QFT ASSIGNMENT II

DUE DATE: 25.09.2024

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(1) Consider two complex Klein-Gordon fields of the same mass. Label each of the fields as $\phi_a(x)$, where $a = 1, 2$.

(a) Show that you have four conserved charges given by

$$Q^\mu = \int d^3x \frac{i}{2} \left(\phi_a^\star (A^\mu)_{ab} \pi_b^\star - \pi_a (A^\mu)_{ab} \phi_b\right)$$

where $\mu = 0, \ldots 3$ and $A^0 = 1, A^i = \sigma^i$ with σ^i being the Pauli sigma matrices.

(b) Show that the three charges Q^i with $i = 1, 2, 3$ have the commutation relations of angular momentum (SU(2)).

(c) Generalise the result for n identical scalar fields.

(2) Evaluate the function

$$\langle 0|\phi(x)\phi(y)|0\rangle = D(x-y) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_p} e^{-ip.(x-y)},$$

for spacelike $(x-y)$, so that $(x-y)^2 = -r^2$, explicitly in terms of Bessel functions.

(3) Consider the spinless Schrodinger field given by the Lagrangian

$$\mathcal{L} = i\psi^\dagger \frac{\partial \psi}{\partial t} - \frac{1}{2m} \nabla \psi^\dagger \cdot \nabla \psi - V(\mathbf{r})\psi^\dagger \psi$$

(a) Express the free fields $\psi, \psi^\dagger$ in terms of creation and annihilation operators and find the commutation relations between them.

(b) Calculate the Green's function

$$G(x_0, \mathbf{x}, y_0, \mathbf{y}) = -i\langle 0|\psi(x_0, \mathbf{x})\psi^\dagger(y_0, \mathbf{y})|0\rangle \theta(x_0 - y_0).$$

(c) Show that this satisfies the equation

$$\left(i\frac{\partial}{\partial t} + \frac{1}{2m}\nabla^2\right) G(t, \mathbf{x}, 0, 0) = \delta(t)\delta^{(3)}(\mathbf{x}).$$

(4) Calculate the vacuum expectation value

$$\langle 0|\{\phi(x), \phi(y)\}|0\rangle$$

for a massless free scalar field, where $\{,\}$ is the anticommutator. Show that this satisfies the KG equation.

(5) Calculate

$$\langle 0|\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)|0\rangle$$

for a free scalar field by plugging in the expansion of the fields in terms of creation and annihilation operators.