Aalto University
School of Science
Master's Programme in Computer, Communication and Information Sciences

Ananth Mahadevan

Inferring Voting Networks in Online Elections

Master's Thesis
Espoo, March 14, 2020

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Supervisor: Professor Aristides Gionis
Advisor: Blank M.Sc. (Tech.)

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Chapter 1

Introduction

In recent years, researchers have become increasingly interested in understanding the behaviour of voters in social networks. Knowledge of the factors that motivate voters is of great importance in selecting successful policies or candidates. This is a classic problem and has been studied in the fields of game theory and political science [15, 21, 25]. More recently, there is a focus on using information from the network of voters to model their behaviour. This provides an insight into the interactions and effect of influence on voters in a community. For example, voting for bills in the United States Congress [14] or electing administrators in Wikipedia [2, 13, 17].

Votes can be represented as a *signed* network with positive or negative links. Finding groups using clustering techniques [1, 4, 20] and predicting signed links [5, 18, 19] in these networks is well researched. These approaches provide an ability to understand the group dynamics at play and predict votes and in such a network. However, they do not consider the iterative and chronological nature of the voting that takes place in these networks. In cases where research does focus on voter models, they rely on external features to build machine learning models that are task-specific and static [13, 14].

In this thesis, we propose a model that creates a local signed network consisting of the neighbours of the current voter and the preceding votes. It will then predict the vote, which when added to the network will comply the most with concepts of balance and status in signed networks. After all the votes are cast in a session, the model can be easily updated to improve quality and is, therefore, iterative and dynamic. The model is also flexible and can incorporate external features to build the local signed network of a voter. The results for Wikipedia administrator elections shows that our model outperforms machine learning based models and traditional signed link prediction solutions.

1.1 Thesis Outline

The rest of the thesis is organized in the following manner. We discuss the background relating to signed graphs, hierarchy in directed networks in Section 2. In Section 3 we describe the vote prediction problem and approaches to solving it. Section 4 provides a comprehensive view of Wikipedia and the election process for administrators. In Section 5 we explain the datasets used, construction of the model and evaluation criteria. We report our findings in Section 6 and discuss their implications. Finally, we conclude the thesis and present future work in Section 7.

Chapter 2

Graph Theory

In this chapter, we will provide the fundamentals of the graph theory concepts required to understand the rest of the thesis. In Section 2.1 we cover the basic definitions, terminologies used to describe different types of graphs. Then we define a signed graph and discuss its unique properties in Section 2.2. We outline the theory of balance in signed networks and methods to measure it in Section 2.2.1. Next, we discuss the theory of status and illustrate the differences to balance in Section 2.2.2. Lastly, we explain techniques of finding hierarchies in directed networks and the concept of agony in Section 2.3.

2.1 Preliminaries

In this section, we define the various types of graphs and their basic properties. The notation and terminologies used closely follow those used in Diestel [6]. Graphs are structures that describe relationships between entities. These entities are called *vertices* and entities related to one another are joined by edges. The terms graph, vertices and edges can be used interchangeably with *network*, *nodes* and *links* respectively.

Graphs can be classified broadly into two types based on whether the edges posses a direction or not. We now go on to define them in detail.

2.1.1 Undirected Graphs

An undirected graph is pair $G = (V, E)$, where V is the set of vertices and E is the set $E \subseteq \{(u, v) \mid u, v \in V\}$ of unordered pairs of vertices called edges. In this thesis we will deal only with *simple graphs*, i.e. no self loops, $(u, v) \in V \times V,\ u \neq v$ and there is at most one edge between vertices u and v.

The number of the vertices in a graph is called the *order* of the graph and is denoted by $n = |G|$. The *size* of a graph is the number of edges denoted by $m = \|G\|$ or $m = |E|$. A vertex u is *adjacent* to v if they are the end points of an edge, $(u, v) \in E$. All the vertices adjacent to a vertex v is called the *neighbourhood* of v and is denoted by $N(v)$. The *degree* of a vertex v is the number of nodes adjacent to that vertex and is denoted by $d(v) = |N(v)|$.

The edges of an undirected graph can also have an associated value. This value can indicate the distance or similarity between a pair of vertices. These values are called *weights* and the corresponding graph is called a *weighted undirected graph*. Therefore, a weighted graph is defined as a triple $G = (V, E, w)$, where $w : E \to \mathbb{R}^+$ is a function that maps an edge e to a positive real weight $w(e)$. Now an *unweighted graph* is simply a weighted graph where the function w is defined as: if $e \in E$ then $w(e) = 1$ else $w(e) = 0$. The degree of a a vertex v in a weighted graph is the sum of the weights to all the neighbours of v and is defined as $d(v) = \sum_{u \in N(v)} w((u, v))$ An example of a weighted undirected graph is shown in Figure 2.1.

