Agenda :- Intro to Conditional Probability

a. Probability

b. Experiment

c. Outcome

d. Sample Space

e. Union, Intersection, Complements.

1. Experiement or Random Experiement :-

→ Are defined as uncertain situations, which can have multiple results

eg:- ① Tossing a coin is an experiement.

② Whether it will rain today ?

Yes or No

③ Roll a dice.

2. <u>Outcome</u> :- The results of the experiement

eg:- ① Toss a coin → experiement

$\underbrace{\{H\} \quad \{T\}}$ → <u>outcomes</u>

② Ind vs Pak (Whether Kohli will score a century?)

<u>Outcomes</u> ← $\boxed{\{Yes\} \quad \{No\}}$

(3) Ind vs Pak → experiment → Outcome

win

$\boxed{\{Ind\} \quad \{Pak\}}$ ← Outcome

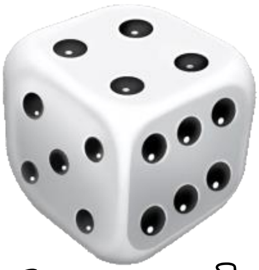
3. Sample Space :- Collection of all outcomes of an experiement

eg:- Toss a coin → experiement

{H} {T} → Outcomes

$S = \{H, T\}$ ← Sample Space

Experiement
↓
Rolling a dice :-



Outcome Sample Space

Outcomes :- {1} {2} {3} {4} {5} {6}

Sample Space :- {1, 2, 3, 4, 5, 6}

Event :- Is defined as a subset of Sample Space.

Roll a dice
Sample Space = $\{1, 2, 3, 4, 5, 6\}$

* To get an outcome which is divisible by 2?

Event A

Outcomes :- $\{2\}$, $\{4\}$, $\{6\}$

$S(A)$:- $\{2, 4, 6\}$ $S(A)$:- Sample Space for event A.

Exp :- Toss a coin

Outcomes :- $\{H\}$ & $\{T\}$

S :- $\{H, T\}$

$\underline{A}$:- Getting a head when a coin is tossed :-

$$\boxed{\underline{S(A)} :- \{H\}}$$

10 April 2023 20:07

Exp :- Tossing a coin & Rolling a dice

Outcomes :- $\{H\}$, $\{T\}$ & $\{1\}$ $\{2\}$ $\{3\}$ $\{4\}$ $\{5\}$ $\{6\}$

$$\text{Sample Space} = \{(H,1), (H,2), (H,3), (H,4), (H,5), (H,6)$$
$$(T,1), (T,2), (T,3), (T,4), (T,5), (T,6)\}$$

Event :- A: Coin turn in to head.

$S(A)$:- $\{(H,1), (H,2), (H,3), (H,4), (H,5), (H,6)\}$

B :- Dice number is 3.

$S(B)$:- $\{(H,3), (T,3)\}$

Experiment :- Two coins are tossed simultaneously

Outcomes :- 1st coin $\{H\}$ $\{T\}$ 2nd coin $\{H\}$ $\{T\}$

S :- $\{\{HH\}, \{HT\}, \{TH\}, \{TT\}\}$

A :- Atleast one head

S(A) :- $\{\{HH\}, \{HT\}, \{TH\}\}$

B :- Both tosses are same

S(B) :- $\{HH\}$ & $\{TT\}$

10 April 2023 20:16

Universal Sample Space

Dice :- S :- $\{1, 2, 3, 4, 5, 6\}$

A :- All odd outcomes

$S(A)$:- $\{1, 3, 5\}$ ← Sample Space of Event

$\boxed{\text{Probability}(A) :- \frac{3}{6}}$, → $\frac{1}{2}$

$$\boxed{P(\text{Event}) = \frac{\text{no. of element in } S \text{ of event}}{\text{total Sample Space}}}$$

1 Toss a coin :- $\underline{\underline{S}}$:- $\{H, T\}$

A :- of Getting only H.

$$S(A) = \underline{\underline{\{H\}}}.$$

$$\boxed{P(A) :- \frac{1}{2}} \leftarrow$$

Dice :- S :- $\{1, 2, 3, 4, 5, 6\}$

$S(A)$:- $\{2, 4, 6\}$ $\boxed{P(A) = 3/6}$

$S(A^c) \rightarrow \{1, 3, 5\}$ $\boxed{P(A^c) = 3/6}$

$$\boxed{\text{Complement of an Event} = \text{Compare } S(\text{Event}) \text{ with Universal Sample Space}}$$

Dice :- S :- $\{1, 2, 3, 4, 5, 6\}$

$S(A)$:- $\{2, 3\}$

$S(A^c)$:- $\{1, 4, 5, 6\}$

$P(A) = 2/6$

$P(A^c) = 4/6 \Leftarrow 1 - \frac{2}{6} \Rightarrow 1 - P(A)$

$$\boxed{P(A^C) = 1 - P(A)}$$

Dice :- S :- $\{1, 2, 3, 4, 5, 6\}$

$S(A) = \{3, 4, 5\}$ $\quad P(A) = 3/6$

$S(B) = \{1, 5, 6\}$ $\quad P(B) = 3/6$

$S(\underline{A \cap B}) = \{5\}$

$$\boxed{P(A \cap B) = 1/6}$$

$S(A \cup B) :- \{1, 3, 4, 5, 6\}$

$\boxed{P(A \cup B) = 5/6}$

Dice :- S :- $\{1, 2, 3, 4, 5, 6\}$

$S(A)$:- $\{\underline{2}, 4, 6\}$ $P(A)$:- $3/6$

$S(B)$:- $\{1, \underline{2}\}$ $P(B)$:- $2/6$.

$S(A \cup B) \Rightarrow \{1, 2, 4, 6\}$

$$\boxed{P(A \cup B) \Rightarrow 4/6}$$

Why can't we say that

$$P(A \cup B) = P(A) + P(B)?$$

$$S(A \cap B) = \{2\} \qquad P(A) = 3/6$$

$$P(A \cap B) = 1/6. \qquad P(B) = 2/6.$$

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

$$P(A \cup B) = 3/6 + 2/6 - 1/6 = 4/6$$

10 April 2023 20:40

Dice :- S :- $\{1, 2, 3, 4, 5, 6\}$

$S(A)$:- $\{2, 4, 6\}$

$S(C)$:- $\{1, 3, 5\}$

$S(A \cap C) = \{\ \}$ ← empty set or null set

$S(A \cap C) = \{\ \}$ ← $\underline{A}$ & $\underline{C}$ are "mutually exclusive" or "disjoint sets"

$\boxed{P(A \cap C) = 0/6}$

Recap :-

1. Sample Space
   "Collection of all outcomes"
2. Event – "A subset of the Sample Space"
3. $$\text{Probability}(\text{Event}) = \frac{n(S(\text{Event}))}{n(S)}$$
4. $$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A') = 1 - P(A)$$

Mutually Exclusive or Disjoint set.

☆ $S(\underline{A} \underline{\cap} \underline{B}) = \underline{\underline{\{\}}}$

☆ $P(A \cap B) = \underline{\underline{0}}$

1. Two events
2. Intersection of two events $= \{\}$
3. P(A Intersection) $= 0$

Tossing a coin :-

$A = \{\ \}$ ← Not mutually exclusive

10 April 2023 20:49