IKS (HS271TA) Unit 2 : Foundational Concepts of Mathematics and Science & Technology: Linguistics; Pānini's work on Sanskrit Grammar, Phoenetics, Ashtadhyayi's vyakarana. Number System and Units of Measurement: concept of zero, Piṅgala and the Binary system. Knowledge: Framework & Classifications: Prameya, Pramana, Samsaya.

Sl no	Subtopic	YouTube Video Links
1	**Linguistics;** Pānini's work on Sanskrit Grammar, Phoenetics, Ashtadhyayi's vyakarana.	**Mod-01 Lec-4 Panini's Astadhyayi** : https://www.youtube.com/watch?v=qZsdAXF9DTE https://www.youtube.com/watch?v=qZsdAXF9DTE&t=2335s **Phonetics : Lecture 02** : The Sounds of Sanskrit: Its Alphabet : https://www.youtube.com/watch?v=UgVwzueOKRU&list=PLbRMhDVUMngfYG2GVf2bQnIgsI0Y923g3&index=2 **Lecture 03 : Sentence Construction and its underlying logic** : https://www.youtube.com/watch?v=4ombhCLE604&list=PLbRMhDVUMngfYG2GVf2bQnIgsI0Y923g3&index=3
2	**Number System and Units of Measurement:** concept of zero, Piṅgala and the Binary system.	3.1 **Number systems in India - Historical evidence** - IKS_CH06_C01 : https://www.youtube.com/watch?v=fuOZPZlzJj8&t=137s 3.2 **Salient aspects of Indian Mathematics,** Part-1 : IKS_CH06_C02 : https://www.youtube.com/watch?v=sFqZwOt_PO8 3.3 **Salient aspects of Indian Mathematics, Part-2** : IKS W06 C03 : https://www.youtube.com/watch?v=s0ADeq3m3vM IKS_CH08 _C07 : **Binary mathematics and combinatorial problems in Chandaḥ Śāstra :** https://www.youtube.com/watch?v=s_RbFa9Z0y0 https://www.youtube.com/watch?v=k794egUhgKo&t=427s Concept of zero : https://www.youtube.com/watch?v=sFqZwOt_PO8&t=67s
3	**Knowledge: Framework & Classifications**: Prameya, Pramana, Samsaya.	IKS_CH07_C01 : Indian scheme of knowledge : https://www.youtube.com/watch?v=n0OQgqDmumg&t=447s https://www.youtube.com/watch?v=N05kLLpg-AI IKS_CH07_C02 : The knowledge triangle IKS_CH07_C03 : Prameya – A vaiśeṣikan approach to physical reality IKS_CH07_C07 : Pramāṇa – the means of valid knowledge IKS_CH07_C06 : Sāmānya, viśēṣa, samavāya : https://www.youtube.com/watch?v=f_Tq8TCDTgo IKS_CH07_C08 : Saṃśaya – ambiguities in existing knowledge : https://www.youtube.com/watch?v=x5CZXEWC0yE IKS_CH07_C09 : Framework for establishing valid knowledge https://www.youtube.com/watch?v=gWKt9mz270s IKS_CH07_C13 : Summary of the chapter https://www.youtube.com/watch?v=0T8P2j9-sM0&t=5s

Linguistics; Pānini's work on Sanskrit Grammar, Phoenetics, Ashtadhyayi's vyakarana.

Introduction : In learning Sanskrit, the study of grammar plays a very important part. Among various Sanskrit grammatical systems, Panini's is the most significant and the most authoritative. Pānini, an influential figure in ancient Indian linguistics, is best known for his seminal work, the Aṣṭādhyāyī, composed around the **5th or 6th century BCE.**

The original work of Panini's grammar is called Astadhyayi. Previously students of Sanskrit used to commit to memory the whole work. Panini composed **3983 sutras** (rules) to accommodate all the patterns and variations in Sanskrit language.

Pāṇini was an ancient Sanskrit philologist, grammarian, and a revered scholar in ancient India. Pāṇini's theory of morphological analysis was more advanced than any equivalent Western theory before the 20th century.

Pāṇini is considered the **father of linguistics**. A consequence of his grammar's focus on brevity is its highly unintuitive structure, reminiscent of modern notations such as the "**Backus–Naur form" (BNF).** His sophisticated logical rules and technique have been widely influential in ancient and modern linguistics.

Born in the ancient province of **Gandhara**, Pānini's contributions stemmed from a need to preserve the correct interpretation of religious texts. His grammar emphasizes the importance of oral tradition, as students memorized the sūtras, often without the aid of written texts. The Aṣṭādhyāyī includes a unique metalanguage, which enables a precise description of Sanskrit that reflects both its sounds and grammatical structures. Pānini's work has had a profound and lasting impact on the study of language, influencing later grammarians and shaping linguistic thought in India and beyond.

Birch bark manuscript from Kashmir of the Rupavatara, a grammatical textbook based on the Sanskrit grammar of Panini. It was composed by Dharmakirti, a Buddhist monk from Ceylon. The manuscript was transcribed in 1663 Wellcome Images Keywords: ORIENTAL; birch bark; panini; kashmir; rupavatra

Panini's work serves as a great way to understand the Indian contribution to linguistics.

