IC 021 Computational Techniques in Control Engineering

End Semester Exam, Apr 28, 2017

Time: 3 hours **Answer all the questions** **Max. Marks: 50**

1. [**10 marks**] *The angle inscribed by any semicircle is a right angle.*

   How do you model this using linear algebra? Prove the above result from geometry using the notions of linear algebra.

2. [**10 marks**] A discrete-time dynamical system is given by

$$x(k+1) = \begin{bmatrix} 0 & 0 & 0.33 \\ 0.18 & 0 & 0 \\ 0 & 0.71 & 0.94 \end{bmatrix} x(k)$$

   For a given initial state $x(0)$, determine $x(6)$. How do you modify RSD based algorithm to compute e^A to solve this problem?

3. [**10 marks**] In the question above, does $x(\infty) \to 0$? Justify your answer.

4. [**10 marks**] For the autonomous system of question 2 above, propose a two-input state-feedback controller $K \in \Re^{2\times 3}$ such that the eigenvalues are now placed at 1, $0.01 \pm j0.01$. Check your result by computing $x(\infty)$.

5. [**10 marks**] For a given linear system $b = Ax + e$, where $A \in \Re^{m\times n}$, $b \in \Re^{m\times 1}$, $m \gg n$ and rank$(A) = n$, the least squares problem is $\min\limits_{x} \|Ax - b\|_2$. If A has QR factorization, then show how the factors would help compute the least squares solution $\hat{x}$. What would be the total flop count (including the QR factorization)?

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K E Y for End Semester

1. Assume that the cirle has center at the origin and radius r in $\Re^2$. The diametrically opposite points are given by x and $-x$ for some x with $\|x\| = r$. Let y be any point on the circle, i.e., $\|y\| = r$. The angle inscribed is the angle between the lines $\mathcal{L}(-x, y)$ and $\mathcal{L}(x, y)$ at y. Their direction vectors are $x + y$ and $x - y$. Thus, you are expected to show that $\|x\| = \|y\|$ if and only if the inner product $< x + y,\, x - y >= 0$. You should have no difficulty in proving this result.

   Remember, you need to argue on either side to prove the equivalence, i.e., "if and only if".

2. You need to compute A^6. Intuitively, after diagonalizing, simply raise the diagonal elements to the power 6, and then de-diagonalize to get A^6

3. The system is asymptotically stable - solve the discrete-time Lyapunov equation, and you'll get the result.

4. Originally, all the eigenvalues are within the unit circle; now you move one of them to the circumference, and you notice that the response $x(\infty)$ approaches a constant.

5. First, recall that if Q is an orthonormal matrix, $\|Qx\| = \|x\|$. Consequently, if $A = QR$

   $$\|Ax - b\|_2 \;=\; \|QRx - b\|_2 \;=\; \|Q^TQRx - Q^Tb\|_2 \;=\; \|Rx - Q^Tb\|_2$$

   The algorithm is as follows:

   Step 1 Obtain $A = QR$, where $Q \in \Re^{m\times m}$ and $R \in \Re^{m\times n}$

   R has two partitions - $\begin{bmatrix} R_1 \\ -- \\ 0 \end{bmatrix}$

   i.e., $R_1 \in \Re^{n\times n}$ is above the $(m-n)\times n$ zero matrix

   Step 2 Form $Q^Tb \;=\; \begin{bmatrix} c \\ -- \\ d \end{bmatrix}$, with $c \in \Re^{n\times 1}$

   Step 3 Obtain the solution $\hat{x}$ by solving the upper triangular system $R_1\hat{x} = c$ where R_1 is the upper partition of R.

   The flop count is $\mathcal{O}\left(mn^2\right)$.

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