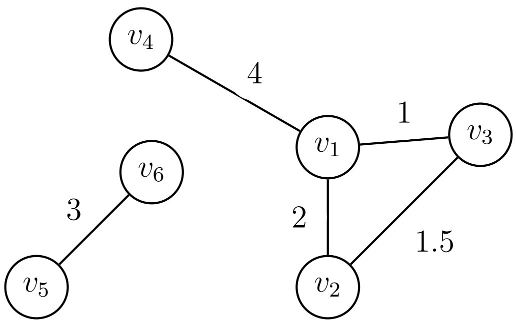


Figure 2.1: An example of a weighted undirected graph

2.1.2 Directed Graphs

The main distinction regarding a *directed graph* (or *digraph*) is that the edges are ordered pairs, i.e.$(u, v) \neq (v, u)$. Therefore, a directed graph has a similar definition: a pair $G = (V, E)$, where V is the set of vertices and E is the set of *ordered* pairs of vertices. Now given an edge $e = (u, v)$ we can define a source function $\mathrm{src} : E \to V$ such that $\mathrm{src}(e) = u$ and a destination function $d : E \to V$ where $\mathrm{dest}(e) = v$. These functions classify the vertices in an edge e as either the source or the destination. In this thesis, we deal only with *simple directed graphs*, i.e. no self-loops and there can be at most one edge from u to v.

As the edges now have an inherent direction we can define the *successors* and *predecessors* of a node v. A vertex u is called the *successor* of a node v is there exists a directed edge from v to u, therefore the set of successors for a vertex v can be defined as $S(v) = \{u \mid (v, u) \in E\}$. A *predecessor* of a node v is a vertex u such that there exists a directed edge from u to v, the set of predecessors for a vertex v can de defined as $P(v) = \{u \mid (u, v) \in E\}$. Now, a vertex u that is either a successor or a predecessor of a vertex v can be called a neighbour of the vertex v. Therefore, we define the *neighbourhood* of a vertex v as the set of vertices in the union of successors and predecessor, i.e. $N(v) = S(u) \cup P(v)$. This definition is also compatible with undirected graphs because if $(u, v) \in E$ then $(v, u) \in E$.

Directed graphs can also have values associated with each directed edge called a *weight*. A *weighted directed graph* can be defined as a triple $G = (V, E, w)$, where the weight function $w : E \to \mathbb{R}^+$ that maps each edge e to a weight $w(e)$. The indegree of a vertex v is defined as the sum of the edge weights from the predecessors of v and is denoted as $\mathrm{d_{in}}(v) = \sum_{u \in P(v)} w((u, v))$. Similarly, the outdegree of a vertex v is defined as the sum of the edge weights to the successors of v and is denoted by $\mathrm{d_{out}}(v) = \sum_{u \in S(v)} w((v, u))$. Figure 2.2 shows an example of a weighted directed graph.

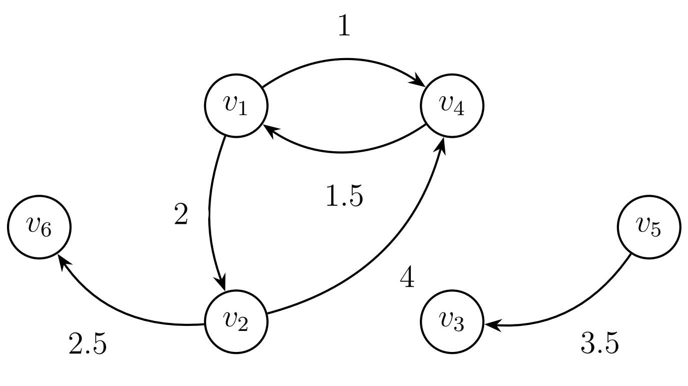
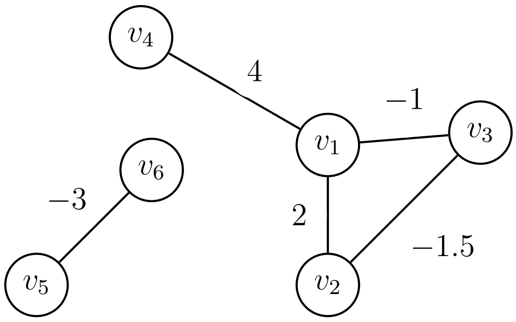