Aşţādhyāyi:

- "**Ashta**" means eight, and "**adhyaya**" means chapter.
- The work consists of eight chapters.
-

The Lineage of Saṃskṛta Grammar and Tri-Muni

- Aṣṭādhyāyī built upon centuries of prior grammatical study
- Provided a standardized and universally accepted model
- Surpassed earlier works due to its logical organization and precision
- Marked the high point in India's grammatical tradition
- Pāṇini: creator of Aṣṭādhyāyī, codification of grammar rules
- Kātyāyana (4th century BCE): wrote Vārtikas, clarifying Pāṇini's sūtras
- Patañjali (2nd century BCE): authored Mahābhāṣya, a monumental commentary
- Together called the "Tri-Muni" (three sages) of Saṃskṛta grammar

Structure of Aşţādhyāyi

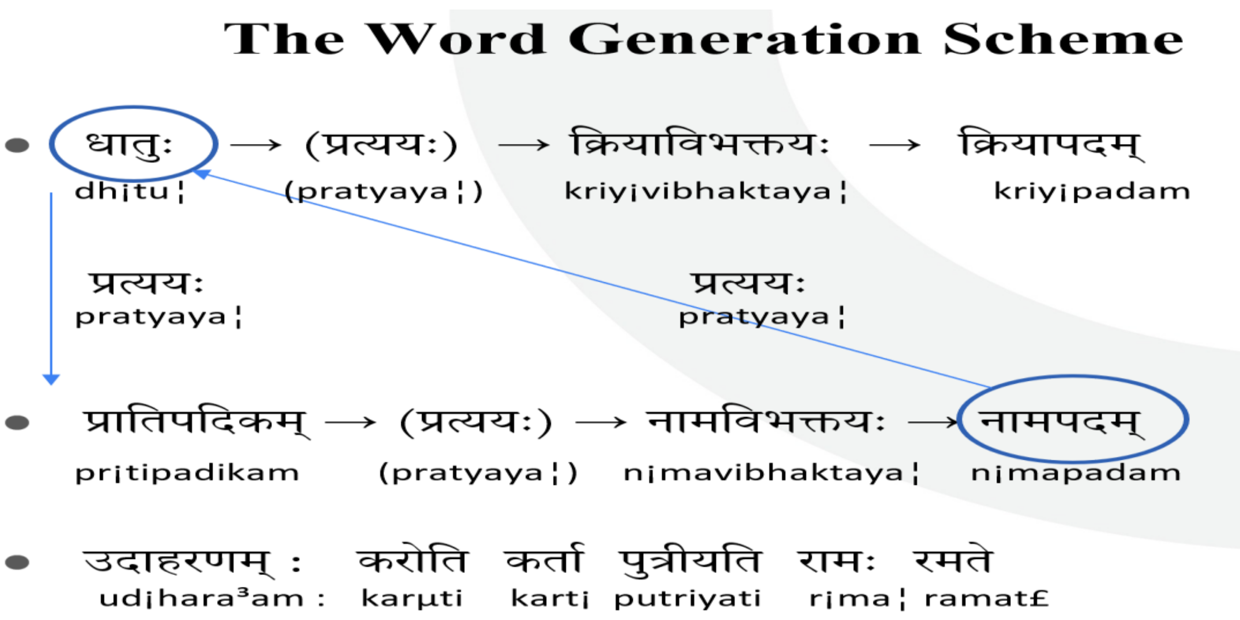
- The 3983 rules are arranged into 8 chapters.
- Each chapter is further divided into 4 quarters.
- 8 chapters x 4 quarters = 32 sections.

Language as Assemblage of Words

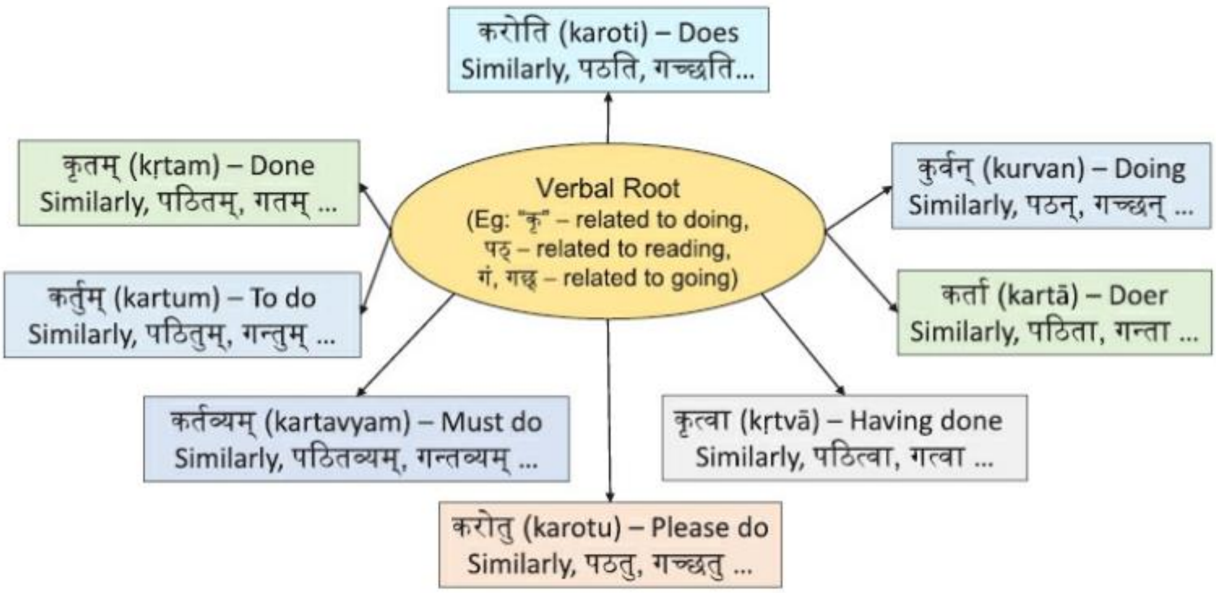
- Language is an assemblage of words.
- Words are combined in various ways to communicate ideas and transact knowledge.
- Understanding word formation and generation is central to language mastery.

