Vote Prediction Models for Signed Social Networks

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Overview

Voting and Signed Networks

Voting in Communities

- Communities need to take collective action
- Voting is a popular method
- Members of the community vote on the agenda
- E.g.
  - Politicians voting for bills in the parliament
  - Wikipedia users voting for promoting administrators
- Understanding voting behaviour is beneficial
- Can propose agendas items which will be be successful
- Identify ideological fault lines amongst members

Votes as Signed Graphs

- Votes are usually for or against an agenda
- Intuitively maps to positive and negative edges in signed graphs
- More tools to analyse voting patterns, e.g.,
  - Correlation clustering [Brito et al., 2020, Arinik et al., 2017]
  - Balance and Status [Levorato and Frota, 2016, Derr and Tang, 2018]
- Two main prediction tasks exist with regard to voting
  1. Predicting the Result
  2. Predicting an individual vote
- We focus on predicting votes

Predicting Votes

Predicting a vote can be split into two phases

1. **Who** will vote next
   - Same as link prediction task
   - Is trivial when voting order is known, e.g., parliament roll calls
   - Combinatorial if no known underlying process
2. **How** they will vote
   - Same as sign prediction task
   - Triad features encode balance and status theory
   - Train a supervised ML model using network and triad features [Leskovec et al., 2010a, Leskovec et al., 2010b]

We propose an *unsupervised iterative model* to predict the sign of a vote using balance and status theory.

Local Signed Network

Terminology

- Current voting session is a signed graph $S = (V_S, E_S, w_S)$
- It contains current voter v, candidate c and prior set of voters U
- We also have a signed *Relationship Graph*, $R = (V_R, E_R, w_R)$
- It is created from the history of voting sessions H
- The *Local Signed Network* is the intersection of these two graphs $LSN = S \cap R$
- Essentially the subgraph of the voter's neighbours in R who have already voted in S
- Can use balance and status theory in the LSN to predict votes

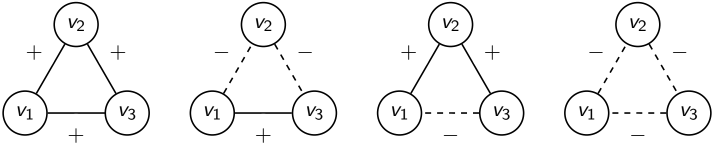
Wikipedia Elections

- RfAs are an election-esque process to gain admin rights
- Anyone can nominate a **candidate** or self-nominate
- 7 day long period of voting and discussion
- Any registered editor can comment and vote :
  - Support
  - Oppose
  - Neutral
- Highly intense process with Q&A with the candidate about Wiki policy etc
- Final decision is made by a bureaucrat
- Decision is based on if a **consensus** has been reached
- Successful RfAs usually have 75% support

Balance Theory

Balance Theory

- Balance theory works for undirected signed graphs
- A cycle is balanced if it has even number of negative links
- A graph is balanced if every cycle is balanced
- Triads B_1, B_2 are balanced
- Traids B_3 and B_4 are unbalanced



B_1 B_2 B_3 B_4

Agreement Graph

- Define a symmetric relationship based on **agreement**
- Subtract 0.5 from agreement ratio to get a signed measure
- Create an *Agreement Graph*, $A = (V_A, E_A, w_A)$

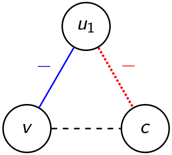
Signed Symmetric Measure

$$w_A((u, v)) = \frac{\text{Number of times } u \text{ and } v \text{ have voted similarly}}{\text{Number of common RfAs for } u \text{ and } v} - 0.5$$

How to measure balance?

