Soft Computing:ISCO630E
Report

Assignment 4

Anant Chaturvedi(IIT2016506)

Question Description

1. Using the data set of two examination results design a predictor using logistic regression for predicting whether a student can get an admission in the institution. Use regularizer to further tune the parameters. Use 70 % data for training and rest 30% data for testing your predictor and calculate the efficiency of the predictor/hypothesis.

2.Hints: 1. You can pre process the data for convenience

2. You must use Python program for evaluating parameters using batch gradient descent algorithm (GDA). No function should be used for GDA.

Introduction

We are given a dataset comprising of marks obtained by various students in two subjects and a binary result of 0 if the student was not admitted based on his marks and 1 if the student was admitted.We have to use Newton's Method to set the parameters theta.

Concepts Used

Hypothesis Function

$$h(x) = \sum_{i=0}^{n} \theta_i x_i = \theta^T x, \qquad X = \begin{bmatrix} — (x^{(1)})^T — \\ — (x^{(2)})^T — \\ \vdots \\ — (x^{(m)})^T — \end{bmatrix}. \qquad \vec{y} = \begin{bmatrix} y^{(1)} \\ y^{(2)} \\ \vdots \\ y^{(m)} \end{bmatrix}.$$

A Linear Regression model can be represented by the equation.

$$h(x) = \theta^T x$$

We then apply the sigmoid function to the output of the linear regression

$$h(x) = \sigma(\theta^T x)$$

where the sigmoid function is represented by,

$$\sigma(t) = \frac{1}{1 + e^{-t}}$$

The hypothesis for logistic regression then becomes,

$$h(x) = \frac{1}{1 + e^{-\theta^T x}}$$

$$h(x) = \begin{cases} > 0.5, & \text{if } \theta^T x > 0 \\ < 0.5, & \text{if } \theta^T x < 0 \end{cases}$$

If the weighted sum of inputs is greater than zero, the predicted class is 1 and vice-versa. So the decision boundary separating both the classes can be found by setting the weighted sum of inputs to 0.

Cost Function(Regularized)

The cost function for a single training example can be given by:

$$cost = \begin{cases} -log(h(x), & \text{if } y = 1 \\ -log(1 - h(x)), & \text{if } y = 0 \end{cases}$$

We can combine both of the equations using:

$$cost(h(x), y) = -ylog(h(x)) - (1 - y)log(1 - h(x))$$

Then with regularization the cost function for m samples becomes

$$J(\theta) = \frac{1}{m}\sum_{i=1}^{m}\left[-y^{(i)}\log(h_\theta(x^{(i)})) - (1-y^{(i)})\log(1-h_\theta(x^{(i)}))\right] + \frac{\lambda}{2m}\sum_{j=1}^{n}\theta_j^2.$$

Newton's Method

$$\theta^{(t+1)} = \theta^{(t)} - H^{-1}\nabla_\theta J$$

In logistic regression, the gradient and the Hessian are

$$\nabla_\theta J = \frac{1}{m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})x^{(i)}$$

$$H = \frac{1}{m}\sum_{i=1}^{m}\left[h_\theta(x^{(i)})\left(1-h_\theta(x^{(i)})\right)x^{(i)}\left(x^{(i)}\right)^T\right]$$

RESULTS

75% of the data was used for training while the remaining was used for testing the predictor.

The convergence appeared to be faster for

Error_Threshold=0.0000001

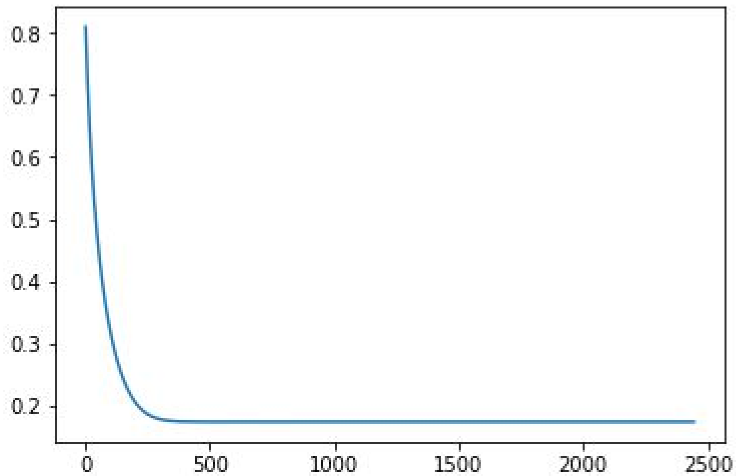
λ =0.03

For the approach of Gradient Descent it took 2445

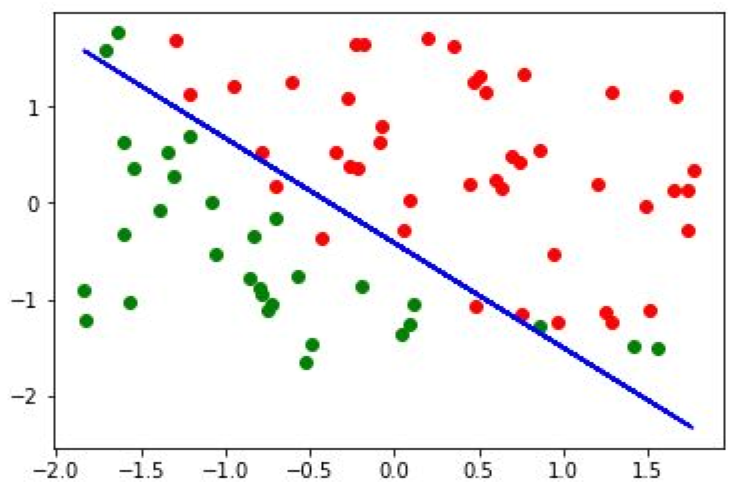
iterations to arrive at a final cost of 0.17496378154397854

The final theta values using Newton's Method obtained were

Final theta values :[1.78870753 3.65845292 3.76341692]



The Data Points was obtained as follows with decision boundary shown in the figure.



%accuracy for train data is 89.33333333333333

% accuracy for test data is 92.0