Rule-Based Nature of Saṃskṛta :

- Word formation is entirely rule-governed
- Process involves:
  - Starting with a verbal or nominal root
  - Adding appropriate suffixes to form words
- Correct word always emerges from correct rule application
- Eliminates ambiguity and ensures precision
- Word derivation is step-by-step and recursive
- At each step, a rule is applied if conditions are satisfied
- The process continues until no more rules apply
- Results in the final, grammatically valid word form
- System is modular: base + suffix combinations create patterns across vocabulary



Word Patterns in Saṃskṛta Language – an example



Dynamic Vocabulary and Modern Significance

- Vocabulary is not fixed; new words can be generated
- As long as rules are followed, new valid forms are possible
- Ensures adaptability of Saṃskṛta for changing needs
- Made Saṃskṛta eternal and relevant across centuries
- Important implication for lexicography and word creation
- Aṣṭādhyāyī resembles a formal generative grammar
- Rule-driven, logical, and algorithmic in nature
- Highly suitable for computer-based language processing
- Inspired modern fields such as mathematics, computational linguistics, and AI
- Stands as the world's oldest and most complete descriptive model of language

Example: Verbal Root 'kru' (to do)

- Let's use 'kru' (to do) as an illustrative example.
- 'Dukrn': Panini attaches indicators to the front and back of verbal roots for mathematical transformation purposes.
- In Panini's Dhatupatha (list of verbal roots), the root is represented as 'Dukrn'.
- By removing 'du' and '[FL]', we get the substantive part of the verb, 'kr'.
- 'Kru' means something to do with doing.
- Example: Karma comes from kru.

Word Derivatives of 'kru'

- **Karoti:** Does (present simple tense).
- **Kurvan:** Doing (continuous).
- **Karta:** Doer.
- **Krtva:** Having done.
- **Karotu:** Please do.
- **Kartavyam:** It must be done.
- **Kartum:** To do.
- **Krtam:** Done (past tense).

Other Verbal Roots: 'pat' (to read) and 'gam' (to go)

- We see two more verbs: 'pat' (meaning reading) and 'gam' (gacch related to going).

Similar Word Patterns

- The equivalent words for the two roots follows a similar pattern like for kru:
    - Karoti: Patathi, Gacchati
    - Kurvan: Pathan, Gacchan
    - Kartha: Pathita, Ganta
    - Krtva: Pathitva, Gatva
    - Karotu: Pathtu, Gacchatu
    - Kartavyam: Pathtavyam, Gantavyam
    - Kartum: Pathitum, Gantum
    - Krtam: Pathitam, Gatam
- If you know the process for a verb, you can apply it to approximately 2200 verbs.

Word Pattern Formation Logic

- There is a consistent logic for creating patterns of words in Sanskrit.

Base and Suffix

- "kr", "path", "gach" are called **Base**.
- In Sanskrit, one start with the **Base**.
- The basic building block in Sanskrit, you start with the Base, then add a suffix to it and then get a word, but in the process of getting the word, some more rules will be triggered

Suffix Example

- Take the verb 'kr', then add the suffix 'tu' > "karotu". The Suffix "tu" triggers karotu.

All Possibilities in Word Generation

- To make rich combination generation of words in Sanskrit Language, a few things need to be kept in mind.
- Start with a verbal root.
- Add suffixes.
- Add more suffixes, leading to a verb form and finally a complete word.

Noun Root and Suffix

- Starting with a noun root, you add suffixes.
- Some suffixes can give you masculine, and some suffixes can give you feminine.
- Start from Noun Root and add Suffix and get Noun Form.

Verb to Noun Conversion

- Add a suffix to a verb to convert it into a noun (e.g., "Do" -> "Doer").
- Convert noun to verb is also possible.

Example: Nominal Root 'Ram'

- Take Nominal Root called Ram.
- 7 x 3 = 21 suffixes are available.
- Using these, 21 different word forms can be created.
- For example adding 'su' to "Ram" gives you "Ramah".

Combine Root with Suffix with rules

- Add any of the suffixes to a nominal root to get a valid noun form.
- Rules will govern the combination.
- Take a verbal root like 'path' and add suffixes like 'tip'.

Understand the Logic

- Take a nominal root and add suffixes, resulting in valid nouns.
- Take a verbal root and add suffixes, resulting in a verb form.

The Numbers: Cases, Singular, Dual, and Plural

- 7 cases and singular, dual, and plural (3), this defines the number of suffix options.
- First, second, and third person and singular, dual, and plural (3 x 3 = 9) defines the additional types of suffix combinations.

Word Generation Logic

•The word generation is algorithmic in nature.

•Take something (the root), add something on top of it (suffixes), apply some rules, and derive the final word.

•Rule-based mechanism which will be discussed again when looking at computational elements of Ashtadhyayi.

Simple Sentence Formation

Sanskrit:
रामः फलम् खादति (*Rāmaḥ phalam khādati* – "Rama eats the fruit")

Pāṇinian components:

- *Rāmaḥ* → noun (*prātipadika* + *suP* suffix → nominative singular)
- *Phalam* → noun (*prātipadika* + *suP* → accusative singular)
- *Khādati* → verb (*dhātu* + *tiṅ* suffix → present tense, 3rd person singular)

Equivalent CFG:

```
S → NP VP
NP → N SUFFIX
VP → V NP
N → 'rāma' | 'phala'
SUFFIX → 'ḥ' | 'm'
V → 'khādati'
```

Generated sentence:
`rāmaḥ phalam khādati`

Observation:
The CFG derivation mimics Pāṇini's derivational process of combining *stem* + *suffix* and *subject* + *object* + *verb*.

Compound Word Formation (Samāsa)

Pāṇini describes:

"tatpuruṣaḥ samāsaḥ" → A compound where one word depends on another.