Figure 2.2: An example of a weighted directed graph

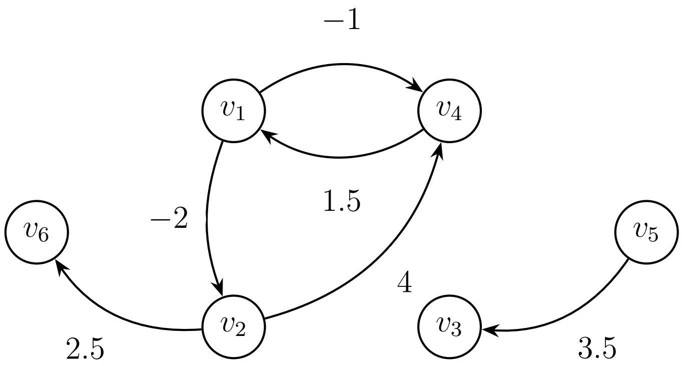
2.2 Signed Graphs

The simple weighted graphs we have defined so far only have non-negative edge weights that can represent similarity or closeness. In the 1950s social psychologists found it desirable to express liking, disliking or indifference in social interactions. This was formalized by Harary [10] using graphs with weights $(-1, 0, 1)$. These graphs are therefore called *signed graphs*, where a negative edge weight can denote dissimilarity between a pair of vertices. In this thesis we will use notations and terms similar to Gallier [7], Kunegis et al. [16], Hou [12] and Zaslavsky [24].

A signed graph is a triple $G = (V, E, w)$, where V is the set of vertices, E is the set of pair of vertices and the weight function $w : E \to \mathbb{R}$. The weight function now takes an edge e and maps it to a signed weight $w(e)$. We can partition the edges into positive and negative edges, $E = E^+ \cup E^-$, where $E^+ = \{e \mid w(e) > 0\}$ and $E^- = \{e \mid w(e) < 0\}$. Similar to Zaslavsky [24], we consider undirected signed graphs and directed signed graphs as distinct and separate entities. We can see some examples of signed graphs in Figure 2.3.



(a) A undirected signed graph



(b) A directed signed graph

Figure 2.3: Examples of Signed Graphs

We can now proceed to define a few more terms. The degree of a vertex v is now the sum of the absolute edge weight of its neighbours is called the *signed degree* and is defined as

$$\overline{d}(v) = \sum_{u \in N(v)} |w((u, v))|$$

This can also be extended to *signed indegree* and *signed outdegree* denoted by $\overline{\mathrm{d}_{\mathrm{in}}}(v)$ and $\overline{\mathrm{d}_{\mathrm{out}}}(v)$ and defined as

$$\overline{\mathrm{d}_{\mathrm{in}}}(v) = \sum_{u \in P(v)} |w((u, v))|$$

$$\overline{\mathrm{d}_{\mathrm{out}}}(v) = \sum_{u \in S(v)} |w((v, u))|$$

We can create a $n \times n$ square weight matrix W, where each entry w_{ij} is defined as

$$w_{ij} = \begin{cases} w((v_i, v_j)) & \text{if } (v_i, v_j) \in E \\ 0 & \text{if } (v_i, v_j) \notin E \end{cases}$$

The signed degree matrix $\overline{D}$ is a diagonal matrix consisting of the signed verted degrees, $\overline{D} = \mathrm{diag}(\overline{d}(v_1), \ldots, d(v_n))$. We can now define the *signed Laplacian*, $\overline{L}$ as

$$\overline{L} = \overline{D} - W$$

The signed Laplacian along with results from spectral analysis of signed graphs [12, 16], will be useful for balance theory of signed graphs.

2.2.1 Balance Theory

Heider [11] proposed in 1940's that when there are either *positive relations* (friendship, love, support) and *negative relations* (enmity, hate, oppose) in a group, the group tends towards *balance*. Balance was defined as there being all positive relations or one positive and two negative relations for a group of 3 people. Harary and Cartwright [3] generalized this notion of balance by using signed graphs. As these relations are typically symmetric, balance is usually defined for undirected signed graphs. This can been seen in Figure 2.4 where solid edges are positive and dashed edges are negative. Social psychology also showed that balanced triads B_1 and B_2 mirror aphorisms such as "the friend of my friend is also a friend" and "the enemy of my enemy is a friend" respectively.

