**PHY681(A): End-term Examination**

November 21, 2023.

Total Marks: 90.

Answer all questions.

1. Consider the free Dirac Lagrangian:
2. Show that is a symmetry of the Lagrangian. Find the corresponding conserved current and charge.
3. Under what conditions is the transformation ( , ) a symmetry of the Lagrangian? In this special case, again find the corresponding conserved current and charge.

5 + 5 10

1. Consider free Dirac theory. Wick contractions in the fermionic theory comes hand in hand with a rule for the sign of a particular contraction.

(a) State the rule to evaluate the sign.

(b) Consider the following process:

By using fermionic Wick contraction rules, find the expression of the above process.

2 + 5 7

1. Consider the following Lagrangian

where .

1. Write down the momentum space Feynman rules for the above theory.
2. Draw the Feynman diagrams which contribute to the leading corrections of the photon and the scalar propagators.

9 + 6 15

1. Consider a massive complex scalar field in four spacetime dimensions.

(a) Write the Lagrangian for the theory and show that this Lagrangian has a global symmetry.

(b) Now promote this global symmetry to a local symmetry and show how the Lagrangian has to change in order for this to still be a symmetry of the system.

(c) Add appropriate kinetic terms to any other field that appears. What does this new Lagrangian describe?

(d) Write down the Feynman rules for this theory.

3 + 6 + 3 + 6 18

1. Consider a complex scalar field with the following Lagrangian:

Notice the “wrong” sign in front of the mass term.

1. Write the potential of the Lagrangian in terms of real fields . Putting , draw the potential as a function of
2. Consider and show the extrema of the potential are at and .
3. Choose in the second solution and show this is the minimum. By computing the value of at the extrema, verify your answer.
4. Quantum mechanically, this means giving some vacuum expectation values to the fields, i.e.

This means the perturbation theory needs to be done about this minima and hence the fields need to be shifted as

Plug this back into your initial Lagrangian and expand.

1. What are the masses of your fields and now? Does the “wrong sign” still persist?
2. What is the physics lesson that you derived out of this exercise?

4 + 2 + 4 + 4 + 4 + 2 20

6. Prove the following identities:

(a)

(b)

(c)

3 + 3 + 4 10

7. Write short answers to the following questions:

(a) Do fermions live on the spacetime manifold? Explain.

(b) How are gamma matrices related to the Lorentz algebra?

(c) How are spinors of different from spinors of ?

(d) What is the difference between a global symmetry and a gauge symmetry?

(e) What is the main difficulty in quantizing the electromagnetic theory?

2 × 5 10