E.g., *Rāja-putraḥ* (king's son)

Pāṇini-like rule:

```
Samāsa → N1 N2   [where N1 modifies N2]
```

CFG Analogy:

```
COMPOUND → N N
N → 'rāja' | 'putra'
```
Output: `rāja-putra`

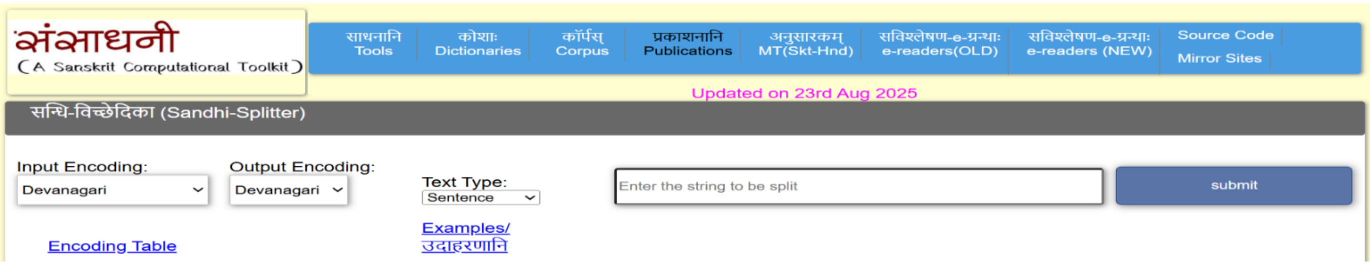
Extra Information : (Computer Science point of view)

Conceptual Mapping

Aspect	Pāṇini's Grammar	Context-Free Grammar (CFG)	Mapping / Analogy
Basic Unit	*Sūtra* (rule)	*Production rule*	Each Pāṇinian sūtra acts like a CFG production.
Non-terminals	Abstract categories (e.g. *Prātipadika*, *Dhātu*, *Sup*, *Tiṅ*)	Non-terminal symbols (e.g. *NP*, *VP*)	Represent parts of speech or word forms.
Terminals	Sounds or morphemes (phonemes, roots)	Terminal symbols	The smallest linguistic units (letters or tokens).
Derivation process	Sequential + recursive rule application (*Anuvṛtti*)	Recursive expansion of non-terminals	Both generate valid strings from abstract rules.
Rule type	Generative and transformative	Generative	CFG captures only generative aspect (Pāṇini includes transformations too).
Recursion	Inherent in compound formation and syntax	Built-in	e.g., embedding of subordinate clauses.
Meta-rules	*Paribhāṣā-sūtras* define how to interpret other rules	Meta-grammar (not in simple CFG)	Closer to *meta-level CFG* or *attribute grammar*.

Learn further :

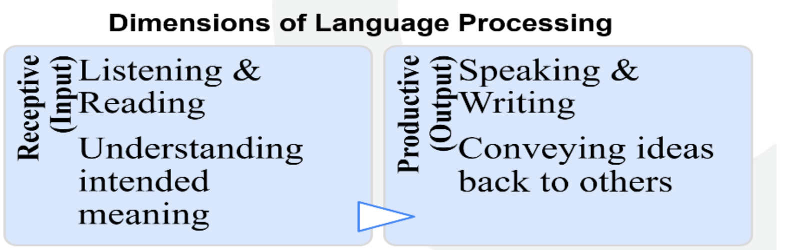
https://sanskrit.uohyd.ac.in/scl/#



Phoenetics

Importance of Communication

Dimensions of Language Processing

<table>
<tr><td>
</td><td>

- **Two Perspectives:**
  - **Sound → Listening & Speaking**
  - **Script → Reading & Writing**
</td></tr>
<tr><td colspan="2">Language Processing Framework (2×2)</td></tr>
<tr><td>

- **Sound:** **Phonetics (listening & speaking)**
- **Script:** **Syntax & Grammar (reading & writing)**
</td><td>Linguistics addresses all aspects of sound & script</td></tr>
</table>

Linguistics: scientific study of language (Provides systematic methods for word forms & meaning)

- Speech sounds
- Grammatical structures
- Meaning & syntax

Language and Vedic Tradition :

- Preservation of the Vedas was considered sacred duty
- Vedāṅgas supported Vedic study, with three focusing on language:
  - *Śikṣā* – Phonetics (pronunciation & articulation)
  - *Nirukta* – Etymology (origin & meaning of words)
  - *Vyākaraṇa* – Grammar (rules of usage and structure)
- Mastery of Saṃskṛta ensured clear understanding of epics, Purāṇas, and Vedas

Sounds

नासिका (nāsikā) (Nasal effort) → ङ् ञ् ण् न् म्

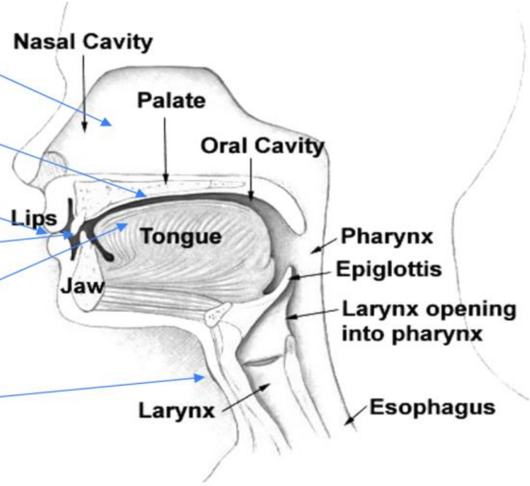
मूर्धा (mūrdhā) Upper palate → ऋ ट् ठ् ड् ढ् ण् र् ष्

