# $\begin{array}{c} \textbf{Summer Internship Project} \\ \textbf{Report} \end{array}$

# Detection of Cosmic Muons Using Gas Detectors

Submitted by

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### Acknowledgements

I would like to express my sincere gratitude to Dr. Sanjib Muhuri, my guide and mentor, for his invaluable support, guidance, and expertise throughout my internship on the simulation and experimental detection of cosmic muons using the ALICE detector. His vast knowledge and constant encouragement have been instrumental in shaping my understanding of this complex field of study. I am also grateful to the High Energy Group at VECC (Variable Energy Cyclotron Centre) for providing me with the necessary resources, infrastructure, and access to the ALICE detector. Their assistance and cooperation have been vital in carrying out the experimental phase of this internship. I extend my heartfelt appreciation to the Department of School of Physical Sciences at NISER (National Institute of Science Education and Research) for providing me with the opportunity to pursue this internship. The conducive academic environment and the support of the faculty have greatly contributed to my overall learning experience. I would like to acknowledge the collective efforts of the researchers, technicians, and staff members who have been associated with the project. Their contributions, suggestions, and collaboration have significantly enriched my internship journey. Lastly, I express my gratitude to my fellow interns and friends for their camaraderie, stimulating discussions, and continuous encouragement. Their presence has made this internship an enjoyable and enriching experience. In conclusion, I am immensely thankful to all individuals and institutions mentioned above for their unwavering support, guidance, and contributions, which have been instrumental in the successful completion of this internship.

# Variable Energy Cyclotron Centre, Kolkata

# Certificate

This is to certify that Mr. Anantha Padmanabhan M Nair, a student of the National Institute of Science Education and Research (NISER), has successfully completed a summer internship in the field of Cosmic Muons and its Detection by ALICE Detectors. The internship was conducted from 5/6/2023 to 29/7/2023, under the guidance and supervision of Dr. Sanjib Muhuri.

During the internship, Mr. M Nair demonstrated exceptional dedication, enthusiasm, and competence in conducting simulations, experimental data collection, and analysis related to cosmic muons and their detection using the ALICE detector. Through their hard work and perseverance, they contributed significantly to the understanding of cosmic muons' behavior and their energy deposition within the detector setup.

We commend Mr. M Nair for their exemplary performance, commitment to scientific inquiry, and collaborative spirit. Their active participation and insightful contributions have been instrumental in the success of the internship project.

We extend our best wishes to Mr. M Nair for a bright and successful future, both in their personal life and professional career. May they continue to excel in their academic pursuits and make significant contributions to the field of research.

VECC Kolkata, Bidhannagar, West Bengal

> Dr Sanjib Muhuri (Project Guide)

Date:

#### Abstract

This report focuses on the simulation and experimental detection of cosmic muons using the ALICE (A Large Ion Collider Experiment) detector, employing the Geant4 simulation framework. The study aims to understand the behavior of cosmic muons and their energy deposition within the detector setup, consisting of a honeycomb gas detector filled with Argon and  $CO_2$ . The simulation phase involved utilizing Geant4 to replicate the laboratory setup and simulate the interaction of cosmic muons with the detector. By accurately modeling the trajectory and behavior of cosmic muons, the simulation facilitated the calculation of their energy deposition within the ALICE detector. This process provided valuable insights into the expected behavior of cosmic muons within the simulated environment. Subsequently, the actual experimental phase involved collecting data from the ALICE detector in the laboratory. The detector, filled with Argon and CO2 gases, accurately captured cosmic muons as they traversed through it. The collected data allowed for a comparison between the simulated and experimental results, validating the accuracy of the Geant4 simulation model and providing further insights into the behavior of cosmic muons. The internship report outlines the methodology employed during both the Geant4 simulation and the experimental data collection phases, including the parameters and variables considered. It discusses the data analysis techniques used to evaluate the energy deposition of cosmic muons and presents a comprehensive comparison between the simulated and experimental results.

