Weekly Presentation
DeltaGrad: Rapid retraining of machine learning models

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Overview

Motivation

Retaining Problem

Regular Pipeline:

1. Train a ML model from data using a learning algorithm
2. Small change in training data occurs (deletions or additions)
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Research Question

Can we retrain models in an efficient manner?

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- **Bias Reduction**: Speeds up jackknife resampling that requires retrained model parameters

Related Work

Prior Work

- Prior work for specialized problems and ML models, usually for deletion
  - Provenane Based deletions for linear and logistic regression [WTD20]
  - Newton step and noise for *certified data removal* [GGHv20]
  - K-means clustering [GGVZ19]

DeltaGrad

Gradient Descent

- Objective function

$$F(\mathbf{w}) = \frac{1}{n} \sum_{i=1}^{n} F_i(\mathbf{w})$$

- Stochastic Gradient Descent update rule, $\mathcal{B}_t$ is randomly sampled mini-batch of size B

$$\mathbf{w}_{t+1} \leftarrow \mathbf{w}_t - \frac{\eta_t}{B} \sum_{i \in \mathcal{B}_t} \nabla F_i\left(\mathbf{w}_t\right)$$

- Full-batch gradient descent (GD) is on entire data

$$\mathbf{w}_{t+1} \leftarrow \mathbf{w}_t - \frac{\eta_t}{n} \sum_{i=1}^{n} \nabla F_i\left(\mathbf{w}_t\right)$$

Removal of data

- After training, $R = \{i_1, i_2, \ldots, i_r\}$ is removed, where $r \ll n$
- Naive retraining is applying GD over remaining samples, $\mathbf{w}^U$ is resulting parameters

$$\mathbf{w}^U{}_{t+1} \leftarrow \mathbf{w}^U{}_t - \frac{\eta_t}{n-r} \sum_{i \notin R} \nabla F_i \left(\mathbf{w}^U{}_t \right) \tag{1}$$

- The explicit gradient computation $\sum_{i \notin R} \nabla F_i \left(\mathbf{w}^U{}_t \right)$ is expensive
- Instead rewrite (1) as follows

$$\mathbf{w}^U{}_{t+1} = \mathbf{w}^U{}_t - \frac{\eta_t}{n-r} \left[\sum_{i=1}^{n} \nabla F_i \left(\mathbf{w}^U{}_t \right) - \sum_{i \in R} \nabla F_i \left(\mathbf{w}^U{}_t \right) \right]. \tag{2}$$

- $\sum_{i \in R} \nabla F_i \left(\mathbf{w}^U{}_t \right)$ is cheaper to compute

Etymology

- After a small change to the data we need to redo the SGD computations
- We can achieve this by understanding the *delta* of the Gradient Descent

$$n\nabla F(\mathbf{w}) = \sum_{i=1}^{n} \nabla F_i(\mathbf{w}_t) \quad \& \quad n\nabla F(\mathbf{w}^U) = \sum_{i=1}^{n} \nabla F_i \left(\mathbf{w}^U{}_t\right)$$

- Hence, the approach is called *DeltaGrad*

Aprroximating $\nabla F(\mathbf{w}^U)$

- $\mathbf{w}_0, \ldots, \mathbf{w}_t$ and $\nabla F(\mathbf{w}_0), \ldots, \nabla F(\mathbf{w}_t)$ are cached from training on initial dataset
- By Cauchy mean-value theorem[1]

$$\nabla F({\mathbf{w}^U}_t) - \nabla F(\mathbf{w}_t) = \mathbf{H}_t \cdot ({\mathbf{w}^U}_t - \mathbf{w}_t)$$

  Where $\mathbf{H}_t = \int_0^1 \mathbf{H}(\mathbf{w}_t + x({\mathbf{w}^U}_t - \mathbf{w}_t))dx$ is the integrated hessian
- This requires a hessian $\mathbf{H}_t$ at each step, which is expensive to maintain and evaluate
- Leverage classical L-BFGS algorithm to approximate $\mathbf{H}_t$

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[1] Seems to be a consequence of Fundamental theory of Calculus and mean-value theorem

Review of L-BFGS

- Traditional L-BFGS updates gradients using

$$\nabla F(\mathbf{w}_{t+1}) - \nabla F(\mathbf{w}_t) = \mathbf{B}_t \cdot (\mathbf{w}_{t+1} - \mathbf{w}_t)$$

Where, $\mathbf{B}_t$ is the approximation of the hessian

Traditional L-BFGS

$$\nabla F(\mathbf{w}_{t+1}) - \nabla F(\mathbf{w}_t) \approx \mathrm{B}_t (\mathbf{w}_{t+1} - \mathbf{w}_t)$$
$$\mathrm{B}_t \approx \mathrm{H}_t$$
$$= \int_0^1 \mathrm{H}(\mathbf{w}_t + x(\mathbf{w}_{t+1} - \mathbf{w}_t))\, dx$$
$$\mathbf{s}_t = \mathbf{w}_{t+1} - \mathbf{w}_t$$
$$\mathbf{y}_t = \nabla F(\mathbf{w}_{t+1}) - \nabla F(\mathbf{w}_t)$$