ओष्ठौ (oṣṭhau) Lips→ उ प् फ् ब् भ् म्

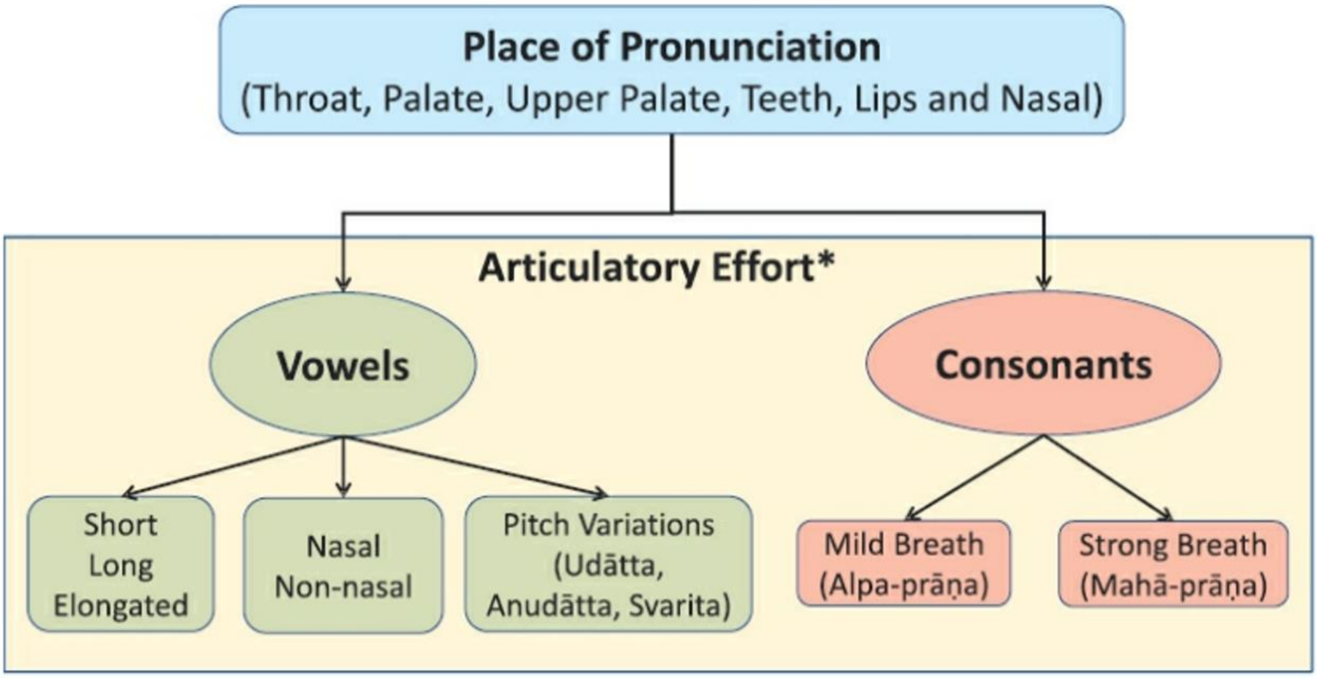
दन्ताः (dantāḥ) Teeth→ लृ त् थ् द् ध् न् ल् स्

तालु (tālu) Palate→ इ च् छ् ज् झ् ञ् य् श्

कण्ठः (kaṇṭhaḥ) Throat→ अ क् ख् ग् घ् ङ् : ह्



Sthana and Prayatna



* This presents only a partial categorisation of the articulatory effort

Number System and Units of Measurement: concept of zero, Piṅgala and the Binary system

यथा शिखा मयूराणां नागानां मणयो यथा। तद्वद् वेदांगशास्त्राणां गणितं मूर्ध्नि संस्थितम्॥

https://hear2read.org/sanskrit-text-to-speech-demo

	Just as the **crest on the heads of peacocks** and the **gems** on the **heads of serpents** is in the **highest position,** in the same way the **place of mathematics** in the Vedangashastras is at the top.	

Brief History :

Pioneering Sages: Sūrya, Pitāmaha, Vaśiṣṭha, Medhātithi, Garga, Lomasa, Cyavana.

Astronomer-Mathematicians: Varāhamihira, Āryabhaṭa, Bhāskarācārya I & II, Brahmagupta, Mahāvīrācārya, Kālakācārya, Pṛthuyasas, Pāvulūri Mallanna.

Invention of the number system and concept of zero.

Development of basic operations: addition, subtraction, multiplication, division.

Insights into solar system and planetary motion.

Advances in algebra, calculus, geometry, astronomy (Post-Vedic period).

Global Influence:

Translations of Sanskrit texts into Arabic by Kaṅka.

Inspiration for Al-Khwārizmī and adoption of Sanskrit numerals.

Spread to Spain, Germany, France, England, and beyond.

Mathematics on the Indian subcontinent has a rich and long history going back over 5,000 years and thrived for centuries before advances were made in Europe. Its influence spread to China, Southeast Asia, the Middle East, and Europe. Apart from introducing the concept of zero, Indian mathematicians made seminal contributions to the study of geometry, arithmetic, binary mathematics, the notion of negative numbers, algebra, trigonometry, and calculus among other areas. The decimal place value system that is employed worldwide today was first developed in India. Classical period (400–1600) This period is often known as the golden age of Indian Mathematics. This period saw mathematicians such as Aryabhata, Varahamihira, Brahmagupta, Bhaskara I, Mahavira, Bhaskara II, Madhava of Sangamagrama and Nilakantha Somayaji give broader and clearer shape to many branches of mathematics.

Concept of Zero:

'Zero' was discovered by an anonymous Indian. The concept of zero in India holds significant historical and mathematical importance. The numeral zero, as we understand it today, is an integral part of the decimal numeral system, and it was developed in ancient India.

1. Early Development: India's earliest recorded use of a decimal system and zero can be traced back to the Indus Valley Civilization, which existed around 2500-1500 BCE. However, the concept of zero as a number with its own value and significance started to evolve later.

2. Brahmagupta's Contributions: One of the key figures in the development of the concept of zero in India was the mathematician Brahmagupta, who lived around 598-668 CE. He made significant contributions to the understanding of zero. He defined zero as a number and discussed its mathematical properties. He introduced rules for arithmetic operations involving zero, such as the addition and subtraction of zero, which are now fundamental concepts in mathematics.

3. Aryabhata's Work: Aryabhata, an Indian mathematician and astronomer who lived around 476-550 CE, also played a role in zero development. In his work, Aryabhata used a symbol to represent zero and recognized its importance in mathematical calculations.

4. Spread to the Islamic World and Europe: The concept of zero and the decimal numeral system with its place-value notation spread from India to the Islamic world. Islamic scholars like Al-Khwarizmi further developed these ideas, and they eventually made their way to Europe during the Middle Ages.