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### 1 Introduction

The detection and study of cosmic muons hold great significance in the field of particle physics and high-energy physics. Cosmic muons, which are highly energetic charged particles originating from cosmic rays, provide valuable insights into the properties of elementary particles and the fundamental forces governing our universe. To explore the behavior of cosmic muons and their interaction with matter, we are using the CoFPMD (Cosmic flux Photon Multiplicity Detectors) detectors.

The internship comprises two main phases: simulation and experimentation. In the simulation phase, the Geant4 framework is utilized to model the laboratory setup and replicate the interaction of cosmic muons with the CoFPMD. Geant4, a widely used toolkit in high-energy physics, provides a comprehensive platform for simulating the passage of particles through matter, accurately capturing their interactions and energy deposition.

In this report, we will outline the methodology employed during the simulation and experimental phases, discuss the data analysis techniques used to evaluate the energy deposition of cosmic muons, present the comparison between simulated and experimental results, and provide a comprehensive analysis of the overall internship experience.

## 2 Cosmic Muons and its Properties

Cosmic rays are high-energy particles that originate from various sources beyond our solar system, such as distant stars, supernovae, and active galactic nuclei. They consist of protons(87%), Alpha Particles(12%), and atomic Heavy Nuclei (1%), some of which can have energies millions or even billions of times greater than those produced in the most powerful particle accelerators on Earth. When cosmic rays enter the Earth's atmosphere, they interact with air molecules, producing a cascade of secondary particles, including muons, neutrinos, and gamma rays.

### 2.1 Production of Cosmic Muons

When there Cosmic Rays Reach the Earths atmosphere, it collides with the air molecules and produces many particles, mostly Pions and Kaons. These are the primary Particles. These particles then decay to produce a wide variety of particles most of which are muons. Also, more than 90% of the cosmic muons are produced from Pions.

From the Kaons, We can see that the muons are produced by the weak interactions and it also produces

neutrinos given by:

$$K^+ \longrightarrow \mu^+ + \nu_\mu \tag{1}$$

From the decay of Pions, 64% of the time, disintegrates directly into  $\mu^+$ ; 21% of the time into  $\pi^0$  and  $\pi^+$  and Only 6% of the disintegrations produce three particles- $\mu^+, \mu^+, \mu^-$ . These pions then decay to produce muons according to:

$$\pi^+ \longrightarrow \mu^+ + \nu_\mu$$
 (2)

$$\pi^- \longrightarrow \mu^- + \bar{\nu_\mu}$$
 (3)

While the decay of these primary particles produces mostly muons, Many other elementary particles are also produced within this process. During the process of generation of primary particles that is the Kaons and pions,  $K^0$  and  $\pi^0$  are also produced which on decay produces  $\gamma$ -particles. These Muons and other particles further decay to produce Electrons and Positrons.

### 2.2 Properties of Muons

Muons are unstable particles having intermediate mass between that of an electron and a proton, just like charged pions. Compared to pions, they are a little lighter. The muons carry one unit of electrical charge, either positive or negative, and are electrically charged. The life time of the muon is  $2.2\mu s$ . The properties of the muons are tabulated below in Table-1

Properties		Values
Mass	$m_{\mu}$	$206.7686m_e$
	$m_{\mu}c^2$	$105.659 \mathrm{MeV}$
Mean Life	$ au_{\mu}$	$2.197 \mu s$
Spin	$s_{\mu}$	1/2
Magnetic Moment	$\mu_{\mu}$	$\frac{eh}{4\pi m_{\mu}}$

Table 1: Properties of Muons

We know that the half life of muons are  $1.56\mu s$ . But we are able to observe the muons coming from outer space on the earth surface. This is due to the relativistic effects. Now we know that the total energy of the cosmic muon is in the range of 4GeV. Let us calculate the value of  $\gamma$ . Considering the relativity:

$$\gamma m_{\mu}c^2 = 4GeV = 6.4 \times 10^{-10}J \tag{4}$$

Substituting  $m_{\mu} = 1.883 \times 10^{-28} Kg$  we get:

$$\gamma = 37.754770 \tag{5}$$

Now, let  $t_{1/2}$  be the half life of muon when it is at rest WRT the earth and  $t'_{1/2}$  be its half life when its moving. So, from time dilation, we have  $t'_{1/2} = \gamma t_{1/2}$ . on substituting the values we get:

$$t'_{1/2} \approx 0.058s$$
 (6)

As  $\gamma$  is very high, the speed of the muons are very close to the speed of the light. So time taken for the muons to reach the earth surface from the outer atmosphere is  $\approx$  distance/c which is  $\approx 10^7 m/c \approx 0.03s$ 

So, this is why we are able to observe the cosmic muons incident on the earths surface even if the half of the muons are in the range of micro seconds. The Calculations are based on an average scale.

