Linear Block Codes:

Transmission Error in digital communication System are caused by the noise in comm^n channel

Gaussian Noise
includes → Thermal noise
→ shot noise

& Impulse Noise:- Characterized by long quiet intervals followed by high amplitude noise burst. (natural & manmade).

In Binary Symmetric channel, the Tx^n errors are due to white Gaussian noise are reffered as random errors.

⇒ When noise burst occurs, it affects more than 1 symbol or bit, it depends on errors in Successive transmitted symbols. Thus errors are reffered as burst.

Types of Codes:
- Block Codes
- Convolutional Codes.

→ In Block codes:

Block of 'k' information bits is followed by a group of r check bits that are derived from block of Information bits. At Rx check bits are used to verify.

→ In Convolutional Codes:

Check bits are continuously interleaved with info^n bits

Matrix description.

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Linear Block Codes:

|← Codeword length = n bits →|

$m_0\, m_1\, m_2 \ldots m_{k-1}$	$c_0\, c_1 \ldots c_{n-k-1}$

|← k msg bits →|← n−k parity bits →|

① → In (n, k) linear block code.

$k \rightarrow$ no of bits identical to the msg sequence which is to be Tx'd

② → Remaining $(n-k)$ number of bits are called generalized parity check bits / parity bits.

③ → These bits are computed from the msg bits.

④ → A code word consists of 'k' messages bits which are denoted by $m_0, m_1, \ldots m_{k-1}$ and $(n-k)$ parity bits denoted by $c_0, c_1 \ldots c_{n-k-1}$.

⑤ → Sequence of msg bits is applied to a linear block encoder to produce an 'n' bit codeword. elements in this code word are $x_0, x_1, \ldots x_{n-1}$.

⑥ → As shown in fig., the first 'k' bits of the code word are identical to the corresponding parity bits $(c_0, c_1 \ldots)$. We can express this mathematically as under

$$x_i = \begin{cases} m_i & \text{for } i = 0, 1 \ldots k-1 \\ c_{i-k} & \text{for } i = k, k+1, \ldots n-1 \end{cases} \quad \text{—①}$$

a(2)

(7) The (n-k) parity bits are linear sums of the k-msg. bits.

(8) The code vector represented by eqn (1) is represented as: $X = [M : C]$; X – code vector.

where: M = k – msg vectors.

C = (n-k) parity vectors.

(9) A block code generator generates the parity bits required to be added to the msg bits to generate the codewords.

$$X = MG$$

where $X \rightarrow$ code vector of $1 \times n$ size.

$M \rightarrow$ msg vector of $1 \times k$ size

$G \rightarrow$ Generator Matrix of $k \times n$ size

$$[X]_{1\times n} = [M]_{1\times k} \times [G]_{k\times n}.$$

(10) Generator Matrix is dependent on the type of linear block code used.

Generator Matrix is $[G] = [I_k | P]$

$I_k = k \times k$ identity Matrix

$P = k \times (n-k)$ Coefficient Matrix.

$$I_k = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & & & \vdots \\ 0 & 0 & \cdots & \cdots & 1 \end{bmatrix}$$

$$P = \begin{bmatrix} P_{11} & P_{12} & P_{13} & \cdots & P_{1q} \\ P_{21} & P_{22} & P_{23} & \cdots & P_{2q} \\ \vdots & & & & \\ P_{k1} & P_{k2} & P_{k3} & \cdots & P_{kq} \end{bmatrix}_{k \times q}$$

$q = n - k$

(ii) parity vector $\Rightarrow C = MP$

$$\begin{bmatrix} C_1 & C_2 & \cdots & C_q \end{bmatrix}_{1 \times q} = \begin{bmatrix} m_1 & m_2 \cdots & m_k \end{bmatrix}_{1 \times k} \begin{bmatrix} P_{11} & P_{12} & P_{13} \cdots & P_{1q} \\ P_{21} & P_{22} & \cdots & P_{2q} \\ \vdots & & & \\ P_{k1} & P_{k2} & \cdots & P_{kq} \end{bmatrix}$$

$C_1 = m_1 P_{11} \oplus m_2 P_{21} \oplus m_3 P_{31} \oplus \cdots + m_k P_{k1}$

$C_2 = m_1 P_{12} \oplus m_2 P_{22} \oplus m_3 P_{32} \oplus \cdots \oplus m_k P_{k2}$

$C_3 = m_1 P_{13} \oplus m_2 P_{23} \oplus m_3 P_{33} \oplus \cdots \oplus m_k P_{k3} \ldots$

* All additions are mod-2 additions.