5. Importance in Modern Mathematics: Zero is now considered one of the most fundamental concepts in mathematics. It serves as a placeholder in the decimal system, enabling us to represent numbers of varying magnitudes efficiently. It is also crucial in algebra, calculus, and many other branches of mathematics.

The concept of zero in India has a long and rich history, with mathematicians like Brahmagupta and Aryabhata making significant contributions to its development.

Number System:

India has a rich history of number systems, with significant contributions to the development of various numeral systems. One of the most influential contributions is the decimal numeral system, which is now the most widely used number system globally. Overview:

1. Indus Valley Civilization: The earliest known evidence of a numerical system in the Indian subcontinent can be traced back to the Indus Valley Civilization (around 2500-1500 BCE). Archaeological findings indicate the use of symbols for counting and recording quantities.

2. Brahmi Numerals: The Brahmi script, dating back to around the 3rd century BCE, played a significant role in the development of numeral systems in India. It had symbols for numbers, and these symbols are considered some of the earliest written representations of numerical values in India.

3. Kharosthi numeral system: (3rd century BCE to 3rd century CE) The Kharosthi numeral system was a positional decimal system similar to the more widely known Indian numeral system. It used symbols to represent numbers, and the position of a symbol in a number determined its value, just as in the Indian numeral system. The Kharosthi numeral symbols included various lines and curves to represent different numbers. They had symbols for numbers from 1 to 9, and a symbol for zero, making it a decimal system. The Kharosthi script and numerals have been found on inscriptions, coins, and other ancient artifacts.

4. Decimal System: The most significant contribution of India to the world of mathematics is the decimal numeral system. This system is based on the number ten and uses ten symbols (0-9) to represent all possible numbers. The concept of zero (0) as a placeholder and as a number itself was developed in India. The decimal system, with its place-value notation, simplifies arithmetic calculations and is the foundation of modern mathematics.

5. Invention of Zero: The concept of zero as both a placeholder and a number is attributed to Indian mathematicians. The use of zero revolutionized mathematics and made complex calculations much more manageable

6. Influence on Other Cultures: The decimal system with zero travelled from India to various parts of the world, including the Islamic world and, eventually, Europe. Islamic scholars like Al-Khwarizmi played a crucial role in transmitting Indian mathematical knowledge to the West, where it became the basis for modern mathematics.

7. Other Numeral Systems: India also had regional numeral systems. e.g. The Gupta numeral system was used in the Gupta Empire (around 4th to 6th century CE). These systems had unique symbols for numbers but were gradually replaced by the more efficient decimal system.

In summary, India's contribution to the development of the decimal numeral system with zero is one of its most significant intellectual achievements. This system has had a profound and lasting impact on mathematics and has become an integral part of global numerical and mathematical notation.

Ancient Indian Mathematician and Their Salient Contributions

	https://mathigon.org/timeline 		
Sr. No.	**Detail of the Work/ Mathematician**	**Period, Location**	**Salient Contributions**
1	**Vedic Texts**	3000BCE or earlier	The earliest recorded mathematical knowledge, number system, Pythagorean type triplets; Decimal system of naming numbers, the concept of infinity.
2	**Sulba-sutras-** Baudhayana, Apasthamba, Katyayana and Manava Sulba-sutras	800-600 BCE	Earliest text of geometry; Approximate value of the square root of 2, and π. Exact procedures for the construction and transformations of squares, rectangles, trapezia, ect.

3	**Aryabhata**-Aryabhatiyam	476-550 CE; Kusumapura, near Pataliputra, Bihar	Concise verses; Algorithm for square root, cube root, Place value system; Sine table; geometry; quadratic equations; Linear indeterminate equations; Sums of squares and cubes of numbers; Planetary astronomy; Plane and spherical trigonometry.
4	**Varaha Mihira**- Brhat Samhita, Brhat-jataka, Panca-siddhantika	482-565 CE; Ujjain, Madhya Pradesh	Summary of five ancient siddhantas; Sine table, trigonometric identities; $\sin^2 + \cos^2$; Combinatorics; Magic squares.
5	**Bhaskara I**- Commentary on Aryabhatiya, Laghu-bhaskariyam and Maha-bhaskariyam.	600-800 CE; Vallabhi region, Saurashtra, Gujarat	Expanded Aryabhata's work on Integer solution for indeterminate equations; Approximate formula for the sine function, Planetary Astronomy.
6	**Brahmagupta**- Brahmasphuta-siddhanta, Khandakhadyaka	598-668 CE; Bhillamala in Rajasthan	Rules of arithmetic operations with zero and negative numbers, Algebra (Bijaganita); linear and quadratic indeterminate equations; Pythagorean triplets, Formula for the diagonals and area of a cyclic quadrilateral; notion of arithmetic mean.
7	**Mahaviracarya**- Ganita-sara- sangraha	800-870 CE; Gulbarga, Karnataka	A comprehensive, exclusive textbook on mathematics covering arithmetic-geometry-algebra, continuing the ancient Jain mathematics tradition; permutations and combinations; arithmetic and geometric series; the sum of squares and cubes of numbers in arithmetic progression.
8	**Shripati**- Ganita-tilaka, Siddhanta-sekhara, Dhikotidakarana, ect.	1019-1066 CE; Rohinikhanda, Maharashtra	Planetary Astronomy

9	**Bhaskaracarya (Bhaskara-II)**- Lilavati on arithmetic and geometry; Bijaganita on algebra; Siddhanta-siromani on astronomy; Vasanabhasya on Siddhanta-siromani.	1114-1185 CE; Hailed from Bijjadavida	Canonical textbooks used all over India, Detailed explanations including Upapatti (demonstration or proof); addition formula for sine function, Surds; permutations and combinations; Solution for indeterminate equations, Ideas of calculus, including mean value theorem, planetary astronomy; construction of several instruments.
10	**Kamalakara**- Siddhanta-tattva-viveka	1616-1700 CE; Varanasi, Uttara Pradesh	Addition and subtraction theorems for the sine and the cosine; Sines and cosines of double, triple, etc., angles.

