

Agenda :-

Naive Bayes :- Entirely about probability

↳ Intro to Probability

Probabilities :- 1. Experiment or Random Experiment  
2. Outcome of an experiment  
3. Sample Space  
4. Event.  
5. Probability Rules  
(Intersection, Union, Complement)

1. Experiment or Random Experiment:-


Defined as [Uncertain] situations which can have [multiple] results.


eg:- ① Tossing a coin

② Rolling a dice

③ Whether it will rain today?

② Rolling a dice :-

Outcome 4 ← 

Outcome 3 ← 

outcome

3. Sample Space :- Collection of all outcomes of the experiment

① Toss a coin :- H, T  
↓

Sample Space :-  $\{H, T\}$

② Roll a dice :-  $\{1, 2, 3, 4, 5, 6\}$

Write the sample space for :- 2 coins tossed.

$$S = \left\{ \overset{\textcircled{1}}{\downarrow} \overset{\textcircled{2}}{\downarrow} HH, \overset{\textcircled{1}}{\downarrow} \overset{\textcircled{2}}{\downarrow} HT, \overset{\textcircled{1}}{\downarrow} \overset{\textcircled{2}}{\downarrow} TH, \overset{\textcircled{1}}{\downarrow} \overset{\textcircled{2}}{\downarrow} TT \right\}$$

Write the sample space for :- 2 dice rolled together

$$S = \left\{ \overset{D^1}{\downarrow} \overset{D^2}{\downarrow} (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), \right. \\ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), \\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), \\ (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), \\ \left. (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \right\}$$

④ Event :- It is defined as subset of Sample Space

eg:- Toss a coin :- H

Tossing a coin and getting head in the output combined is called as an Event.

eg:- Toss a coin

$$S = \{H, T\}$$

Event Focus is only on tossing a coin & getting head in the output

$$\underline{S(\text{Event})} = \{H\}$$



eg:- 2: Experiment :- Rolling a dice

Outcomes :- 1, 2, 3, 4, 5, 6.

Sample Space :-  $\{1, 2, 3, 4, 5, 6\}$   
(S)

Event :- Roll a dice & get only odd nos.

Outcome(Event) :-  $\{1, 3, 5\}$

$S(\text{Event})$  :-  $\{1, 3, 5\}$

eg:- 3. Experiment :- Roll a dice & Toss a coin simultaneously

Possible outcomes :-  $\{1, 2, 3, 4, 5, 6\}$   $\{H, T\}$

Sample Space :-  $\{(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6),$   
 $(T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)\}$

Event (A) :- Got only Tails :-

$S(A) :- \{(T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)\}$

Event (B) :- Got only dice where divisible by 2.

$S(B) :- \{(H, 2), (H, 4), (H, 6),$   
 $(T, 2), (T, 4), (T, 6)\}$

What is Probability?

Prob =  $\frac{\text{no. of elements in Sample Space of that Event}}{\text{no. of elements in the Sample Space}}$

$\therefore$  Experiment :- E

S :- { ... }

Event :- A

S(A) :- { ... }

$$P(A) = \frac{S(A)}{S}$$

eg:- Toss a coin.

$$S :- \{ \underline{H}, \underline{T} \} = \underline{n(S) = 2}$$

A :- Getting only Head

$$S(A) = \{ \underline{H} \} = \underline{n(S(n)) = 1}$$

$$P(A) :- \frac{1}{2}$$

eg:-2:- Roll a dice

$$S :- \{1, 2, 3, 4, 5, 6\} \Rightarrow n(S) = 6$$

A :- Get all nos. divisible by 3.

$$S(A) = \{3, 6\} \Rightarrow n(S(A)) = 2$$

$$P(A) = \frac{2}{6} = \frac{1}{3}$$

eg:- Roll a dice

$$S := \{1, 2, 3, 4, 5, 6\} \Rightarrow n(S) = 6$$

A:- All odd outcomes.