The concept of balance can be generalized to state that an undirected signed graph $G = (V, E, w)$ is balanced iff every cycle in G has an even number of negative edges. This leads to result from Harary [10] that states that if a graph G is balanced, then there is a partition of the vertices $V = V_1 \cup V_2$ such that edges within the vertices of each set is positive and edges between the sets are negative. This means that we can have a bipartite

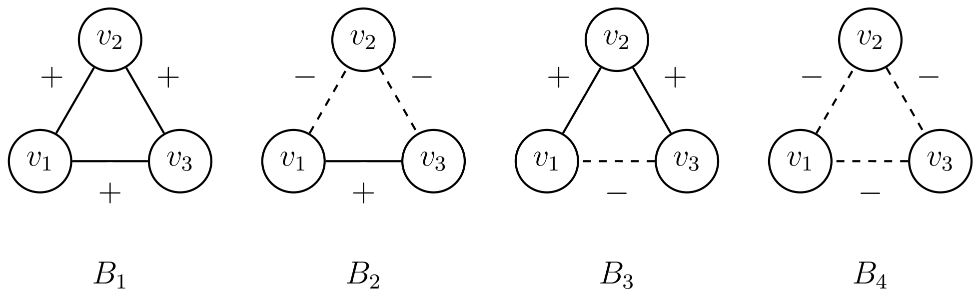


Figure 2.4: Triads in undirected signed graphs. B_1 and B_2 are *balanced* triads as they have even number of negative edges. B_3 and B_4 are *unbalanced* as they have odd number of negative edges.

graph when we delete the positive edges and negative edges span between the two sets of vertices. An example of a balanced signed graph is shown in Figure 2.5. Here the partition of the vertex set is $V_1 = \{v_1, v_3, v_4, v_7, v_8\}$ and $V_2 = \{v_2, v_5, v_6, v_9\}$.

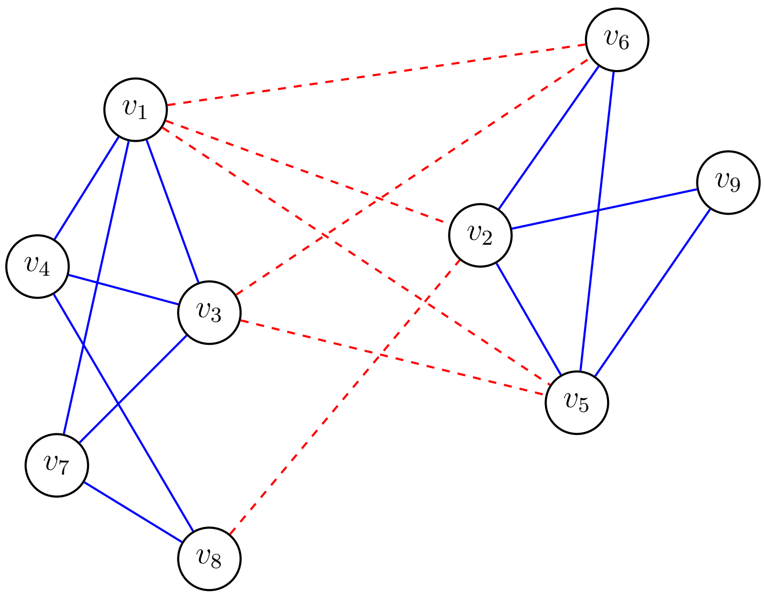


Figure 2.5: A balanced signed graph. Solid blue edges are positive and dashed red edges are negative. Every cycle in this graph contains an even number of negative edges.

The signed Laplacian matrix $\overline{L}$ of a signed graph G is always positive-semidefinite and $\overline{L}$ is positive-definite iff G is *unbalanced* [12, 16, 24]. If the smallest eigenvalue of a graph G is denoted by $\lambda_1(G)$, then G is balanced iff

$\lambda_1(G) = 0$. Hou [12] provides bounds on the value of $\lambda_1(G)$ which means that we can use the term as a measure of amount of imbalance in the signed graph G.

2.2.2 Status Theory

Guha et al. [8] mention implicitly that a signed edge from u to v can be interpreted in a asymmetric manner different from "friend" or "enemy". Leskovec et al. [18, 19] introduce the concept of *status* to contextualize directed singed edges. A positive edge $u \xrightarrow{+} v$ indicates that v has a higher status than u and a negative edge $u \xrightarrow{-} v$ means that v has a lower status than u. This concept of relative status can be propagated transitively along multiple steps which might lead to contradictions with balance theory [19].