Origin of Number Systems in India : The roots of the Indian number system can be traced back to the Vedic period (circa 1500–500 BCE).

The Rigveda, one of the earliest Indian texts, contains references to numbers and arithmetic operations used in rituals and measurements.

Numerical concepts were deeply embedded in religious and cultural practices, such as the construction of altars with precise geometrical shapes and dimensions.

During the later Vedic period, the Shulba Sutras (texts related to geometry and mathematics) demonstrated advanced numerical skills, including fractions, proportions, and square roots.

These texts indicate that ancient Indians had a profound understanding of numbers well before the formalization of the decimal system.

Decimal System and Place Value Notation :

The most significant contribution of ancient India to the world of numbers is the decimal place value system.

This system, which uses ten symbols (0–9), revolutionized arithmetic, making calculations simple and efficient. The earliest evidence of the decimal system appears in inscriptions from the 3rd century BCE, such as the Ashokan Edicts.

The concept of zero (śūnya), first formalized in India, was a groundbreaking innovation. The Indian mathematician Brahmagupta(598–668 CE) was the first to describe zero as a number and laid down rules for arithmetic operations involving zero in his seminal work Brahmasphuta siddhanta.

Zero, coupled with the place value system, allowed for infinite scalability in numerical representation and calculations.

The Indian system of numbers was later transmitted to the Islamic world through scholars like Al-Khwarizmi and Al-Samawal, and eventually reached Europe, where it became the foundation of modern mathematics.

Large Numbers :

The Shukla Yajurveda (17.2) [Chapter 17 mantra 2], has a list of names for powers of ten up to 1012. The list given in the Yajurveda text is:

eka (1)

daśa(10)

mesochi/ shatam(100)

sahasra(1000)

ayuta(10000)

niyuta(100000)

prayuta(1000000)

arbuda(10000000)

nyarbuda(100000000)

saguran(1000000000)

madhya(10000000000)

anta (100000000000)

parârdha(1000000000000)

During Rigveda time, the Sanskrit language has unique names for the numbers:

1-ekam, 2-dve,3- tr"i" ̇ni, 4-catv"a" ̄ri, 5-pa"n" ̄ca, 6-sat, 7-sapta, 8-asta, 9-nava

There are unique names for numbers from 10 to 100 in steps of ten (dasa, vimsati, trimsat, catvarimsat, pancasat, sasti, saptati, asiti, navati, sata).

Later Hindu and Buddhist texts have extended this list, but these lists are no longer mutually consistent and names of numbers larger than 108 differ between texts.

For example, the Panchavimsha Brahmana lists 109 as nikharva, 1010 vâdava, 1011akṣiti, while ŚâṅkhyâyanaŚrauta Sûtrah as 109 nikharva, 1010samudra, 1011 salila, 1012 antya, 1013 ananta. Such lists of names for powers of ten are called daśaguṇottarra saṁjñâ.

Entire universe measured in meters and then squared would be around 10^53.

This is just one of a series of counting systems that can be expanded geometrically. The last number at which he arrived at after going through nine successive counting systems was10421, that is, a 1 followed by 421 zeros!

Siddharth mentioned that this number tallakshana forms part of one counting system. He further went on to show that eight more such counting systems are available to enumerate higher numbers. In each system there are 23 terms starting from the previous system.

Text	Context	Number quoted
Lalitavistara-sutra	Siddhartha during swayamvara	10^{421}
Kaccayana's Pali Grammar		10^{140}
Ramayana	Size of Rama's army	10^{62}
Jain canonical works	Sirsaprahelika-an estimate of time	$(8,400,000)^{28}$
Anuyogadvara-sutra	Jain canonical text	10^{28}
Taittiriya upanisad	An inquiry into bliss	10^{21}
Lilavati	Description of place value number system	10^{17}

Three Systems of Numeration

Numeration can be defined as the process of expressing numbers in words, alphabets, sentences or verses. Different kind of mathematical formulae and calculations and well-defined number systems and units of measurement for length, weight, time and motion, etc. were used in Astronomy.

Bhūtasaṅkyā System: Represents numbers by equivalent names. Large numbers were represented by using decimal system.

Kaṭapayādi System: Uses ten symbols, each with a place and absolute value.

Āryabhaṭīya System: Assigns a different absolute value to each alphabet.

Bhūtasaṅkyā System

Definition: The term "Bhūtasaṅkhyā" combines "bhūta" (being) and "saṅkhyā" (number), which translates to "number associated with a being."

https://sanskrit.iitk.ac.in/jnanasangraha/sankhya/bhoota

Categories of Entities Used in Bhūta-Saṃkhyā

Number Names: Words for the numbers themselves, such as Shunya (0), Ekam (1), Dvi (2), Trini (3), Chatwaari (4), and so on.

Physical Entities: Earth, moon, stars, mountain, fire, sky, etc.

Animals: Elephant (representing 8), horse, snake (also representing 8).

Body Parts: Eyes, limbs, 7 Dhatus (constituents of the body).

Gods: Shiva, Vishnu, Indra, Rama (representing 3), Manu (representing 14), Agni (representing 3).

Concepts: Seasons (6), months (12), days,5 Bhutas (elements).

Bhūta-saṃkhyā system

Number	Represented by (partial list only)*
0	Śūnya, Pūrṇa, Kha
1	Ādi, Candra, Prithivī, Eka
2	Aśvin, Pairs Of Limbs, Ayana, Dvandva, Dvi
3	Rāma, Guṇa, Loka, Kāla, Agni, Trinetra
4	Veda, Sṛuti, Yuga, Āśrama, Varṇa, Samudra, Kṛta
5	Bhūta, Śāstra, Bāṇa, Pāndava, Indriya
6	Aṅga, Ṛtu, Darśana, Ṣanmukha, Ṣaṭ
7	Ṛṣi, Adri, Svara, Dhātu, Chandas
8	Vasu, Bhujaṅga, Siddhi, Dik, Kuñjara, Nāga
9	Gṛha, Aṅka, Nanda
10	Dik, Aṇgulī, Avatāra, Rāvaṇaśira
11	Rudra
12	Āditya, Rāśi
13	Viśva, Kāma
14	Manu
15	Tithi, Dina

अङ्कानां वामतो गतिः. "Ankaanaam Vamatho Gatihi"

(meaning "**start from the right side and move in reverse order**").