Example for linear block:

Q The Generator Matrix for a (6,3) block code is given below. find all code vectors of the code.

$$G = \begin{bmatrix} 1 & 0 & 0 & : & 0 & 1 & 1 \\ 0 & 1 & 0 & : & 1 & 0 & 1 \\ 0 & 0 & 1 & : & 1 & 1 & 0 \end{bmatrix}$$

Sol^n Given that $n = 6$, $k = 3$, & $I = I_{3\times3}$

$$P = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

Code vectors can be obtained through following steps:

(1) Determine the P submatrix from generator matrix.

(2) Obtain Equations for check bits using $C = MP$.

(3) Determine check bits for every msg vector

⇒ (1) step 1: To obtain P Sub matrix.

$$G = [I_K : P_{K\times q}]$$

$$I_K = I_{3\times3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P_{K\times q} = P_{3\times3} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

Obtain the eqn for check bits.

$k = 3$, $q = 3$ & $n = 6$.

Block size of msg vector is 3 bits. Hence there will be '8' possible msg vectors.

$m_1 m_2 m_3$

: 000

⋮

: 111

P submatrix is given in example as
$$\begin{matrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{matrix}$$

⇒ For check bit vector, there will be 3 bits.

$$[C_1\ C_2\ C_3]_{1\times 3} = [m_1\ m_2\ m_3]_{1\times 3} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}_{3\times 3}$$

$$\Rightarrow C_1 = 0\times m_1 \oplus 1\times m_2 \oplus 1\times m_3 = m_2 \oplus m_3$$
$$C_2 = 1\times m_1 \oplus 0\times m_2 \oplus 1\times m_3 = m_1 \oplus m_3$$
$$C_3 = 1\times m_1 \oplus 1\times m_2 \oplus 0\times m_3 = m_1 \oplus m_2$$

Step 3

To determine check bits and code vectors for every msg vector:

→ Consider first block msg. $m_1\ m_2\ m_3 = 000$

$$C_1 = 0 \oplus 0 = 0$$
$$C_2 = 0 \oplus 0 = 0$$
$$C_3 = 0 \oplus 0 = 0$$

i.e. $C_1\ C_2\ C_3 = 0\ 0\ 0$.

⇒ Consider 2nd block msg $m_1 m_2 m_3 = 001$.

$$C_1 = 0 \oplus 1 = 1$$
$$C_2 = 0 \oplus 1 = 1$$
$$C_3 = 0 \oplus 0 = 0$$

i.e. $C_1\ C_2\ C_3 = 110$

$[(d_{min}-1)/2]$.

Hamming Codes:

Hamming codes are (n, k) linear block codes, those which satisfies the conditions

1. No of check bits $(q) \geq 3$.
2. Block length $n = 2^q - 1$
3. Number of msg bits $(k) = n - q$
4. Min^m distance $d_{min} = 3$.

ie, Code rate $(r) = k/n$ $\Rightarrow r = \frac{n-q}{n}$ for H.C $(k) = n-q$

$r = 1 - q/n$.

By substituting $n = 2^q - 1$ $\Rightarrow r = 1 - \frac{q}{2^q - 1}$

It is observed that $r \cong 1$ if $q \gg 1$.

Error Detection & Error Correction Capabilities of Hamming Codes:

→ Min^m distance (d_{min}) of Hamming code is 3.

→ It can be used to detect double errors (or) to correct single error.