Given three vertices v_1, v_2 and v_3, the presence of an edge $v_1 \xrightarrow{+} v_2$ indicates that v_1 thinks v_2 has higher status, the edge $v_2 \xrightarrow{+} v_3$ indicates that v_2 thinks v_3 has higher status. Now we wish to to close this triad with an edge from v_3 to v_1. Status theory would say that through transitivity v_1 has lower status than v_3, therefore the prediction is $v_3 \xrightarrow{-} v_1$. Whereas, in balance theory we would predict a positive edge $v_3 \xrightarrow{+} v_1$ to make the cycle have even number of negative edges. This example can be seen in triads S_1 and S_2 shown in Figure 2.6.

There are also cases when status theory is ambivalent to the edge that closes a triad. Consider the example when we have the edges $v_1 \xrightarrow{+} v_2$ and $v_2 \xrightarrow{-} v_3$. If we were to indicate the status of a vertex v using the function $s(v)$, then the edges describe the following : $s(v_2) > s(v_1)$ and $s(v_2) > s(v_3)$. Therefore, we have no knowledge of the relative difference in status between the vertices v_1 and v_3. Hence, both edges $v_3 \xrightarrow{+} v_1$ $(s(v_3) > s(v_1))$ and $v_3 \xrightarrow{-} v_1$ $(s(v_1) > s(v_3))$ are equally valid for status theory. Balance theory on the other hand can only predict $v_3 \xrightarrow{-} v_1$ to keep the triad balanced. This case is shown in Figure 2.6 as triads S_3 and S_4.

Each positive link inwards $(\mathrm{d_{in}}^+(v))$ and negative link outwards $(\mathrm{d_{out}}^-(v))$ increases status. Each positive link outwards $(\mathrm{d_{out}}^+(v))$ and negative link inwards $(\mathrm{d_{in}}^-(v))$ decreases status. Therefore, $\sigma(v) = \mathrm{d_{in}}^+(v) + \mathrm{d_{out}}^-(v) - \mathrm{d_{out}}^+(v) - \mathrm{d_{in}}^-(v)$ is a heuristic for the status of a node [18]. An interesting fact is that the edge $u \xrightarrow{-} v$ can be converted into positive edge in the opposite direction $u \xleftarrow{+} v$. This fact reduces the number of unique triads that can be formed using status theory and will be used in edge prediction tasks that will be discussed in coming chapters.

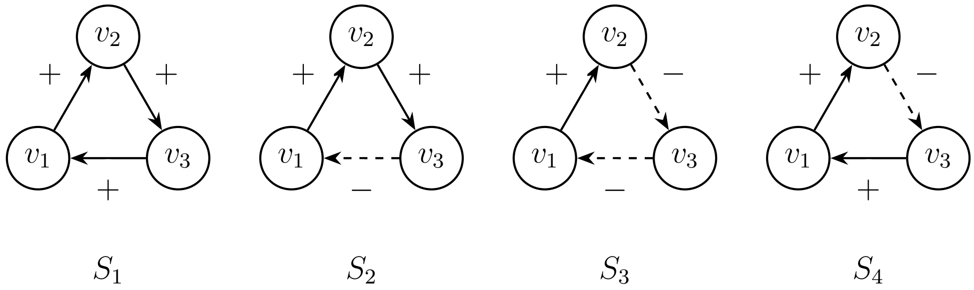


Figure 2.6: Triads in directed signed graphs. Triads S_2, S_3 and S_4 are compliant with status theory. Only triads S_1 and S_3 are compliant with balance theory.

2.3 Hierarchy in directed networks

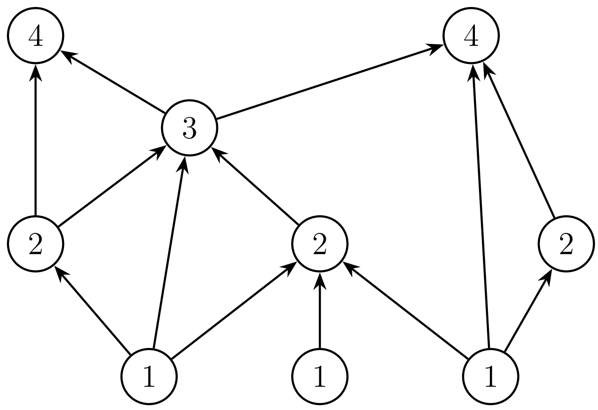
Hierarchies exist in all social structures, from the explicit levels found in nature such as the food chain or organizational structures in businesses to more implicit stratification that occurs on social media or online networks. A common method to represent such hierarchies is through a tree, for example, the chain of command in the military or within governments. Trees have well defined levels and a single person at the top. If we generalize this structural concept then we get a *Directed Acyclic Graph* (DAG) which represents a partial ordering set. DAGs have perfect hierarchy while structures such as cycles tend to have no hierarchy. Other directed graphs occur somewhere between these two extremes.