$$S(A) := \{1, 3, 5\} \Rightarrow n(S(A)) = 3$$

$$\underline{\underline{P(A)}} := \frac{3}{6} = \frac{1}{2}$$

## Probability Rules :-

### 1. Complement :-

eg :- Rolling a dice

$$S :- \{1, 2, 3, 4, 5, 6\} \Rightarrow n(S) = 6$$

A :- Getting all nos. divisible by 3

$$S(A) = \{3, 6\} = n(S(A)) = 2$$

$$P(A) = \frac{2}{6} = \frac{1}{3}$$

$$P(A^c) = 1 - P(A)$$

$$P(A^c) = \left(1 - \frac{2}{6}\right) = \frac{4}{6}$$

B :- Getting all nos. not divisible by 3.

$$S(B) = \{1, 2, 4, 5\}$$

$$P(B) = \frac{4}{6} = \frac{4}{6} \Rightarrow P(A^c)$$

eg:- Roll a dice

$$S \Rightarrow \{1, 2, 3, 4, 5, 6\}$$

A:- Get all nos. div. by 3

$$S(A) = \{3, 6\}$$

$$P(A) = \frac{2}{6} = \frac{1}{3}$$

$$P(A^c) = 1 - P(A)$$

$$= \left(1 - \frac{2}{6}\right) = \frac{4}{6}$$

B:- Get all nos. not  
div. by 3

$$S(B) = \{1, 2, 4, 5\}$$

$$P(B) = \frac{4}{6}$$



Dice :-  $S := \{1, 2, 3, 4, 5, 6\} \Rightarrow n(S) = 6$

$$S(A) = \{3, 4, 5\}$$

$$P(A) = 3/6$$

$$S(B) = \{1, 5, 6\}$$

$$P(B) = 3/6$$

∩  
↓  
intersection

$$S(\underline{A \cap B}) = \{5\}$$

$$n(S(A \cap B))$$

$$= 1$$

Intersection =

Common in both

$$P(A \cap B) = 1/6$$

3. Union :- Not common.

Dice :-  $S :- \{1, 2, 3, 4, 5, 6\} \Rightarrow 6$

$S(A) :- \{2, 4, 6\}$

$S(B) :- \{1, 2\}$

$\{ S(A \cup B) :- \{1, 4, 6\} \Rightarrow 3$

$\boxed{P(A \cup B) :- \underline{\underline{3/6}} = 1/2}$

$\cup \Rightarrow$  or  
 $\cap \Rightarrow$  and

\* Law of probability:-

$$\checkmark P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

eg:- Dice:-  $S:- \{1, 2, 3, 4, 5, 6\}$

$$S(A) = \{2, 4, 6\} \Rightarrow \underline{P(A)} = 3/6$$

$$S(B) = \{1, 2\} \Rightarrow \underline{P(B)} = 2/6$$

$$S(A \cap B) = \{2\} \Rightarrow \underline{P(A \cap B)} = 1/6$$

$$P(A \cup B) = \frac{3}{6} + \frac{2}{6} - \frac{1}{6} = \frac{4}{6}$$

Dice :-  $S := \{1, 2, 3, 4, 5, 6\}$

$S(A) := \{2, 4, 6\}$

$S(B) := \{1, 3, 5\}$

$S(C) := \{\}$  ←

null set or  
empty set.

$$S(A \cap C) = \{\}$$

$$P(A \cap C) = 0$$

For 2 Events, eg :- A & C

$$\boxed{\begin{aligned} S(A \cap C) &= \{\} \\ P(A \cap C) &= 0 \end{aligned}}$$

← Event A & Event C are mutually exclusive events. or disjoint events

eg :- Tossing a coin

A = {} ← Mutually exclusive ? ⇒ No

Recap :- Experiment

Outcome

Sample Space

Event

Prob (Event)

Complement  $\Rightarrow$

$$P(A^c) = 1 - P(A)$$

Intersection

Union  $\Rightarrow$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Mutually Exclusive Event or Disjoint Event