## 3 Passage of Radiation Through Matter

Naturally, penetrating radiation views matter as a collection of electrons and nuclei along with their subatomic particles, which are its fundamental building blocks. Reactions with the atoms or nuclei as a whole, or with each of their individual constituents, may take place through any channels that are permitted, depending on the type of radiation, its energy, and the type of material. The Coulomb force, electromagnetic collisions with atomic electrons, elastic scattering from a nucleus, absorption in a nuclear reaction, and other processes can all occur when an alpha particle enters a gold foil, for instance. These occur with a certain probability that is determined by the fundamental interactions that are involved, as well as by the laws of quantum mechanics.

# 3.1 The Cross section of the Interactions

The cross section is a common way to describe how two particles collide or interact. If the fundamental interaction between the particles is known, this quantity can be calculated and serves as a gauge of the likelihood that a reaction will take place. According to formal definitions, the cross-section is defined as follows. Think about a particle beam that hits a target particle 2, as seen in Fig. 2.1. Assume that the target is much farther away from the beam and that the beam's particles are evenly spaced out in time. After that, we can talk about a flux of incident particles per unit of space and time.

Now, considering the amount of particles scattering into an angle  $d\Omega$  per unit time, the number of particles is not constant if we measure more number of time. Let the average no of particles be  $N_s$  and let F be the flux. The differential cross section is defined as:

$$\frac{d\sigma}{d\Omega}(E,\Omega) = \frac{1}{F} \frac{N_s}{d\Omega} \tag{7}$$

that is,  $d\sigma/d\Omega 2$  is the average fraction of the particles scattered into df2 per unit time per unit flux F. In terms of a single quantum mechanical particle, this may be reformulated as the scattered probability

current in the angle  $d\Omega$  divided by the total incident probability passing through a unit area in front of the target.

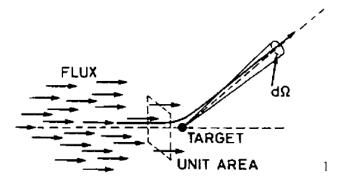


Figure 1: CDiagram showing the cross section-[3]

So, the total cross section is given by:

$$\sigma(E) = \int d\Omega \frac{d\sigma}{d\Omega} \tag{8}$$

Assuming that the target centers are uniformly distributed and the slab is not too thick so that the likelihood of one center sitting in front of another is low, the number of centers per unit perpendicular area which will be seen by the beam is then  $N\delta x$  where N is the density of centers and  $\delta x$  is the thickness of the material along the direction of the beam. If the beam is broader than the target and A is the total perpendicular area of the target, the number of incident particles which are eligible for an interaction is then FA. The average number scattered into  $d\Omega$  per unit time is then:

$$N_s(\Omega) = FAN\delta x \frac{d\sigma}{d\Omega} \tag{9}$$

and,

$$N_{tot} = FAN\delta x\sigma \tag{10}$$

And the probability of interaction in  $\delta x$  is  $N\sigma\delta x$ -[3]

### 3.2 Interaction Probability

Here we will calculate what id probability that a particle does not involve in an interaction for a distance of x, this probability is known as the survival probability P(x). Let the probability of having an interaction between x and dx be wdx. Then we have the probability of not having an interaction between x and x + dx as:

$$P(x+dx) = P(x)(1-wdx) \tag{11}$$

On solving, we get the P(x) as:

$$P(x) = C \exp{-wx} \tag{12}$$

C turns out to be 1 while substituting the usual probability properties.