$d_{min} \geq s+1 \quad s=2$

For detecting double (2) errors $\Rightarrow d_{min} \geq 2+1$ i.e. $d_{min} \geq 3$

For correcting upto one (1) errors $\Rightarrow d_{min} \geq 2t+1$ i.e. $d_{min} \geq 3$.

— x —

$d_{min} \geq 2t+1$

Ex:

Q Parity check matrix of a particular (7, 4) linear block code is given by $[H] = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$

(i) Find Generator Matrix (G)

(ii) List all the code vectors.

(iii) What is the minm distance b/n code vectors?

(iv) How many errors can be detected & corrected?

Sol^n: $(7, 4) = (n, k)$

$\Rightarrow n = 7, \quad k = 4.$

q = No of check bits $(n-k) = 3$. ; $\boxed{q=3}$

$n = 2^q - 1 = 2^3 - 1 = 8-1 = 7$

$\boxed{n=7}$

Determine 'P' submatrix:—

→ The parity check matrix is of $q \times n$ size & is given by its written as.

$q = 3,\ n = 7\ \&\ k = 4.$

Parity Check Matrix. $[H]_{3\times7} = \begin{bmatrix} P_{11} & P_{21} & P_{31} & P_{41} & \dots & 100 \\ P_{12} & P_{22} & P_{32} & P_{42} & \dots & 010 \\ P_{13} & P_{23} & P_{33} & P_{43} & \dots & 001 \end{bmatrix}$

$$[H] = [P^T : I_3]$$

Parity check matrix $P^T = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}_{3\times4}$ → Given.

$$P\text{- Submatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}_{4\times3}$$

(1) Determine Generator Matrix (G):

$$G = [I_k : P_{k\times q}]_{k\times n}$$

$k = 4,\ q = 3\ \&\ n = 7.$

$$G = [I_4 : P_{4\times3}]_{4\times7}.$$

$$G = \left[\underbrace{\begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}}_{4\times4} \;\vdots\; \underbrace{\begin{matrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{matrix}}_{4\times3}\right]_{4\times7}.$$

(2). To find all the Codewords:

To obtain eqns for check bits.

$$C = MP$$

$$[c_1 \ c_2 \ c_3]_{1\times 3} = [m_1 \ m_2 \ m_3 \ m_4]_{1\times 4} [P]_{4\times 3}$$

$$[c_1 \ c_2 \ c_3] = [m_1 \ m_2 \ m_3 \ m_4] \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$c_1 = (1\times m_1) \oplus (1\times m_2) \oplus (1\times m_3) \oplus (0\times m_4)$$

$$c_2 = (1\times m_1) \oplus (1\times m_2) \oplus (0\times m_3) \oplus (1\times m_4)$$

$$c_3 = (1\times m_1) \oplus (0\times m_2) \oplus (1\times m_3) \oplus (1\times m_4)$$

$$c_1 = m_1 \oplus m_2 \oplus m_3$$

$$c_2 = m_1 \oplus m_2 \oplus m_4$$

$$c_3 = m_1 \oplus m_3 \oplus m_4$$

Consider a msg ~~vecm~~ vector of $(\underset{m_1}{1} \ \underset{m_2}{0} \ \underset{m_3}{1} \ \underset{m_4}{1})$

$$c_1 = 1 \oplus 0 \oplus 1 = 0$$

$$c_2 = 1 \oplus 0 \oplus 1 = 0$$

$$c_3 = 1 \oplus 1 \oplus 1 = 1$$

the check bits are $(c_1 c_2 c_3) = (0 0 1)$.

Codeword is $mc = 1011 : 001$

$m_1\, m_2\, m_3\, m_4 \Rightarrow 0000$ to 1111 (16) for $C_1\, C_2\, C_3$ (5)

Sr. No	Msg vector $m_1\, m_2\, m_3\, m_4$	Check bits $C_1\, C_2\, C_3$	Code word (or) Code vector	Weight of code vector $w(x)$
1	0 0 0 0	0 0 0	$m_1 m_2 m_3 m_4 c_1 c_2 c_3$ 0 0 0 0 0 0 0	0
2	0 0 0 1	0 1 1	0 0 0 1 0 1 1	3
.				
16	1 1 1 1	1 1 1	1 1 1 1 1 1 1	7

(3) Minimum distance b/n Code Vectors:

In table $2^k = 2^4 = 16$ Code vectors along with their weights.

