

Assignment 5.2

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Question Description

2. Using Naive Bayesian classifier predict river non river using Satellite data set of Hooghly river (unstructured data set).

Concepts Used:

GAUSSIAN CLASS CONDITIONAL DENSITIES

Here, $\mathcal{X} = \mathbb{R}^d$ and $\mathcal{Y} = \{1, \dots, K\}$. Estimate Bayes classifier via MLE:

- **Class priors:** The MLE estimate of π_y is $\hat{\pi}_y = \frac{1}{n} \sum_{i=1}^n \mathbb{1}(y_i = y)$.
- **Class conditional density:** Choose $p(x|Y = y) = N(x|\mu_y, \Sigma_y)$.
The MLE estimate of (μ_y, Σ_y) is

$$\hat{\mu}_y = \frac{1}{n_y} \sum_{i=1}^n \mathbb{1}(y_i = y) x_i,$$

$$\hat{\Sigma}_y = \frac{1}{n_y} \sum_{i=1}^n \mathbb{1}(y_i = y) (x_i - \hat{\mu}_y)(x_i - \hat{\mu}_y)^T.$$

This is just the empirical mean and covariance of class y .

- **Plug-in classifier:**

$$\hat{f}(x) = \arg \max_{y \in \mathcal{Y}} \hat{\pi}_y |\hat{\Sigma}_y|^{-\frac{1}{2}} \exp \left\{ -\frac{1}{2} (x - \hat{\mu}_y)^T \hat{\Sigma}_y^{-1} (x - \hat{\mu}_y) \right\}.$$

Introduction

We are given 4 images denoting the R band, G Band, B Band and I Band Images of an image with river and non river points. We have to classify every point in a 512X512 image into either a river class or a non river class.

Steps Followed:

- Four satellite Images of Kolkata (Rband, Gband, Bband and Iband) are given to you with equal image size (512 * 512).
 - The feature vector dimension is 4
 - Each pixel location we have four values.
 - Two Classes are given (River and NonRiver)
 - Take 50 sample points (Pixel location's corresponding pixel values) from river class for training for each band
 - Take 100 sample points (Pixel location's corresponding pixel values) from non river class for training for each band.
 - Take (512 * 512) sample points (Pixel location's corresponding pixel values) for testing for each band.
 - Apply baye's decision rule to classify all the test sample either in river or nonriver class denoting 0 and 255 at corresponding pixel locations.
 - Show the result in image form with black and white image (either 0 and 255)
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- Step 1: Calculate Mean of River Class : $T1 = [\text{Mean1}; \text{Mean2}; \text{Mean3}; \text{Mean4}]$
Mean1 = mean of Rband image for 50 sample points
Mean2 = mean of Gband Image for 50 sample points
Mean3 = mean of Bband image for 50 sample points
Mean4 = mean of Iband image for 50 sample points

•Step 2: Calculate Mean of NonRiver Class : $T2 = [\text{Mean1}; \text{Mean2}; \text{Mean3}; \text{Mean4}]$;

Mean1 = mean of Rband image for 100 sample points

Mean2 = mean of Gband Image for 100 sample points

Mean3 = mean of Bband image for 100 sample points

Mean4 = mean of Iband image for 100 sample points

•Step 3: Calculate the Covariance Matrix for River Class for 50 samples which is 4×4

dimensions. Basically $(X - T1)$ deviation and $(Y - T1)$ deviation and multiply it and summing up

where X and Y represents all the sample points considered for training (R, G, B and I band

image) we will get $2^4 = 16$ values in the covariance matrix for possible combinations of 4 band

images. We are doing the deviation of sample points from the mean vector.

(Apply covariance matrix calculation formula)• Step 4: Calculate the Covariance Matrix for Non River Class for 100 samples which is 4×4

dimensions also by applying same process explained in step 3.

• Step 5: Take whole image for test data where : $\text{test_data} = [\text{Rband_img}(i,j) \text{ Gband_img}(i,j)$

$\text{Bband_img}(i,j) \text{ Iband_img}(i,j)]$; $i = 1$ to 512 ; and $j = 1$ to 512 ;

• step 6: The dimension of test data is $(4 \times (512 \times 512))$;

• Step 7: For each pixel location of test image Run the loop from $i = 1$ to (512×512) Do

• Step 8: For river class calculate $(\text{test_data} - T1)$ deviation and $(\text{test_data} - T1)^T$ Then

Multiply it :

$\text{River_class} = (\text{Test_data} - T1)^T \times \text{Inverse}(\text{Covariance_matrix_Riverclass}) \times (\text{Test_data} - T1)$

• Step 9: For Non_river class calculate $(\text{test_data} - T2)$ deviation and $(\text{test_data} - T2)^T$ Then

Multiply it :

Nonriver class = (Test_data – T2) T * Inverse (Covariance_matrix_NonRiverclass) *(Test_data – T2)• Step 10: Calculate density function p1 for river class where P1 = 0.3 given

$p1 = (-0.5) * 1/\sqrt{\text{Determinant of Covariance_matrix_Riverclass}} * \exp(\text{River_class});$

(Here we apply multivariate Normal Distrubution)

- Step 11: Calculate density function p2 for nonriver class where P2 = 0.7 given

$p2 = (-0.5) * 1/\sqrt{\text{Determinant of Covariance_matrix_nonRiverclass}} * \exp(\text{NonRiver_class});$

- Step 12: For each pixel location of test image apply baye's rule ($P1 * p1$) \geq ($P2 * p2$) then

Out_image(i) = 255 (River class)

Else

Out_image(i) = 0; (Nonriver class)

- Step 13 : Goto step 7;

- Step 14: Show the three output image Image using imshow function for three cases:

Case 1 : River class (Prior Prob:) = 0.3 , Nonriver class(Prior Prob) = 0.7

Case 2 : River class (Prior Prob:) = 0.7 , Nonriver class(Prior Prob) = 0.3

Case 3 : River class (Prior Prob:) = 0.5 , Nonriver class(Prior Prob) = 0.5

RESULTS

For P1=0.7 and P2=0.3 the results are :-

