QFT ASSIGNMENT III

DUE DATE: 02.11.2023

---

(1) Consider the Dirac Lagrangian in dimensions $d = 1 + 3$.

(a) Find the Energy-Momentum tensor using the two different methods done in class.

(b) Is this symmetric? If not, make this symmetric using methods discussed in class.

(c) Use the method of varying the metric to find a symmetric stress tensor. Does this match with your improved stress tensor above?

(2) We will again look at the Dirac theory.

(a) Consider the transformation $\psi(x) \to e^{i\alpha}\psi(x)$. Show that this is a symmetry of the action. Find the conserved current and the conserved charge.

(b) Now consider the transformation $\psi(x) \to e^{i\alpha\gamma^5}\psi(x)$. When is this a symmetry of the theory? Find the conserved current and the conserved charge corresponding to this.

(3) Casimirs of a particular algebra are objects constructed out of the generators of the algebra which commute with all generators of the algebra. E.g. $J^2 = J_x^2 + J_y^2 + J_y^2$ is the (quadratic) Casimir of the $su(2)$ algebra where the generators are J_i.

Show that P^2 and W^2 are the (quadratic and quartic) Casimirs of the Poincare algebra where we have denoted P^a as the momentum generators and L^{ab} as the Lorentz generators and

$$W_a = \epsilon_{abcd} P^a L^{ab}.$$

This means that you would need to show that the commutators of the two Casimirs vanish with all generators of the Poincare algebra.

(4) Find the properties of the following objects under Lorentz transformations and Parity and classify them in terms of scalars, pseudoscalars, vectors etc.

$$\bar{\psi}(x)\psi(x),\ \bar{\psi}(x)\gamma^5\psi(x), \bar{\psi}(x)\gamma^\mu\psi(x),\ \bar{\psi}(x)\gamma^\mu\gamma^5\psi(x),$$
$$\bar{\psi}(x)\{\gamma^\mu, \gamma^\nu\}\psi(x), \bar{\psi}(x)\{\gamma^\mu, \gamma^5\}\psi(x), \bar{\psi}(x)[\gamma^\mu, \gamma^\nu]\psi(x).$$

(5) What are Majorana fermions?

(a) Show that in $d = 1 + 3$ you cannot impose Majorana and Weyl conditions on fermions.

(b) Now, consider $d = 1 + 1$. Show that here it is possible to do this.
(This is going to require understanding how fermions behave in dimensions other than four and is involved!)

(6) Do the following problems from Peskin and Schroeder's book "An introduction to Quantum Field Theory":
(a) Problem 3.4,
(b) Problem 4.2.