→ The smallest weight of any non-zero code vector is 3. $d_{min} = [w(x)]_{min}$; $x \neq (00\ldots0)$

(4) Error detection & Correction Capabilities:

Since $d_{min} = 3$

$$d_{min} \geq s + 1$$

$$3 \geq s + 1$$

$$\boxed{s \leq 2}$$

Thus two errors will be detected

$$d_{min} \geq 2t + 1$$

$$3 \geq 2t + 1$$

$$\boxed{t \leq 1}$$

Thus one error will be corrected.

48, 1, 83
27

The hamming Code. (dmin = 3) always two error can be detected and single error can be corrected by its property.

Hamming EnCoder:

→ Encoder is implemented for generating matrix, of ,

→ lower register contains check bits. C_1 C_2 & C_3.

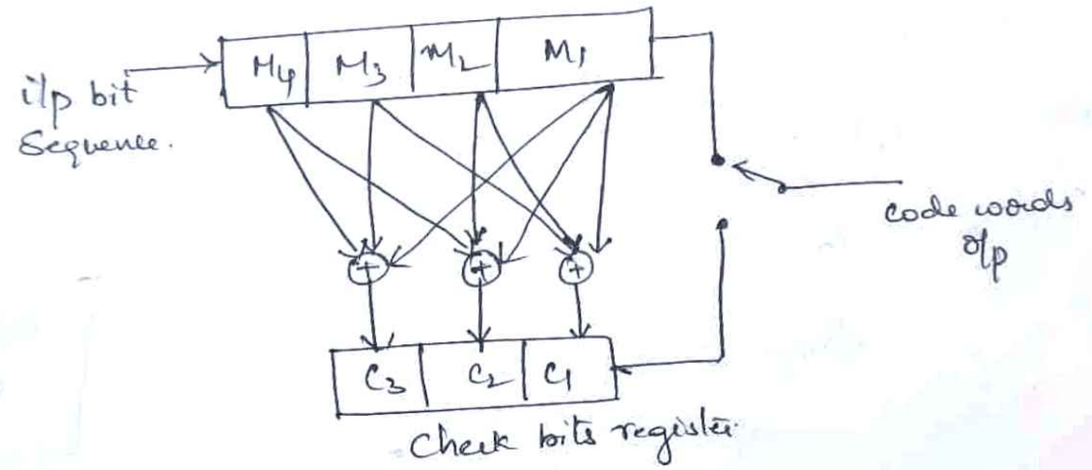

Fig: Hamming Encoder

→ These bits are obtained from the msg bit by mod-2 additions. These mod-2 addition operation is nothing but exclusive OR operation.

Syndrome Technique:

Syndrome Decoding:

The method to correct errors in linear block coding.

→ Let Tx'd code vector be 'X' and corresponding received code vector be represented by 'Y'.

$X = Y \Rightarrow$ no tx'n error.

$X \neq Y \Rightarrow$ errors are created during Tx'n.

→ Decoder detects (or) corrects those errors in Y, using the stored bit pattern in the decoder about the code.

→ For larger block length's more & more bits are required to be stored in the decoder.

→ This ↑es the m/r requirements and adds to the complexity and cost of the system. To avoid these problems, Syndrome decoding is used in linear block codes.

→ The parity check matrix (H) is $H = [P^T / I]_{q \times n}$.

→ The transpose of H is given as:—

$$H^T = \begin{bmatrix} P \\ \hline I_q \end{bmatrix}_{n \times q}.$$ P → Submatrix of size $k \times q$

I_q → identity matrix of $q \times q$

→ The transpose of parity check matrix (H^T). exhibits an important property.

$$X H^T = (0, 0, 0 \cdots)$$

i.e, the product of any code vector X and the transpose of the parity check matrix will be always '0'.