Now as we have the interaction probability, we will calculate the mean free path, which is defined as:

$$\lambda = \frac{\int x P(x) dx}{\int P(x) dx} = \frac{1}{w}$$
 (13)

but the interaction probability depends on the  $\delta x$  intuitively, after approximating to the linear order terms we get:

$$\lambda = \frac{1}{N\sigma} \tag{14}$$

So the survival probability becomes:

$$P(x) = \exp\left(\frac{-x}{\sigma}\right) = \exp\left(-N\sigma x\right) \tag{15}$$

# 3.3 Energy Loss of the penetrating particle by Atomic Collisions

Inelastic collisions with atomic electrons and the elastic scattering from the nuclei are the two main reasons for the energy loss and change in direction of the particle.

Of course, the inelastic collisions are statistical in nature and have a certain quantum mechanical probability of happening. The fluctuations in the total energy loss are, however, small due to their abundance per macroscopic path length, so one can effectively work with the average energy loss per unit path length. Bohr first calculated this quantity often referred to as the stopping power or simply dE/dx—using classical reasoning. Later, Bethe, Bloch, and others did so using quantum mechanics.

#### 3.3.1 Bohr's Calculation

Consider a heavy particle traveling through a material medium with a charge ze, mass M, and velocity v. Assume that an atomic electron is present at a distance and from the particle trajectory as shown in Figure-2 In order to capture the electric field acting on the electron at its initial position, we assume that the electron is free, initially at rest, and that it only moves very slightly during the interaction with the heavy particle. Furthermore, because of its much greater mass  $(M_{\tilde{c}}m)$ , we assume that the incident particle will have essentially maintained its original course after the collision. This is one justification for separating heavy particles from electrons!

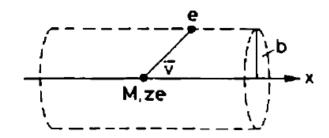


Figure 2: Diagram For Bohr Calculation of Scattering-[3]

Now, we will calculate the energy gained by the electron by finding the momentum impulse it receives by colliding with the heavy particle.so:

$$I = \int F dt = e \int E_{\perp} dt = e \int E_{\perp} \frac{dx}{v}$$
 (16)

By applying the gauss law, we get,

$$\int E_{\perp} 2\pi b dx = 4\pi z e \tag{17}$$

So that we have:

$$I = \frac{2ze^2}{bv} \tag{18}$$

The energy gained by the electron is then:

$$\Delta E(b) = \frac{I^2}{2m_e} \frac{2z^2 e^4}{mev^2 b^2} \tag{19}$$

Now, let the density of particles be  $N_e$ , then the energy lost to all the electrons in the range b to b+db is given by:

$$-dE(b) = \Delta E(B)N_e dV = \frac{4\pi z^2 e^4}{mev^2} N_e \frac{db}{b} dx \qquad (20)$$

On solving with volume element  $dV=2\pi bdbdx$  we get:

$$-\frac{db}{dx} = \frac{4\pi z^2 e^4}{m_e v^2} N_e \ln\left(\frac{b_{max}}{b_{min}}\right) \tag{21}$$

Ideally the limit should be 0 to infinity, but due to our assumption that the collision at large b wont take place over a short period of time, there is an upper bound  $b_{max}$  and also at b=0, the integral diverges so there is also a  $b_{min}$ .

To calculate the  $b_{min}$ , the maximum kinetic energy is transferred when there is a head on collision and the maximum energy it can gain is  $\frac{1}{2}m_e(2v)^2$ , taking relativity we get this energy as  $2\gamma^2 m_e v^2$  substituting this to Equation-19, we get,

$$b_{min} = \frac{ze^2}{\gamma m_e v^2} \tag{22}$$

Now for the calculation of  $b_{max}$ , We should look at the electrons. These electrons are not Free but bound to an atoms with some orbital frequency  $\nu$ . For the electron to absorb some energy, the perturbation caused by the incident particle should be for a short time as compared to the angular frequency  $(1/\nu)$ . Otherwise the perturbation will be adiabatic and there will be no transfer od energy. For Collisions, the interaction time is  $t = b/\nu$ . Considering the relativistic effects, t becomes  $t/\gamma$ . so we can write:

$$\frac{b}{\gamma v} \le \tau = \frac{1}{\bar{\nu}} \tag{23}$$

where  $n\bar{u}$  is the mean frequency averaged over all the states. The the upper limit of b becomes:

$$b_{max} = \frac{\gamma v}{\bar{\nu}} \tag{24}$$

Now substituting this into the Equation-21, we get the classical bohr formula for the Energy Loss as:-[3]

$$-\frac{db}{dx} = \frac{4\pi z^2 e^4}{m_e v^2} N_e \ln{(\frac{\gamma^2 m_e v^3}{z e^2 \bar{\nu}})}$$
 (25)