Gupte et al. [9] provide a method to discern the levels of stratification present in a given directed network when no prior information of the hierarchy exists. They define a measure of the hierarchy of a given directed network G as $h(G)$ along with a polynomial algorithm to find the largest hierarchy in that network. They define a concept of *social agony* which posits that agony is present when a person having a higher rank in network interacts with a person who has a lower rank. Therefore, if we define the rank of a node in graph G as the function $r : V \to \mathbb{N}$, then a directed edge $u \to v$ causes agony when $r(u) \geq r(v)$. The agony for the edge can be quantified as $\max(r(u) - r(v) + 1, 0)$. The agony of the graph G wrt to rank function r is defined as

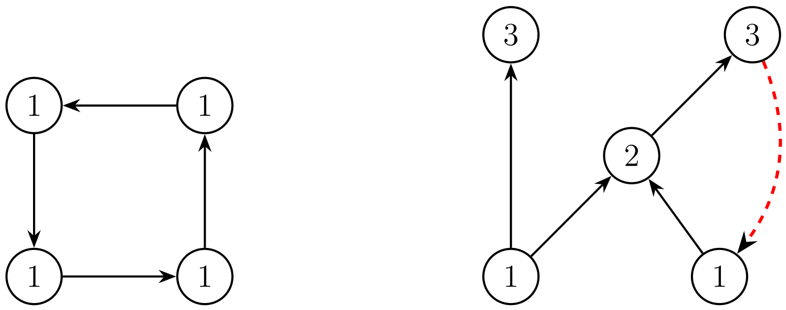
$$A(G, r) = \sum_{(u,v) \in E} \max(r(u) - r(v) + 1, 0)$$

As nodes in a graph tend to minimize the overall agony, the agony of a network G is the smallest possible agony over all possible ranking for r,

$A(G) = \min_{r \in \text{Rankings}} A(G, r)$. The hierarchy of a given network G, denoted by $h(G)$ can now be expressed in terms of the agony of the network, $h(G) = 1 - \frac{1}{m}A(G)$, where m is the number of edges. We can see examples of hierarchy in unweighted directed graph in Figure 2.7.



(a) DAG has perfect hierarchy, $h(G) = 1$ and agony of each edge is 0



(b) Cycle has no hierarchy, $h(G) = 0$ and each edge has agony of 1

(c) Graph with some hierarchy $h(G) = \frac{2}{5}$. Red dashed edge has agony of 3 and solid black edges have 0 agony.

Figure 2.7: Examples of hierarchy in unweighted directed graphs. Numbers inside nodes indicate the rank of vertex.

Therefore, finding a ranking of the nodes that minimizes the agony of the network gives us the optimal hierarchy present in that network. Gupte et al. [9] and Tatti [22] present a polynomial algorithm that can solve the dual of the agony minimization problem to obtain the optimal ranking r for unweighted graphs. Tatti [23] provides an alternate approach using a capacitated circulation solver that can handle weighted digraphs as well as additional cardinality constraints. These algorithms allows us to find the levels of hierarchy present in any given directed social network and analyse the interaction between members belonging to different strata in that community.

We will explain in future chapters how hierarchies in social network is intrinsically linked to the theory of status in directed signed networks. We show one can use the concept of agony of a directed signed graph G to quantify the violation to status theory and use it as a metric to predict an unknown signed edge.

Chapter 3

Vote Prediction

In this chapter, we cover the main motivation behind predicting the vote of an individual voter and present the methods that can be used to solve this task. We discuss the contrast in perspectives that is present when predicting the result compared to predicting a vote in Section 3.1. Next, we explain the link prediction problem in signed networks and the existing approaches as well as limitations to predicting votes in Section 3.2. In Section 3.3, we describe a supervised machine learning framework that can use graphs from voting and non-voting features to predict a vote. Lastly, we present our novel approach of constructing a signed graph from neighbours of the current voter and previous votes and using balance and status theory to predict the vote in Section 3.4.

