27 December 2023 19:27

Recap :-
1. Experiement
2. Outcome
3. Sample Space
4. Event
5. Probability Rules

Agenda :- (1) Conditional probability
(2) Bayes theorem.

Event :- Subset of Sample Space

Events

Independent Events

eg:- (1) Will India win the match?
(2) Will I get a head after tossing a coin.
(3) Will Anil pass the exam?
(4) Will I get 6 when I roll a dice

Dependent Events.

eg:- (1) Anil will get placed [if he has practiced all the coding Assignments]
(2) India will win the match [if Kohli scores a century]
(3) Sandeep will get a [promotion given that he has successfully completed all his assignments and responsibilities]

Eg :- We roll 2 dice simultaneously & we want to calculate the probability that the second dice get a value 2 given that the sum of the numbers on both dice should be less than or equal to 5.

$\overset{D1}{(1}, \overset{D2}{2}), (2, 2), (3, 2)$ $\overset{D1}{(4}, \overset{D2}{2}), (5, 2), (6, 2)$

$\underbrace{1+2}$ ≤ 5 $4 \le 5$ $5 \le 5$ $6 \le 5$

$3 \le 5$

In this case, getting 2 on the second dice is an event & getting the sum of both dice less than or equal to 5 is a condition

$$P(D_2 = 2 \mid D_1 + D_2 \le 5)$$

condition (given that)

Break the problem into 2 events.

Event 1 :- Getting the sum of both dice less than or equal to 5.

Event 2 :- Getting 2 on 2nd dice.

Sample Space of Rolling 2 dice at once =

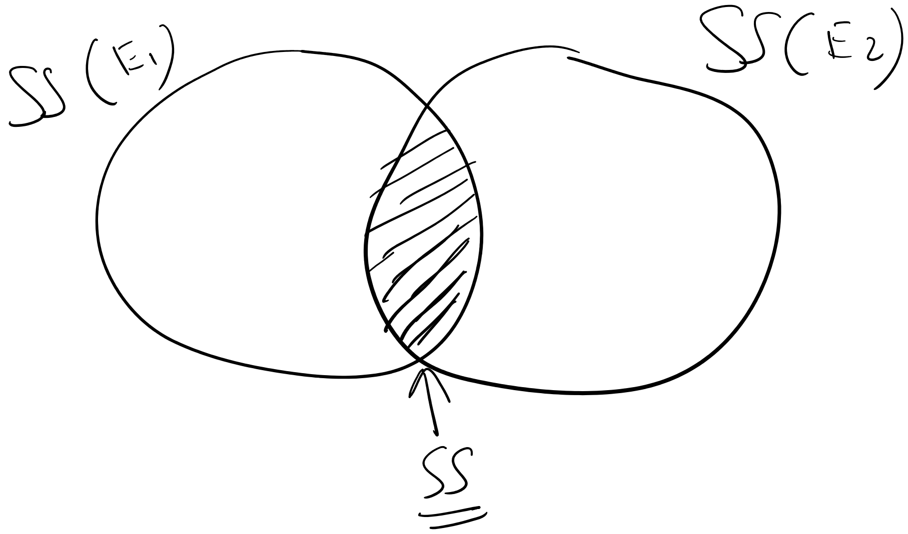
	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

Sum of both dies.

	⚀	⚁	⚂	⚃	⚄	⚅
⚀	2	3	4	5	6	7
⚁	3	4	5	6	7	8
⚂	4	5	6	7	8	9
⚃	5	6	7	8	9	10
⚄	6	7	8	9	10	11
⚅	7	8	9	10	11	12

Event 1 :- $SS = \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (3,1), (3,2), (4,1)\}$

Event 2 :- $SS = \{(1,2), (2,2), (3,2), (4,2), (5,2), (6,2)\}$



Getting a 2 on 2nd dice given that sum of both dice ≤ 5

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$\therefore SS = \{(1,2), (2,2), (3,2)\}$

$SS = \{ (1,2), (2,2), (3,2) \}$

$SS = P(E_1 \cap E_2)$ $\quad P(E_1 \cap E_2) = \frac{3}{36}$

$$\boxed{P(E_2 \mid E_1) = \frac{P(E_1 \cap E_2)}{P(E_1)}}$$

$$P(E_2 \mid E_1) = \frac{3/3\cancel{6}}{10/3\cancel{6}} = \frac{3}{10}$$

$$\boxed{P(E_2 \mid E_1) = \frac{3}{10}}$$

$$P(E_2 | E_1) = \frac{P(E_1 \cap E_2)}{P(E_1)} \Rightarrow \boxed{P(E_1 \cap E_2) = P(E_2 | E_1) * P(E_1)}$$

$$P(E_1 | E_2) = \frac{P(E_2 \cap E_1)}{P(E_2)} \Rightarrow \boxed{P(E_2 \cap E_1) = P(E_1 | E_2) * P(E_2)}$$

$$\Downarrow$$

$$P(E_2 | E_1) * P(E_1) = P(E_1 | E_2) * P(E_2)$$

Bayes Theorem

Bayes Eqn

$$\boxed{P(E_2 | E_1) = \frac{P(E_1 | E_2) * P(E_2)}{P(E_1)}}$$

$P(E_1 | E_2)$ = Likelihood.

$P(E_2)$ = Prior

$P(E_1)$ = Evidence.

Q. You toss 3 coins simultaneously.

a. Prob (HHH)

b. Prob (getting exactly one head)

c. Prob (atleast 2 heads | you have observed atleast one head)

{ HHH, HHT, HTH, HTT, TTH, THT, THH, TTT } $n(S) = 8$

a. $P(HHH) = \frac{\quad}{8}$

b. $S(B) = \{ HTT, TTH, THT \}$

$P(B) = 3/8$.

c. $P(\text{atleast } HH \mid \text{atlest } 1H) = \frac{4/8}{7/8} = 4/7$

1 → $P(\text{atlest } HH) = \{ HHH, HHT, HTH, THH \}$

2 → $P(\text{atleast } H) = \{ HHH, HHT, HTH, HTT, THH, THT, TTH \}$

→ $P(1) = 4/8$) $P(2) = \frac{7}{8}$.