#### 3.3.2 The Bethe-Bloch Formula

The more realistic Quantum mechanical formulation of the Energy loss is carried out by Bethe and Bloch in which the Energy is parametrized in terms of momentum rather than the impact parameter. The formula thus obtained is:

$$-\frac{db}{dx} = 2\pi N_a r_e^2 C^2 \rho \frac{z^2 Z}{A\beta^2} \left( \ln \left( \frac{2m_e \gamma^2 v^2 W_{max}}{I^2} \right) - 2\beta^2 \right)$$
(26)

Where  $r_e$  is the electron radius,  $N_a$  is the Avogadro Number,  $\rho$  is the density of the material,  $m_e$  is the electron mass, I is the mean excitation potential, Z and A are the atomic number and atomic mass of the absorbing material, z is the charge of the incident particle in units of e,  $W_{max}$  is the maximum energy transfer in a single collision,  $\beta$  and  $\gamma$  have the usual definition in terms of velocity of the incident particle.

The maximum Energy Transfer id given by the formula:

$$W_{max} = \frac{2m_e c^2 \eta^2}{1 + 2s\sqrt{1 + \eta^2} + s^2} \tag{27}$$

where  $s = m_e/M$ , where M is the mass of the incident particle and  $\eta = \beta \gamma$ .

Normally, two corrections are also added to the bethe bloch formula which is then given by:

$$-\frac{db}{dx} = 2\pi N_a r_e^2 C^2 \rho \times \frac{z^2 Z}{A\beta^2} \times \left(\ln\left(\frac{2m_e \gamma^2 v^2 W_{max}}{I^2}\right) (28) -2\beta^2 - \delta - 2\frac{C}{Z}\right) (29)$$

Where C is the Shell Correction and  $\delta$  is the density Correction

### 4 Simulation Using Geant4

Geant4-[1], short for "Geometry and Tracking 4," is a powerful and widely-used toolkit for simulating the interactions of particles with matter. Developed by the CERN collaboration and used extensively in high-energy physics, nuclear physics, and other scientific fields, Geant4 is renowned for its accuracy and versatility. The toolkit provides a comprehensive set of tools and libraries that allow researchers to design complex geometries, define various particle sources, and study the interactions of particles with matter in intricate detail. Its modular and extensible architecture makes it adaptable to a wide range of simulation tasks, from studying the behavior of elementary particles in particle accelerators to modeling radiation therapy treatments in medical physics.

### 4.1 The Detector Geometry

The Detector Geometry is defined almost same as that is present in the laboratory to detect the muons. The Detector construction is implemented using G4box, G4Tubs and G4Polyhedra. The logical volumes are crated using the G4LogicalVolume and it is placed in appropriate positions just like what we have in the lab.

The detector that we are using in the lab is a Cosmic flux Photon Multiplicity Detector (CofPMD) in which we will be detecting the Muons using the  $Ar + CO_2$  (9:1) mixture present in the detector. So We will be setting the Sensitive detector as the Logical volume of the Gas mixture. Using this Sensitive Detector, we will Measure the position of incidence and the Total Energy deposition from the Sensitive Detector.

### 4.1.1 The Honeycomb shaped Detector

Our main part of the detector is a honeycomb shaped detector made up of Copper in which at the Centre of each hexagon there is a thin gold wire and this acts as the Anode in the real case and the Cu part acts as the Cathode. This whole honeycomb is filled with a Mixture of Argon and Carbon Dioxide in the ratio 9:1. The Inner Radius of the Hexagon is 2.5mm and its depth is 5mm. The radius of the Gold wire present at the centre is 10 microns. The construction of the honeycomb and the gold wire is done using G4Polyhedra and G4Tubs respectively. The Constructed Hexagonal Honeycomb Structure is shown in the Figure-3.