⇒ ~~the~~ At receiver, the property for the detection of errors in the received code as

$Y H^T = (0, 0, 0 \cdots 0)$ if no error.

$Y H^T \neq (0, 0, \cdots 0)$ if error exists in Rx'd codeword.

Syndrome And its Use for Error detection:

The syndrome is defined as the non zero o/p of the product YH^T. Thus, the non-zero syndrome represents the presence of errors in the received Codeword.

a (7)

The syndrome is represented by 's' and is expressed as $S = YH^T$

$$[S]_{1\times q} = [Y]_{1\times n}[H^T]_{n\times q}.$$

When $S = 0$ —
- $X = Y$ no error.
- $X \neq Y$ Y is some other valid codeword (other than x).

→ <u>Error Vector</u>:

The non zero elements of 's' represent error in o/p.

Consider ~~a~~ n-bit error vector E. Let this vector be position of tx'n errors in Y.

→ $x = (1\ 0\ 1\ 1\ 0)$ → Tx'd vector

$y = (1\ 0\ 0\ 1\ 1)$ → Rx'd vector.

$E = (0\ 0\ 1\ 0\ 1)$ → Error vector.

Mod-2 addition X-OR addition.

~~eg~~. $Y = X \oplus E$

$= (10\ 110) \oplus (00\ 101)$

$= (1\oplus0\ \ 0\oplus0\ \ 1\oplus1\ \ 1\oplus0\ \ 0\oplus1)$

$Y = 1\ \ 0\ \ 0\ \ 1\ \ 1$

lly $x = Y \oplus E \Rightarrow (10011) \oplus (00101)$
$\Rightarrow (10110)$.

Relation b/n syndrome vector s & error vector E.

~~From~~ In Syndrome eqn, $S = YH^T$

If we substitute $Y = X \oplus E$, then we get

$$S = (X \oplus E)H^T$$

$$= XH^T \oplus EH^T$$

From the property we stated earlier $XH^T = 0$

$$\boxed{S = EH^T}$$

Syndrome 's' depends on error pattern.

Example problem:

Q The parity check matrix of a (7,4).

Hamming code is given as.

$$H = \begin{bmatrix} 1 & 1 & 1 & 0 & : & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & : & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & : & 0 & 0 & 1 \end{bmatrix}_{3\times 7}.$$

Calculate syndrome vector for single bit error

Sol^n 7,4 linear block code.

$n = 7, \quad k = 4, \quad q = 3.$

(1): Determine error pattern for single bit errors.

(2) Calculation of Syndromes.

① We know that syndrome vector is given by a⑧

$$S = EH^T$$

$$S_{1\times 3} = E_{1\times 7}\left[H^T\right]_{7\times 3}.$$

Various errors can be taken as $q = 3$.

$n = 2^q - 1 \Rightarrow 2^3 - 1 = 7.$; $\boxed{n = 7}$

Error Vector 'E' is a n bit vector representing error pattern.

S.No	Error Vectors	Bit in error.
1.	$\underline{1}$ 0 0 0 0 0 0	first
2.	0 $\underline{1}$ 0 0 0 0 0	Second
3.	0 0 $\underline{1}$ 0 0 0 0	Third
4	0 0 0 $\underline{1}$ 0 0 0	Fourth
5.	0 0 0 0 $\underline{1}$ 0 0	fifth
6.	0 0 0 0 0 $\underline{1}$ 0	Sixth.
7.	0 0 0 0 0 0 $\underline{1}$	Seventh.