L-BFGS for approximating $\nabla F(\mathbf{w}^U)$

$$\nabla F\left(\mathbf{w}^U{}_t\right) - \nabla F(\mathbf{w}_t) \approx \mathrm{B}_t \left(\mathbf{w}^U{}_t - \mathbf{w}_t\right)$$
$$\mathrm{B}_t \approx \mathrm{H}_t$$
$$= \int_0^1 \mathrm{H}\left(\mathbf{w}_t + x\left(\mathbf{w}^U{}_t - \mathbf{w}_t\right)\right) dx$$
$$\mathbf{s}_t = \mathbf{w}^U{}_t - \mathbf{w}_t$$
$$\mathbf{y}_t = \nabla F\left(\mathbf{w}^U{}_t\right) - \nabla F(\mathbf{w}_t)$$

Using L-BFGS

- Maintain m historical observations of $\mathbf{Y} = (\mathbf{y}_t, \mathbf{y}_{t-1}, \ldots, \mathbf{y}_{t-m})$ and $\mathbf{S} = (\mathbf{s}_t, \mathbf{s}_{t-1}, \ldots, \mathbf{s}_{t-m})$
- Let g be a function defined by L-BFGS, then we can approximate $\mathbf{B}_t \cdot \mathbf{v}$ using

$$g(\mathbf{Y}, \mathbf{S}, \mathbf{v})$$

  Where, $\mathbf{v}$ is an arbitrary vector.
- Therefore,

$$\mathbf{B}_t \cdot ({\mathbf{w}^U}_t - \mathbf{w}_t) = g(\mathbf{Y}, \mathbf{S}, {\mathbf{w}^U}_t - \mathbf{w}_t)$$

- Hence we obtain the approximation as

$$\nabla F({\mathbf{w}^U}_t) \approx \nabla F(\mathbf{w}_t) + \mathbf{B}_t \cdot ({\mathbf{w}^U}_t - \mathbf{w}_t)$$

Rewriting

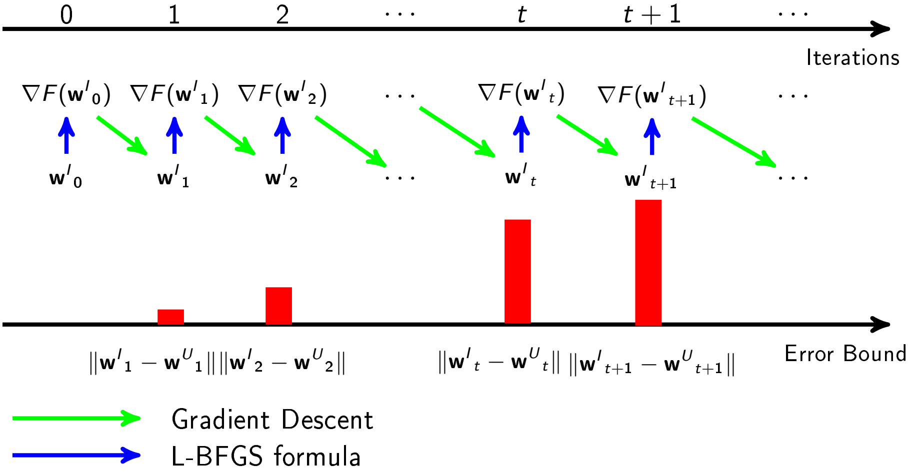
- Denoting $\mathbf{w}^I$ as the approximate $\mathbf{w}^U$ we have

$$\nabla F({\mathbf{w}^I}_t) \approx \nabla F(\mathbf{w}_t) + \mathbf{B}_t \cdot ({\mathbf{w}^I}_t - \mathbf{w}_t).$$

- replacing in (2)