3.1 Result versus Vote Prediction

- Discuss existing result predictions schemes
- Discuss the limitations in understanding voting dynamics through just predicting results
- Describe the process as an information cascade, discuss the potential Game Theory settings
- Show the two parts of the problem from an information cascading perspective
  - Who is going to vote next
  - How they are going to vote
- Discuss the assumptions in usual Independent Cascade (IC) models

- Explain the difficulty of both aspects in the domain of an election
- Motivate the selection of the problem as an **Independent Vote Prediction**

3.2 Signed Link Prediction

- Discuss the existing edge predictions work
- Directly using signed triads as features
- Using triads along with network features
- Using user information and interaction data for predicting votes and/or elections
- The main drawbacks in these methods when considering an election setting

3.3 Linear Combination of Graphs

- Describe the linear combination of graphs derived from user and election data
- Explain topic similarity, follows network, interaction networks and other features
- How it can also incorporate signed features as additional features in prediction

3.4 Local Signed Network

- Explain the concept of the local signed network for a particular user
- Motivate the definition with respect to elections and influence
- Describe how to use balance and status theory to predict the vote
- Clarify the differences to signed edge prediction efforts
- Mention Agony as a way to measure status compliance here?

Chapter 4

Wikipedia

In this section we provide an overview of how Wikipedia is structured, the hierarchy that exists withing editors. We then explain the election process of getting administrator rights in Wikipedia.

4.1 Structure and hierarchy in Wikipedia

4.2 Elections in Wikipedia

- Explain Editors and Administrators in Wikipedia
- Describe the Request for Administrator(RfA) process
- Discuss general trends and patters
- Mention research interest and possible current works?

Chapter 5

Experiments

In this section we first describe the datasets that will be used in building our vote prediction models. Then we discuss the various linear and graphical models that we consider and their implementations details. Lastly we define the metrics and other means of evaluating the models and the results.

5.1 Datasets

- Maybe a short description of existing SNAP datasets and their limitations
- The details of the *Wiki-RfA* data and the *User-Contribution* datasets

5.2 Graphs

- Discuss the process of extraction of the various graphs discussed in the previous sections
- **Agree Graphs and Follows Graph**, where we measure the degree to which one user agrees and follows another user in previous elections
- **Topic similarity** from the top 100 articles edited for each user and the pairwise Jaccard similarity
- **Talk and Interaction graphs**, measures communication between users on their respective user talk pages
- **Signed Graphs**, triad encoding and extracting the triad counts for each voter

5.3 Models

5.3.1 Linear Combination of Graphs

- Discuss the various linear models considered for Graph Combinations
  - Linear Regression
  - Support Vector Classifier
  - Extreme Gradient Boosting (XGBOOOST)
- Discuss how each graph contributes features and the problem is a linear classification problem

5.3.2 Iterative Mode

- Discuss the motivation behind an iterative model versus a static prediction model
- Describe how balance is derived from the Agree Graph in a local signed network
- Discuss how the Agree graph is updated in terms of Balance
- Describe how status is derived from the Follows graph in a local signed network
- Discuss how the Follows graph is updated after every election
- Describe how to make the predictions
  - Deterministic : just decide based on eigen value or agony as support or oppose
  - Probabilistic : provide a probability for predicting a support vote

5.4 Evaluation

- Discuss the issues with the imbalance in the datasets
- Illustrate the issues with pure measures of accuracy
- Define Precision, Recall and Macro F1 score
- Discuss ROC AUC and Precision Recall curves for probability based predictions

Chapter 6

Results and Discussion

In this section we will present the results of the models and discuss their implications.

6.1 Linear Combination of Graphs

- Present results for each linear classifier
- Discuss the different splits of the dataset to check for robustness and chronological consistency
- Show the feature importances and discuss their relevance
- Compare the raw accuracy versus the macro f1 scores
- Highlight the difficulty of predicting negative votes

6.2 Local Signed Network

- Present the Iterative Balance model results
- Discuss quality of predictions using evaluation metrics
- Mention the difference between deterministic and probabilistic prediction accuracies
- Explain the Iterative Status model results
- Discuss the issues with local model of status and the potential reasons for lower score and quality

6.3 Comparison

- Compare results from signed edge prediction and Iterative signed models
- Discuss Static Linear combination predictions versus Iterative signed predictions
- Discuss the assumptions used in the models and limitations

Chapter 7

Conclusions and Future Work

- Explain the quality of results with the election perspective
- Future work is to extend this to other election settings and investigate generality of this approach
- Possible future work in congressional voting data
- Can also tackle the other problem in information cascade theory of how to predict who is most likely to vote next
- This can lead to a complete model of election dynamics and could incorporate elements of game theory and network inference

Bibliography

[1] Brito, A. C. M., Silva, F. N., and Amancio, D. R. A complex network approach to political analysis: Application to the brazilian chamber of deputies. *PLOS ONE 15*, 3 (2020).

[2] Cabunducan, G., Castillo, R., and Lee, J. B. Voting behavior analysis in the election of wikipedia admins. In *2011 International Conference on Advances in Social Networks Analysis and Mining* (2011), pp. 545–547.