② Calculation of syndromes:

$$H^T = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Syndrome for first bit error $\Rightarrow S = EH^T = [1\,0\,0\,0\,0\,0\,0]\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$S = [(1.1 \oplus 0.1 \oplus 0.1 \oplus 0.0 \oplus 0.1 \oplus 0.0 \oplus 0.0), (0.1 \oplus 0.1 \oplus 0.1 \oplus 0.1 \oplus 0.0 \oplus 0.1 \oplus 0.0), (1.1 \oplus 0.1 \oplus 0.0 \oplus 0.1 \oplus 0.0 \oplus 0.0 \oplus 0.1)]$$

$$= [1 \oplus 0 \oplus 0 \oplus 0 \oplus 0 \oplus 0 \oplus 0,\ 0 \oplus 0 \oplus 0 \oplus 0 \oplus 0 \oplus 0 \oplus 0,\ 1 \oplus 0 \oplus 0 \oplus 0 \oplus 0 \oplus 0 \oplus 0).$$

$$S_1 = [1 \quad 0 \quad 1]$$

Syndrome for Second bit in error.

$$S = EH^T = [0100000]\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= [0 \oplus 1 \oplus 0 \oplus 0 \oplus 0 \oplus 0 \oplus 0,\ 0 \oplus 1 \oplus 0 \oplus 0 \oplus 0 \oplus 0 \oplus 0,\ 0 \oplus 1 \oplus 0 \oplus 0 \oplus 0 \oplus 0 \oplus 0]$$

$$= [1 \quad 1 \quad 1]$$

Ily After Calculating other bit errors.

SNo	Error Vector 'E' showing Single bit error.	Syndrome 'S'	
1.	0 0 0 0 0 0 0.	0 0 0.	
2.	1 0 0 0 0 0 0	1 0 1	1st row(H^T)
3.	0 1 0 0 0 0 0	1 1 1	2nd row (H^T)
4.	0 0 1 0 0 0 0	1 1 0	3rd row of H^T
5.	0 0 0 1 0 0 0	0 1 1	4th row of H^T
6	0 0 0 0 1 0 0	1 0 0	5th row of H^T
7.	0 0 0 0 0 1 0	0 1 0	6th row of H^T
8	0 0 0 0 0 0 1	0 0 1	7th row of H^T.

Q. Parity check matrix of a (7,4) Hamming code is

$$H = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}_{3\times 7}$$

— x —

Error Correction Using Syndrome Vector.

Procedure to use Syndrome Vector for the error correction:

For transmitted code vector $X = (0,1,0\,0\,1\,1\,0)$

the received code vector $Y = (0\,1\,1\,0\,1\,1\,0)$.

Assume error to be in 3rd position.

(1) Calculate the corresponding syndrome vector.

$$S = YH^T$$

(2). ~~The error vectors are $S = EH^T$ i.e. $S = E$.~~

(2) Check row of H^T which is same as 'S'.

(3) From Syndrome vector, we obtain error vector.

(4) From error vector, ~~the~~ $X = Y \oplus E$.

Example

$Y = [1\,0\,1\,1\,1\,1\,0\,0]$ $H = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$

(1) obtain Syndrome vector

(2) Determine row of H^T is same as 'S'. (3) To determine 'E'. (4) Obtain correct vector.

(a) Syndrome Vector 's'.

$$S = YH^T = [1\,0\,1\,1\,1\,1\,0] \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow [1 \oplus 0 \oplus 1 \oplus 0 \oplus 1 \oplus 0 \oplus 0, \quad 0 \oplus 0 \oplus 1 \oplus 1 \oplus 0 \oplus 1 \oplus 0,$$
$$1 \oplus 0 \oplus 0 \oplus 1 \oplus 0 \oplus 0 \oplus 0]$$

$$= [1\ 1\ 0]$$

$$S = YH^T = EH^T \Rightarrow 110.$$

(b) row of H^T. In table. if we observe,

$S = 110$ is the 3rd row of H^T

(c) The error pattern corresponding to this syndrome. $E = (0\,0\,1\,0\,0\,0\,0)$

This shows that there is an error in the third bit of 'Y'.

(4) To obtain Correct vector.

from eqn: $X = Y \oplus E$

$$= [1\,0\,1\,1\,1\,1\,0] \oplus [0\,0\,1\,0\,0\,0\,0]$$

$$X = [1\,0\,0\,1\,1\,1\,0]$$

Which is same as trd code vector

Thus a single bit errors can be corrected using syndrome decoding