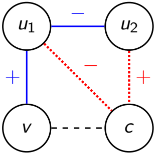
- Triads can be capture local balance of a network
- Need capture information from longer cycles [Chiang et al., 2011]
- Smallest eigenvalue λ_1 of the signed Laplacian $\overline{L}$ is a measure of the imbalance of the network [Hou, 2005]
- We state that a voter chooses to maintain the balance of the LSN
- Therefore, can use λ_1 as basis for predictions in model

Iterative Balance Model



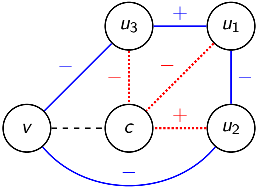
$\lambda_1^+ = 0, \lambda_1^- = 1$

(a) $i = 1$



$\lambda_1^+ = 0.58, \lambda_1^- = 0$

(b) $i = 2$



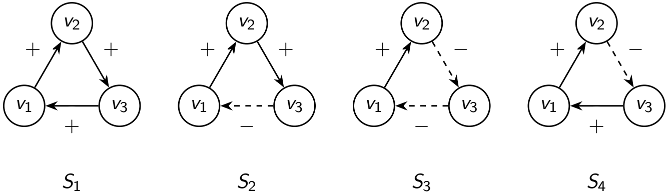
$\lambda_1^+ = 0.55, \lambda_1^- = 0.55$

(c) $i = 3$

Status Theory

Status Theory

- Status theory is based on relative merit of nodes
- Edge $u \xrightarrow{+} v$: v has higher status than u
- Edge $u \xrightarrow{-} v$: v has lower status than u
- Status and Balance theories can contradict each other when used to predict edges
- Triads S_2, S_3 and S_4 adhere to status theory
- Only triads S_1 and S_3 are balanced



Follows Graph

- Define a asymmetric relationship based on **following**
- Create an *Follow Graph*, $F = (V_F, E_F, w_F)$
- An edge $u \rightarrow v$ indicates u votes after v
- Weight is 0.5 subtracted from the *followe ratio*

Signed Asymmetric Measure

$$w_F((u, v)) = \frac{\text{Number of times } u \text{ voted after and agreed with } v}{\text{Number of times } u \text{ voted after } v} - 0.5$$

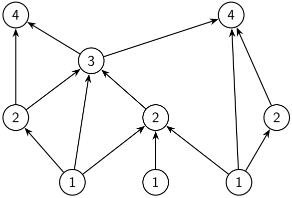
How to measure status?

- Directed triads and other graph motifs can be a proxy for measuring status [Liu et al., 2019]
- Heuristic for status like $\sigma(v) = d^{+}_{\text{in}}(v) + d^{-}_{\text{out}}(v) - d^{+}_{\text{out}}(v) - d^{-}_{\text{in}}(v)$ [Leskovec et al., 2010a]
- These still only capture local effect of status
- Cannot measure violations of status in larger cycles
- The concept of a nodes possessing an implicit status is very similar to a hierarchy

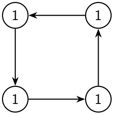
Agony

- Assume a ranking for nodes, $r : V \to \mathbb{N}$
- An edge $u \to v$ causes **agony** if $r(u) < r(v)$
- Quantified as $\max(r(u) - r(v) + 1, 0)$ for an edge
- Agony of a graph G wrt r is
  $A(G, r) = \sum_{(u,v) \in E} \max(r(u) - r(v) + 1, 0)$
- Agony of a graph is lowest possible agony over all rankings
  $A(G) = \min_{r \in \text{Rankings}} A(G, r)$
- Lower agony means better hierarchy $h(G) = 1 - \frac{1}{|E|} A(G)$

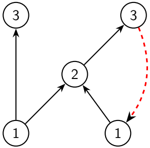
Examples



(a) DAG has perfect hierarchy, $h(G) = 1$ and agony of each edge is 0



(b) A cycle has no hierarchy, $h(G) = 0$ and each edge has agony of 1

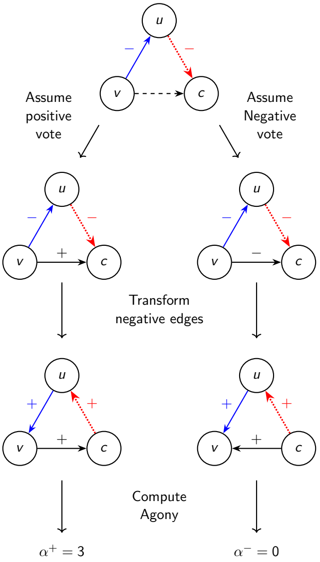


(c) Graph with some hierarchy $h(G) = 2/5$. Red dashed edge has agony of 3 and solid black edges have 0 agony.

