IC 021 Computational Techniques in Control Engineering

Cycle Test II, Mar 23, 2017

Time: 1 hour **Answer all the questions** **Max. Marks: 20**

1. (a) [**5 marks**] Write Padè's scaling and squaring algorithm for computing the matrix exponential e^A.

   (b) [**3 marks**] Demonstrate the algorithm computing e^A if

   $$A = \begin{bmatrix} -3 & 3 \\ -1 & -1 \end{bmatrix}$$

   (c) [**2 marks**] Verify your results.

2. (a) [**5 marks**] Using Kronecker's products, compute $P : A^T P + PA = -I$ and check if the (autonomous) system

   $$A = \begin{bmatrix} -3 & 3 \\ -1 & -1 \end{bmatrix}$$

   is stable.

   (b) [**5 marks**] Demonstrate how Bartels - Stewart algorithm works on the above matrix.

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1. The algorithm is as follows:
   Given A, and an error tolerance δ,

   Chose j such that $\|A\|_\infty \leq 2^{j-1}$. Set $A = A/2^j$.

   Find p such that p is the smallest non-negative integer satisfying

   $$\left(\frac{8}{2^{2p}}\right) \frac{(p!)^2}{(2p)!(2p+1)!} \leq \delta$$

   Set $D = N = Y = I$, $c = 1$

   For $k = 1 \cdots p$ do

   $$\begin{aligned} c &= c(p-k+1)/[(2p-k+1)k] \\ Y &= AY,\ N = N + cY,\ D = D + (-1)^k cY \end{aligned}$$

   Solve for F in $DF = N$

   For $k = 1 \cdots j$ do $F = F^2$

   Typically one can choose $\delta = 0.005$, though this is not needed for the numerical in the following part (b) of the problem, and $\|A\|_\infty$ is the maximum column sum.

2. The system is stable;

   $$P = \begin{bmatrix} \frac{1}{6} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}, \quad \text{+ve definite}$$

   (a) 3 marks for correct set up of the Kronecker's products and the resulting linear system of equations; 2 marks for the final solution P and the verification that it is positive definite.

   (b) Bartels - Stewart algorithm is based on Real Schur decomposition of A to an upper triangular matrix. Once A is upper triangular, the resulting equations are linear, and it is enough to test the sign definiteness of the transformed matrix, instead of the original P matrix.

   5 marks for the complete explanation as outlined above.

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