the kinetic term and pion-pion scattering term (4-point vertex), from the above Lagrangian [1]. (8)

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[1] Please turn over

The following expressions/hints may be useful.

- Question 1: Ignore terms $\mathcal{O}(\eta^2)$ and assume that the total derivative vanishes.
- Question 2: Recall that $C = C^{-1} = -C^{\dagger} = -C^{T}$, and $C^{-1}\gamma^{\mu}C = (-\gamma^{\mu})^{T}$, $C^{-1}\gamma^{\mu}\gamma^{5}C = (\gamma^{\mu}\gamma^{5})^{T}$.
- Question 3: We can use the following expression

$$D_{\mu}\phi_i \supset \frac{g}{\sqrt{2}}\left(T_i^{+}W_{\mu}^{+} + T_i^{-}W_{\mu}^{-}\right)\phi_i + \left(gT_i^{3}W_{\mu}^{3} + \frac{g}{2}Y_i B_{\mu}\right)\phi_i$$

- Question 4: You can use the following trace expression

$$\mathrm{Tr}\left[p_2(1+\gamma_5)\gamma_{\mu}p_1\gamma_{\nu}(1-\gamma_5)\right] = 8\left[p_{1\nu}p_{2\mu} + p_{1\mu}p_{2\nu} - p_1 \cdot p_2 g_{\mu\nu} + i\epsilon_{\mu\nu\rho\sigma}p_1^{\rho}p_2^{\sigma}\right].$$

- Question 5: I am not giving any hints for this one.
- Question 6: You can use the same trace expression as given before.
- Question 7: You can find this expression useful

$$\partial^{\mu}e^{M} = \int_0^1 ds\, e^{sM}\partial^{\mu}M e^{(1-s)M}.$$