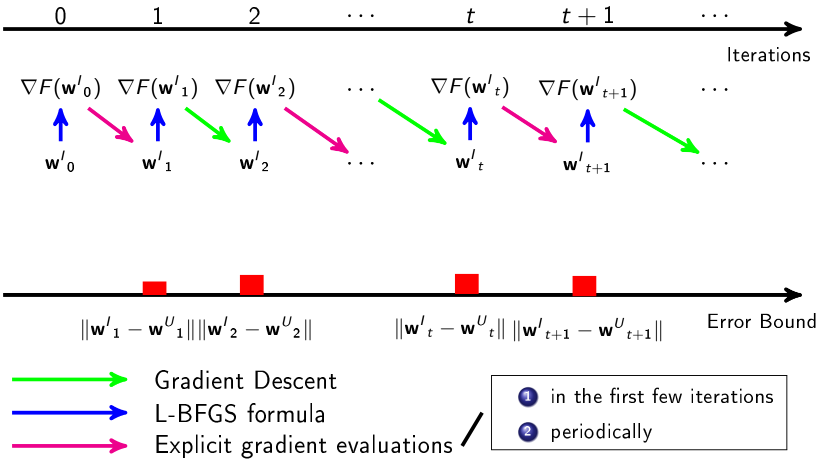
$${\mathbf{w}^I}_{t+1} = {\mathbf{w}^I}_t - \frac{\eta_t}{n-r}\left\{ n[\mathbf{B}_t({\mathbf{w}^I}_t - \mathbf{w}_t) + \nabla F(\mathbf{w}_t)] - \sum_{i \in R} \nabla F({\mathbf{w}^I}_t) \right\}$$

Problem with Error Bound



Controlling the Errors

- Do explicit evaluations for j_0 "burn-in" iterations and then periodically every T_0 iterations



Benefit of DeltaGrad

- DeltaGrad can be extended to when r samples are added rather than deleted
- Change the $+$ to minus in the update formula to get

$$\mathbf{w}^I{}_{t+1} = \mathbf{w}^I{}_t - \frac{\eta_t}{n+r}\left\{n[\mathbf{B}_t(\mathbf{w}^I{}_t - \mathbf{w}_t) + \nabla F(\mathbf{w}_t)] + \sum_{i \in R} \nabla F(\mathbf{w}^I{}_t)\right\}$$

- Here $\sum_{i \in R} \nabla F(\mathbf{w}^I{}_t)$ is the gradient of the added r samples

Algorithm

Algorithm 1: DeltaGrad

Input : The full training set $(\mathbf{X}, \mathbf{Y})$, model parameters cached during the training phase over the full training samples $\{\mathbf{w}_0, \mathbf{w}_1, \ldots, \mathbf{w}_t\}$ and corresponding gradients $\{\nabla F(\mathbf{w}_0), \nabla F(\mathbf{w}_1), \ldots, \nabla F(\mathbf{w}_t)\}$, the indices of the removed training samples R, period T_0, total iteration number T, history size m, "burn-in" iteration number j_0, learning rate η_t

Output: Updated model parameter $\mathbf{w}^I{}_t$

Initialize $\mathbf{w}^I{}_0 \leftarrow \mathbf{w}_0$
Initialize an array $\Delta G = []$
Initialize an array $\Delta W = []$
for $t = 0; t < T; t++$ **do**
    if $[((t - j_0) \mod T_0) == 0]$ *or* $t \leq j_0$ **then**
        compute $\nabla F(\mathbf{w}^I{}_t)$ exactly
        compute $\nabla F(\mathbf{w}^I{}_t) - \nabla F(\mathbf{w}_t)$ based on the cached gradient $\nabla F(\mathbf{w}_t)$
        set $\Delta G[k] = \nabla F(\mathbf{w}^I{}_t) - \nabla F(\mathbf{w}_t)$
        set $\Delta W[k] = \mathbf{w}^I{}_t - \mathbf{w}_t$, based on the cached parameters $\mathbf{w}_t$
        $k \leftarrow k + 1$
        compute $\mathbf{w}^I{}_{t+1}$ by using exact GD update (equation (1))
    else
        Pass $\Delta W[-m:]$, $\Delta G[-m:]$, the last m elements in ΔW and ΔG, which are from the $j_1^{th}, j_2^{th}, \ldots, j_m^{th}$ iterations where $j_1 < j_2 < \cdots < j_m$ depend on t, $\mathbf{v} = \mathbf{w}^I{}_t - \mathbf{w}_t$, and the history size m, to the L-BFGFS Algorithm to get the approximation of $\mathbf{H}(\mathbf{w}_t)\mathbf{v}$, i.e., $\mathbf{B}_{j_m}\mathbf{v}$
        Approximate $\nabla F(\mathbf{w}^I{}_t) = \nabla F(\mathbf{w}_t) + \mathbf{B}_{j_m}(\mathbf{w}^I{}_t - \mathbf{w}_t)$
        Compute $\mathbf{w}^I{}_{t+1}$ by using the "leave-r-out" gradient formula, based on the approximated $\nabla F(\mathbf{w}^I{}_t)$
    end
end
return $\mathbf{w}^I{}_t$

Theoretical Results

Experimental Results

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Certified Data Removal from Machine Learning Models.
arXiv:1911.03030 [cs, stat], August 2020.

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Making AI Forget You: Data Deletion in Machine Learning.
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In *Proceedings of the 2020 ACM SIGMOD International Conference on Management of Data*, pages 447–462, Portland OR USA, June 2020. ACM.

Large Deletions

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