Soft Computing:ISCO630E
Report

Assignment 3.2

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Question Description

Using the data set of two quality test results of a microchip product, design a predictor using logistic regression which will predict the acceptance or rejection of the microchip given the two test results. Use regularizer to further tune the parameters. Use 70 % data for training and rest 30% data for testing your predictor and calculate the efficiency of the predictor/hypothesis.

Hints: 1. You can pre process the data for convenience

2. You must use Python program for evaluating parameters using batch gradient descent algorithm (GDA). No function should be used for GDA.

Introduction

We are given a dataset comprising of quality test results obtained by various microchip products in two tests and a binary result of 0 if the microchip product was not admitted based on its quality test results and 1 if the microchip product was admitted.

Concepts Used

Hypothesis Function

$$h(x) = \sum_{i=0}^{n} \theta_i x_i = \theta^T x, \qquad X = \begin{bmatrix} — (x^{(1)})^T — \\ — (x^{(2)})^T — \\ \vdots \\ — (x^{(m)})^T — \end{bmatrix}, \qquad \vec{y} = \begin{bmatrix} y^{(1)} \\ y^{(2)} \\ \vdots \\ y^{(m)} \end{bmatrix}.$$

A Linear Regression model can be represented by the equation.

$$h(x) = \theta^T x$$

We then apply the sigmoid function to the output of the linear regression

$$h(x) = \sigma(\theta^T x)$$

where the sigmoid function is represented by,

$$\sigma(t) = \frac{1}{1 + e^{-t}}$$

The hypothesis for logistic regression then becomes,

$$h(x) = \frac{1}{1 + e^{-\theta^T x}}$$

$$h(x) = \begin{cases} > 0.5, & \text{if } \theta^T x > 0 \\ < 0.5, & \text{if } \theta^T x < 0 \end{cases}$$

If the weighted sum of inputs is greater than zero, the predicted class is 1 and vice-versa. So the decision boundary separating both the classes can be found by setting the weighted sum of inputs to 0.

Cost Function(Regularized)

The cost function for a single training example can be given by:

$$cost = \begin{cases} -log(h(x), & \text{if } y = 1 \\ -log(1 - h(x)), & \text{if } y = 0 \end{cases}$$

We can combine both of the equations using:

$$cost(h(x), y) = -ylog(h(x)) - (1 - y)log(1 - h(x))$$

Then with regularization the cost function for m samples becomes

$$J(\theta) = \frac{1}{m}\sum_{i=1}^{m}\left[-y^{(i)}\log(h_\theta(x^{(i)})) - (1-y^{(i)})\log(1-h_\theta(x^{(i)}))\right] + \frac{\lambda}{2m}\sum_{j=1}^{n}\theta_j^2.$$

Gradient Descent(Regularized)

$$\theta_j := \theta_j - \alpha\frac{\partial}{\partial\theta_j}J(\theta).$$

$$\frac{\partial J(\theta)}{\partial\theta_0} = \frac{1}{m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})x_j^{(i)} \qquad \text{for } j = 0$$

$$\frac{\partial J(\theta)}{\partial\theta_j} = \left(\frac{1}{m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})x_j^{(i)}\right) + \frac{\lambda}{m}\theta_j \qquad \text{for } j \geq 1$$

Where

$$h(x) = \frac{1}{1 + e^{-\theta^T x}}$$

α = Learning rate

The convergence arises when $J_t(\Theta) - J_{t+1}(\Theta) <=$ ERROR_THRESHOLD

Since the cost function is convex in nature, the local minima obtained for the cost function would be the global minima.

Approach

Here a linear fit of data will lead to a very bad accuracy.So we needed to add additional features to fit the data with high accuracy.Let the test result obtained by a single microchip in the first test be X1 and in the second test be X2.Then we can add three additional features by using the two given features.Those three features will be X1*X2 , X1*X1 , X2*X2 .Appending these feature values into our matrix X along with our original values of X1 and X2 for every sample we will have

the modified X.Now applying the Original Gradient Descent Algorithm to the Modified X will give the values of theta and we can proceed in the same way as we proceeded before.

RESULTS

75% of the data was used for training while the remaining was used for testing the predictor.

The convergence appeared to be faster for

α =0.01

Error_Threshold=0.0000001

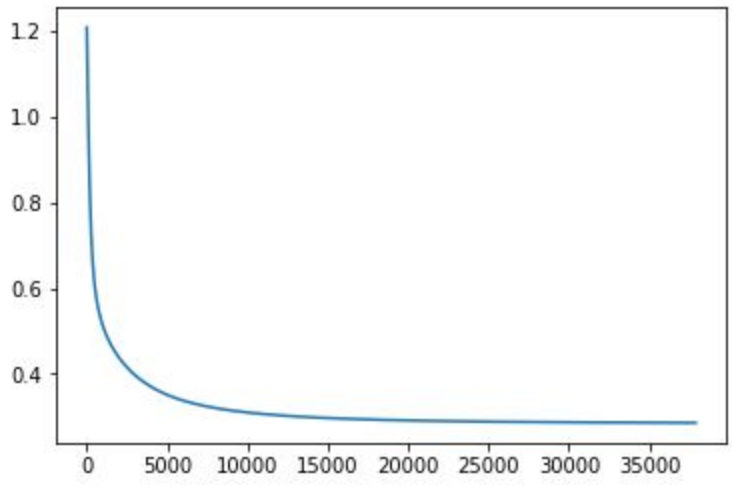
λ =0.03

For the approach of Gradient Descent it took 37881

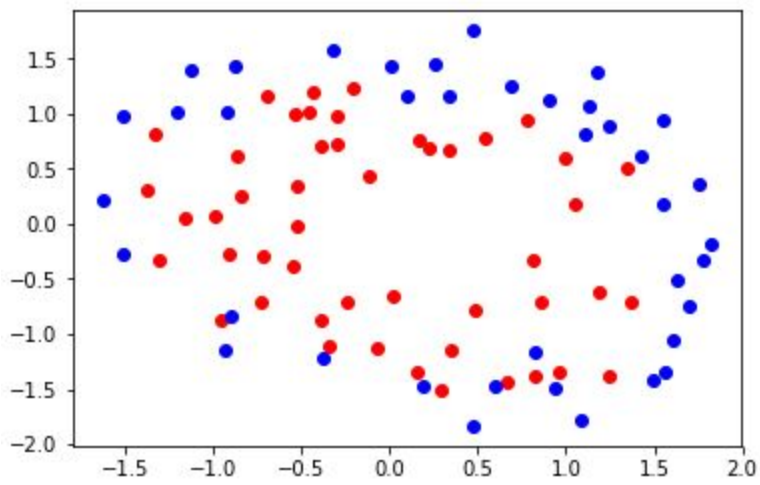
iterations to arrive at a final cost of 0.28628609682282824

The final theta values using Gradient Descent obtained were

Final theta values : [0.13481118 -4.37746783 -3.36633977 -1.58377714 1.55847625 2.30545435]



The Data Points was obtained as follows.Drawing the decision boundary was complex so it was skipped for now.



%accuracy for train data is 86.20689655172413

% accuracy for test data is 76.66666666666667