Figure 3: Honeycomb structure created from GEANT4

Figure 5: The complete Experimental Setup Simulation

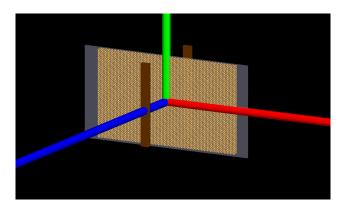


Figure 4: Honeycomb structure and Scintillation detectors created from GEANT4 (The RED BLUE and GREEN Lines are x,z and y axes)

The detector also contains Two scintillation detectors (of Thickness 1 cm and length of 11cm) which are placed one above and one below at equal distances of 11cm from the honeycomb detector. This is shown in the Figure-4

### 4.1.2 The Complete Setup

The complete Setup of the Experiment is implemented using the Detector construction in GEANT4. The Complete construction is Shown in the Figure-5. The Grey parts of the simulations are made up of Aluminum and the green parts are made up of plastic. Also the Electronics and the PMDs are constructed whose material is FR4.

### 4.2 Generation of Particles

The particle generation and its projection is done using the usual fParticleGun with the G4VUserPrimaryGeneratorAction in Geant4. Using this one can generate as many particles as we want in a single run. for our case we will be generating muons only for the single particle case and and for the Actual simulation of the Cosmic Shower, we will be using the EcoMug for the data for different types of particles.

# 4.3 Collection of Energy Deposition Data

The energy Deposit form the Gas Mixture is calculated by the sensitive detector which was assigned for the Argon Gas mixture. Geant4 Calculates the Energy Deposits and then it writes the values for each run (each run consist of shooting of a single muon perpendicular to the Gas detector) into a text file. Which is then read by the Python and ROOT for further analysis. The implementation of the Detection and its calculation is done using G4VSensitiveDetector and G4Track.

## 5 Single Muon Interaction

In this case, we will be using the fParticleGun to Generate  $\mu^-$ -Particles and will project them to the Detector through the scintillation detector parallel to the Z-Axis. We will be defining the momentum in terms of Energy (MeV and GeV) and all of this with Construction of the detectors as mentioned above.

### 5.1 Obtained Energy Deposit data

As mentioned earlier, the  $\mu^-$ -particles are fired at different energies. Then its Most probable Energy Deposit Values are taken and plot against the incident

energy and the graph is Checked by the bethe-bloch formula.

### 5.1.1 Single Energy Muon

At first, we fired one particle at 4GeV perpendicular to the detector through the scintillator in one run (100000 Runs were made). The plot of the range of Energy Deposit is Shown in the Figure-6 where we can see that the distribution is landau. The function is Fitted by the Landau function and MPV of the Energy Deposit is obtained as 677.82eV and the standard deviation is 136.66eV. The corresponding image of the accumulated paths of the muons as shown in the Figure-7

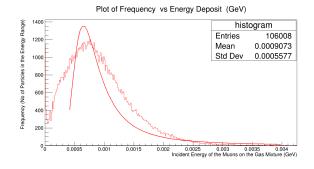


Figure 6: Statistics of Energy Deposit by 4GeV Muon Minus-[2]

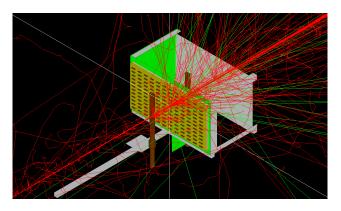


Figure 7: Accumulated paths of the muons for 500 Runs-[1]

### 5.1.2 Muons With Different Energies

Now, we will be calculating the Most Probable Value (MPV) of the Energy Deposit for different values of the incident energy. We will Start from low energy (100MeV) and will end at calculating MPV of each Energy deposition till we reach 100GeV particles. For Each energy we will be firing 100000  $\mu^-$ -particles one by one for each run. The energy Deposit is Calculated by the Sensitive Detector (Argon+ $CO_2$  Mixture).

The histogram showing the distributions for

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