Agony and Status

- [Gupte et al., 2011] and [Tatti, 2017] provide algorithms to compute agony of directed graphs
- Consider rank function r as status function σ
- For edg $u \rightarrow v$, $\sigma(u) < \sigma(v)$ is a status violation
- Therefore, agony is a measure of status violation of an edge
- The agony of a network is the **overall status compliance** of G
- What to do with signed edges?
- Flip edges $u \xrightarrow{-} v$ to $u \xleftarrow{+} v$
- Now, we say a voter chooses to reduce the agony of the LSN

Iterative Status Model



Iterative Prediction

Iterative Predictions and its Advantages

- Predictions with balance and status predictions are unsupervised
- Can be bootstrapped by starting with an empty graph R
- After predicting votes in a session, it updates R
- Iteratively gathers more information in R increasing performance
- Therefore, model can be trained on the entire dataset

Algorithm

Input: Candidate c, Relationship graph $R = (V_R, E_R, w_R)$, Order of voters in current session O and true votes w^*

Result: Predictions for current session

$k \leftarrow |O|$

$u \leftarrow O[1]$ // First voter

$V_S \leftarrow \{c, u\}$ // candidate and first voter

$E_S \leftarrow \{(u, c)\}$ // first vote

$w_S((u, c)) \leftarrow w^*((u, c))$ // Assign true vote

Initialize session graph $S = \{V_S, E_S, w_S\}$

$predictions \leftarrow \emptyset$

for $i \leftarrow 2$ **to** k **do**

  $v \leftarrow O[i]$

  $V_S \leftarrow V_S \cup \{v\}$

  $LSN \leftarrow S \cap R$

  $p \leftarrow Predict(v, c, LSN)$

  $predictions \leftarrow predictions \cup p$

  $E_S \leftarrow E_S \cup \{(v, c)\}$

  $w_S((v, c)) \leftarrow w^*((v, c))$ // Assign true vote

end

$Update(R, S)$ // Update Relationship graph

return $predictions$

Results

Tabular Results

Table: Information of relationship graphs of iterative models using entire WIKI-RFA dataset

Relationship Graph	$\|V\|$	$\|E\|$	density	largest component size
Agreement Graph	11924	2451028	0.0345	11908
Follow Graph	11924	3136303	0.0220	11563

Table: Results of iterative models on the complete WIKI-RFA dataset

Model	AUC-ROC	AUC-PR$_{pos}$	AUC-PR$_{neg}$
Baseline	0.5	0.784	0.216
Iterative Balance	0.835	0.935	0.635
Iterative Status	0.784	0.917	0.502

Plots

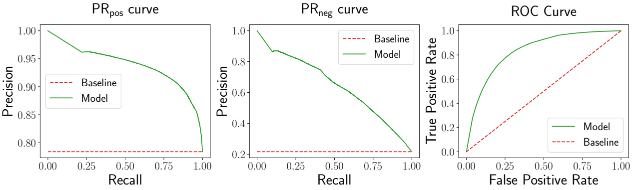


Figure: Plots for the Iterative Balance Model on the complete WIKI-RFA dataset

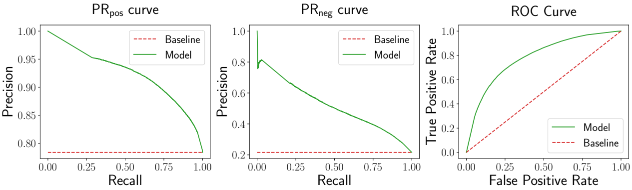


Figure: Plots for the Iterative Status Model on the complete WIKI-RFA dataset

Questions or Comments

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