[3] Cartwright, D., and Harary, F. Structural balance: a generalization of heider's theory. *Psychological Review 63*, 5 (1956), 277–293.

[4] Chiang, K.-Y., Hsieh, C.-J., Natarajan, N., Dhillon, I. S., and Tewari, A. Prediction and clustering in signed networks: a local to global perspective. *Journal of Machine Learning Research 15*, 1 (2014), 1177–1213.

[5] Chiang, K.-Y., Natarajan, N., Tewari, A., and Dhillon, I. S. Exploiting longer cycles for link prediction in signed networks. In *Proceedings of the 20th ACM international conference on Information and knowledge management* (2011), pp. 1157–1162.

[6] Diestel, R. *Graph Theory.* 1997.

[7] Gallier, J. Spectral theory of unsigned and signed graphs. applications to graph clustering: a survey. *arXiv preprint arXiv:1601.04692* (2016).

[8] Guha, R., Kumar, R., Raghavan, P., and Tomkins, A. Propagation of trust and distrust. In *Proceedings of the 13th international conference on World Wide Web* (2004), pp. 403–412.

[9] Gupte, M., Shankar, P., Li, J., Muthukrishnan, S., and Iftode, L. Finding hierarchy in directed online social networks. In *Proceedings of the 20th international conference on World wide web* (2011), pp. 557–566.

[10] Harary, F. On the notion of balance of a signed graph. *Michigan Mathematical Journal 2*, 2 (1953), 143–146.

[11] Heider, F. Attitudes and cognitive organization. *The Journal of Psychology 21*, 1 (1946), 107–112.

[12] Hou, Y. P. Bounds for the least laplacian eigenvalue of a signed graph. *Acta Mathematica Sinica 21*, 4 (2005), 955–960.

[13] Jankowski-Lorek, M., Ostrowski, L., Turek, P., and Wierzbicki, A. Modeling wikipedia admin elections using multidimensional behavioral social networks. *Social Network Analysis and Mining 3*, 4 (2013), 787–801.

[14] Karimi, H., Derr, T., Brookhouse, A., and Tang, J. Multi-factor congressional vote prediction. In *Proceedings of the 2019 IEEE/ACM International Conference on Advances in Social Networks Analysis and Mining* (2019), pp. 266–273.

[15] Kearns, M. J., Judd, J. S., Tan, J., and Wortman, J. Behavioral experiments on biased voting in networks. *Proceedings of the National Academy of Sciences of the United States of America 106*, 5 (2009), 1347–1352.

[16] Kunegis, J., Schmidt, S., Lommatzsch, A., Lerner, J., Luca, E. W. D., and Albayrak, S. Spectral analysis of signed graphs for clustering, prediction and visualization. In *SDM* (2010), pp. 559–570.

[17] Lee, J. B., Cabunducan, G., Cabarle, F. G., Castillo, R., and Malinao, J. A. Uncovering the social dynamics of online elections. *Journal of Universal Computer Science 18* (2012), 487–505.

[18] Leskovec, J., Huttenlocher, D., and Kleinberg, J. Predicting positive and negative links in online social networks. In *Proceedings of the 19th international conference on World wide web* (2010), pp. 641–650.

[19] Leskovec, J., Huttenlocher, D., and Kleinberg, J. Signed networks in social media. In *Proceedings of the SIGCHI Conference on Human Factors in Computing Systems* (2010), pp. 1361–1370.

[20] LEVORATO, M., AND FROTA, Y. Brazilian congress structural balance analysis. *Journal of Interdisciplinary Methodologies and Issues in Sciences* (2016).

[21] TAL, M., MEIR, R., AND GAL, Y. K. A study of human behavior in online voting. *adaptive agents and multi-agents systems* (2015), 665–673.

[22] TATTI, N. Faster way to agony discovering hierarchies in directed graphs. In *ECMLPKDD'14 Proceedings of the 2014th European Conference on Machine Learning and Knowledge Discovery in Databases - Volume Part III* (2014), pp. 163–178.

[23] TATTI, N. Tiers for peers: a practical algorithm for discovering hierarchy in weighted networks. *Data Mining and Knowledge Discovery 31*, 3 (2017), 702–738.

[24] ZASLAVSKY, T. Signed graphs. *Discrete Applied Mathematics 4*, 1 (1982), 47–74.

[25] ZOU, J., MEIR, R., AND PARKES, D. Strategic voting behavior in doodle polls. In *Proceedings of the 18th ACM Conference on Computer Supported Cooperative Work & Social Computing* (2015), pp. 464–472.