Introduction to AI and ML

Matrix Project

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Question

The area(in sq.unit) of the quadrilateral formed by the tangents at the ends of the latera recta of the eclipse $\frac{x^2}{9} + \frac{x^2}{5} = 1$,is:

Given eclipse in matrix form:

$$x^T \begin{bmatrix} 1/9 & 0 \\ 0 & 1/5 \end{bmatrix} x = 1$$

Solution

Conic equation is matrix form:

$$x^T Vx + 2u^T x + F = 0$$

Here,

$$V = \begin{bmatrix} 1/9 & 0 \\ 0 & 1/5 \end{bmatrix}$$

$$u = O$$

$$F = -1$$

The length of semi major axis $a = 3$,the length of semi minor axis $b = \sqrt{5}$

$$F_1 = \begin{bmatrix} \sqrt{a^2 - b^2} \\ 0 \end{bmatrix}$$

$$F_2 = \begin{bmatrix} -\sqrt{a^2 - b^2} \\ 0 \end{bmatrix}$$

$$F_1 = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$F_2 = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$$

The end points of the latera recta can be found from

$$\begin{bmatrix} 2 & y \end{bmatrix} \begin{bmatrix} 1/9 & 0 \\ 0 & 1/5 \end{bmatrix} \begin{bmatrix} 2 \\ y \end{bmatrix} = 1$$

$$\begin{bmatrix} -2 & y \end{bmatrix} \begin{bmatrix} 1/9 & 0 \\ 0 & 1/5 \end{bmatrix} \begin{bmatrix} -2 \\ y \end{bmatrix} = 1$$

From the above equations we get,

$$P_1 = \begin{bmatrix} 2 \\ \frac{5}{3} \end{bmatrix} \qquad P_3 = \begin{bmatrix} -2 \\ -\frac{5}{3} \end{bmatrix}$$

$$P_2 = \begin{bmatrix} 2 \\ -\frac{5}{3} \end{bmatrix} \qquad P_4 = \begin{bmatrix} -2 \\ \frac{5}{3} \end{bmatrix}$$

Tangent of a conic is given by

$$(p^T V + u^T)x + p^T u + F = 0$$

Tangent at P_1

$$\begin{bmatrix} \frac{2}{9} & \frac{1}{3} \end{bmatrix} x = 1$$

Tangent at P_2

$$\begin{bmatrix} \frac{2}{9} & -\frac{1}{3} \end{bmatrix} x = 1$$

Tangent at P_3

$$\begin{bmatrix} -\frac{2}{9} & -\frac{1}{3} \end{bmatrix} x = 1$$

Tangent at P_4

$$\begin{bmatrix} -\frac{2}{9} & \frac{1}{3} \end{bmatrix} x = 1$$

The point of intersection of two lines is given by:

$$x = N^{-T} p$$

Where $N = \begin{pmatrix} n_1 & n_2 \end{pmatrix}$ Intersection points of tangents at P_1 and P_2 is

$$A = \begin{bmatrix} \frac{9}{4} & \frac{9}{4} \\ \frac{3}{2} & \frac{3}{2} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{9}{2} \\ 0 \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{9}{4} & -\frac{9}{4} \\ -\frac{3}{2} & -\frac{3}{2} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -3 \end{bmatrix}$$

$$C = \begin{bmatrix} -\frac{9}{4} & -\frac{9}{4} \\ \frac{3}{2} & \frac{3}{2} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -\frac{9}{2} \\ 0 \end{bmatrix}$$

$$D = \begin{bmatrix} -\frac{9}{4} & \frac{9}{4} \\ \frac{3}{2} & \frac{3}{2} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

Area of quadrilateral formed by tangents,

$$Area = 4 * Area(\triangle AOB)$$

$$Area = 4 * \frac{27}{4}$$

$$Area = 27 sq.units$$