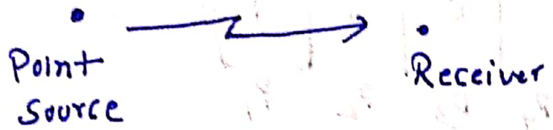
Hamming Code is used for following purpose

ⓐ Error detection

ⓑ Error correction

ⓒ Encoding and decoding



HELLO → HELLO (No Error)

HELLO → BYE (Error)

* In this coding Parity bits are used to detect & Correct the error.

* Parity bits are the extra bit to mix with message bits

* Hamming code is used to detect and correct single bit error

* ~~St~~ Parity bits position is decided by 2^n
where $n = 0, 1, 2, 3$ - - —

* For (7,4) Hamming Code Parity bits positions as follows
$2^0 = 1$, $2^1 = 2$, $2^2 = 4$

7	6	5	4	3	2	1
D_7	D_6	D_5	P_4	D_3	P_2	P_1

* Parity bits ~~are~~ values are decided as

P_1 → check 1 bit and skip 1-bit,
so position (1, 3, 5, 7, 9 - - -)

P_2 → check 2 bit and skip 2-bit
so position (2, 3, 6, 7 - - —)

P_3 → check 4-bit and skip 4 bit
so position (4 5 6 7) (12, 13, 14), (20, 21, 22)

For (7, 4) Hamming code

Lets hamming code (n, K)

$\hookrightarrow$ Total length of message

→ Parity bit $(r) = n - K$

→ Rate $(R) = \dfrac{K}{n}$

→ where $K = 2^r - r - 1$

r represent the number of parity bit $r = (n - K)$

→ Block length $(n) = 2^r - 1$

where $r > 2$

x Hamming distance (d) x

The number of bits in which two codewords vary is called hamming distance

For eg

1	0	1	1	0	0	1
1	1	0	1	1	0	1
no change	change	change	No change	change	No change	No change

Hamming distance $(d) = 3$
(no. of changes in codeword)

Q Let the transmitted message be 1001, using hamming code find out

a) Encode the message and transmit

b) Include error in 6th bit position

c) Correct the error

Sol (a)

7	6	5	4	3	2	1
D_7	D_6	D_5	P_{B4}	D_3	P_2	P_1

or

7	6	5	4	3	2	1
1	0	0	P_{B4}	1	P_2	P_1

① Parity bit P_1 → check 1-bit & skip 1-bit

~~P_1 = check 1-bit~~ (1, 3, 5, 7)

$P_1 = D_3 \oplus D_5 \oplus D_7 = 1 \oplus 0 \oplus 1 = 0$

② Parity-bit P_2 ⇒ check 2-bit, skip 2-bit (2, 3, 6, 7)

$P_2 = D_3 \oplus D_6 \oplus D_7 = 1 \oplus 0 \oplus 1 = 0$

③ Parity-bit P_{B4} → check 4-bit, skip 4-bit (4, 5, 6, 7)

$P_{B4} = \cancel{D_4 \oplus} D_5 \oplus D_6 \oplus D_7 = 0 \oplus 0 \oplus 1 = 1$

Encoded message

7	6	5	4	3	2	1
1	0	0	1	1	0	0

(Three parity bit + four message bit)

(b) Error in 6th bit position

1 0 0 1 1 0 0

↓ error (Invert)

7	6	5	4	3	2	1
1	1	0	1	1	0	0

Received code word

D_7	D_6	D_5	P_4	D_3	P_2	P_1
1	1	0	1	1	0	0
7	6	5	4	3	2	1

(c) 1) $P_1 \rightarrow$ check 1-bit, skip 1-bit (1, 3, 5, 7)

P_1 D_3 D_5 D_7

0 1 0 1 ⇒ even parity means no error

$\boxed{P_1 = 0}$

2) $P_2 \rightarrow$ check 2-bit & skip 2-bit (2, 3, 6, 7)

P_2 D_3 D_6 D_7

0 1 1 1 ⇒ odd parity means $\boxed{P_2 = 1}$

3) ~~P_8~~ $P_4 \rightarrow$ check 4-bit & skip 4-bit (4, 5, 6, 7)

P_4 D_5 D_6 D_7

1 0 1 1 ⇒ odd parity means $\boxed{P_4 = 1}$

P_1, P_2 & P_4 all are not zero means error exist in the received code-word

$$P_4\ P_2\ P_1 = \overset{4}{1}\ \overset{2}{1}\ \overset{1}{0} = (6)_{10}$$

In 6th bit position error, so correct word by simple inverting the 6th bit

1001100 ⇒ Correct code word

⑤ A 7-bit hamming code is received as 1011011. Assume even parity and state wheather the received code is correct or wrong, if wrong locate the bit in error.

Sol Hamming code (7,4)

Hamming code (7,4)	7	6	5	4	3	2	1
Encoded message	1	0	1	1	0	1	1.
	D_7	D_6	D_5	P_4	D_3	P_2	P_1

(Three parity bit + Four message bit)

1) $P_1 \rightarrow$ check 1-bit & skip 1-bit (1,3,5,7)

P_1 D_3 D_5 D_7

1 0 1 1 ⇒ odd parity means $\boxed{P_1 = 1}$

2) $P_2 \rightarrow$ check 2-bit & skip 2-bit ~~4,5,6,~~ (2,3,6,7)

P_2 D_3 D_6 D_7

1 0 0 1 ⇒ even parity means no error

$\boxed{P_2 = 0}$

3) $P_3 \rightarrow$ check 4-bit & skip 4-bit (4,5,6,7)

P_4 D_5 D_6 D_7

1 1 0 1 ⇒ odd parity means $\boxed{P_4 = 1}$

P_1, P_2 & P_4 all are not zero means error

Correcting errors

P_4 P_2 P_1

1 0 1

4 2 1 = (6)

6th bit error so correct by simple inverting the bit