Examples :

1. नेत्र ख भूमि शशि सर्प "**Netra** kha **bhūmi** śaśi **sarpa**"
   2 0 **1** 1 **8** ➔ Required num is **81102**

https://sanskrit.iitk.ac.in/jnanasangraha/sankhya/bhoota

2. **veda-vedāṅka-candrāḥ** : Let us split the words and associate the numbers to them. Veda (4); Veda (4): Anka(9); Candra(1).

   Therefore, the number is **1944.**

3. **khādri-rāmāgnayaḥ:** Kha (0); Adri (7); Râma (3); Agni (3).

   Therefore, the number is **3370.**

4. bhujanga-nanda-dvi-naga-anga-bana-sat-krita-indavaḥ: bhujanga (8), nanda (9), dvi (2), naga (synonym of adri) (7), anga (6), bâṇa (5), sat(6), krta (4), indu (1). Therefore, the number is **146,567,298.**

Kaṭapayādi System

Katapayadi System - Dr.K. Ramasubramanian : https://www.youtube.com/watch?v=z27TWfYeoHI

Katapayadi - an ancient Indian art of encoding : https://www.youtube.com/watch?v=QxTjaK8suMo&t=611s

Simulator : https://sanskrit.iitk.ac.in/jnanasangraha/sankhya/katapayaadi

The **ka·ṭa·pa·yā·di** system (Devanagari: कटपयादि) is an ancient Indian alphasyllabic numeral system to depict letters to numerals for easy rememberance of numbers as words or verses. Assigning more than one letter to one numeral and nullifying certain other letters as valueless, this system provides the flexibility in forming meaningful words out of numbers which can be easily remembered

नञावचश्च शून्यानि सङ्ख्याः कटपयादयः।
मिश्रे तूपान्त्यहल् सङ्ख्या न च चिन्त्यो हलस्वरः॥

~ सद्रत्नमाला (शङ्करवर्मणः)

Katapayadi is a mnemonic numeral system from classical India that encodes numbers into syllables (consonant+vowel units) so that numeric values can be embedded within ordinary Sanskrit (and later Malayalam, Kannada, Tulu) words and verses.

1	2	3	4	5	6	7	8	9	0
क	ख	ग	घ	ङ	च	छ	ज	झ	ञ
ट	ठ	ड	ढ	ण	त	थ	द	ध	न
प	फ	ब	भ	म	-	-	-	-	-
य	र	ल	व	श	ष	स	ह	ळ	-

Examples :

राघवाय *:* र् + आ + घ् + अ + व् + आ + य् + अ

र्	घ्	व्	य्	अङ्कानाम् वामतो गतिः
2	4	4	1	→ 1442

भवति *:* भ् + अ + व् + अ + त् + इ

भ्	व्	त्	अङ्कानाम् वामतो गतिः
4	4	6	→ 644

A famous example

In the **kaṭapayādi** system, this evaluates to *314159265358979324*, denoting the value of **π** (*Pi*) upto 17 decimals.

भद्राम्बुद्धिसिद्धजन्मगणितश्रद्धा स्म यद् भूपगी:

~ सद्रत्नमाला (शङ्करवर्मणः)

Decode text

भद्राम्बुद्धिसिद्धजन्मगणितश्रद्धा स्म यद् भूपगी:

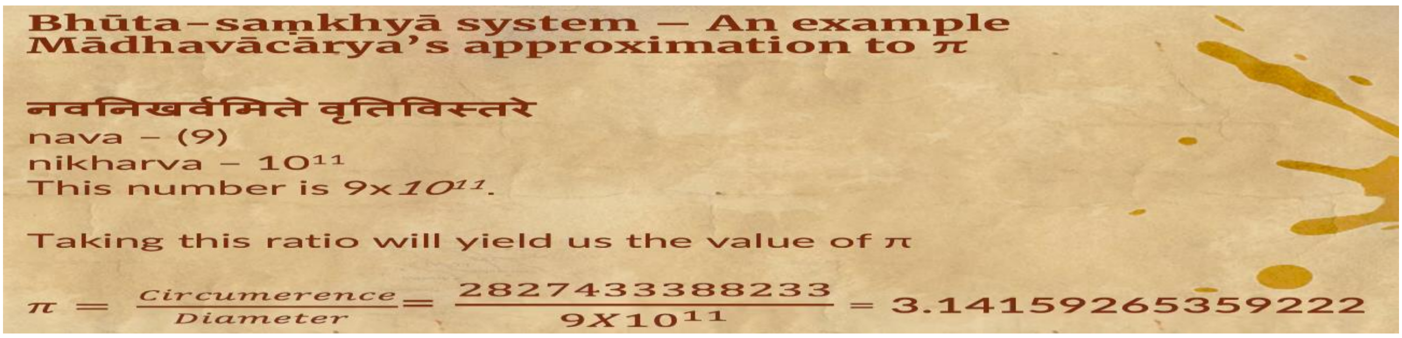
Sadratnamālā Variant   Kerala Variant

Submit

Kaṭapayādi Number

314159265358979324

Split	भ	द्	रा	म्	बु	द्	धि	सि	द्	ध	ज	न्	म	ग	णि	त	श्	र	द्	धा	स्	म	य	द्	भू	प	गी
Relevant	भ		र		ब		ध	स		ध	ज		म	ग	ण	त		र		ध		म	य		भ	प	ग
Numbers	4		2		3		9	7		9	8		5	3	5	6		2		9		5	1		4	1	3

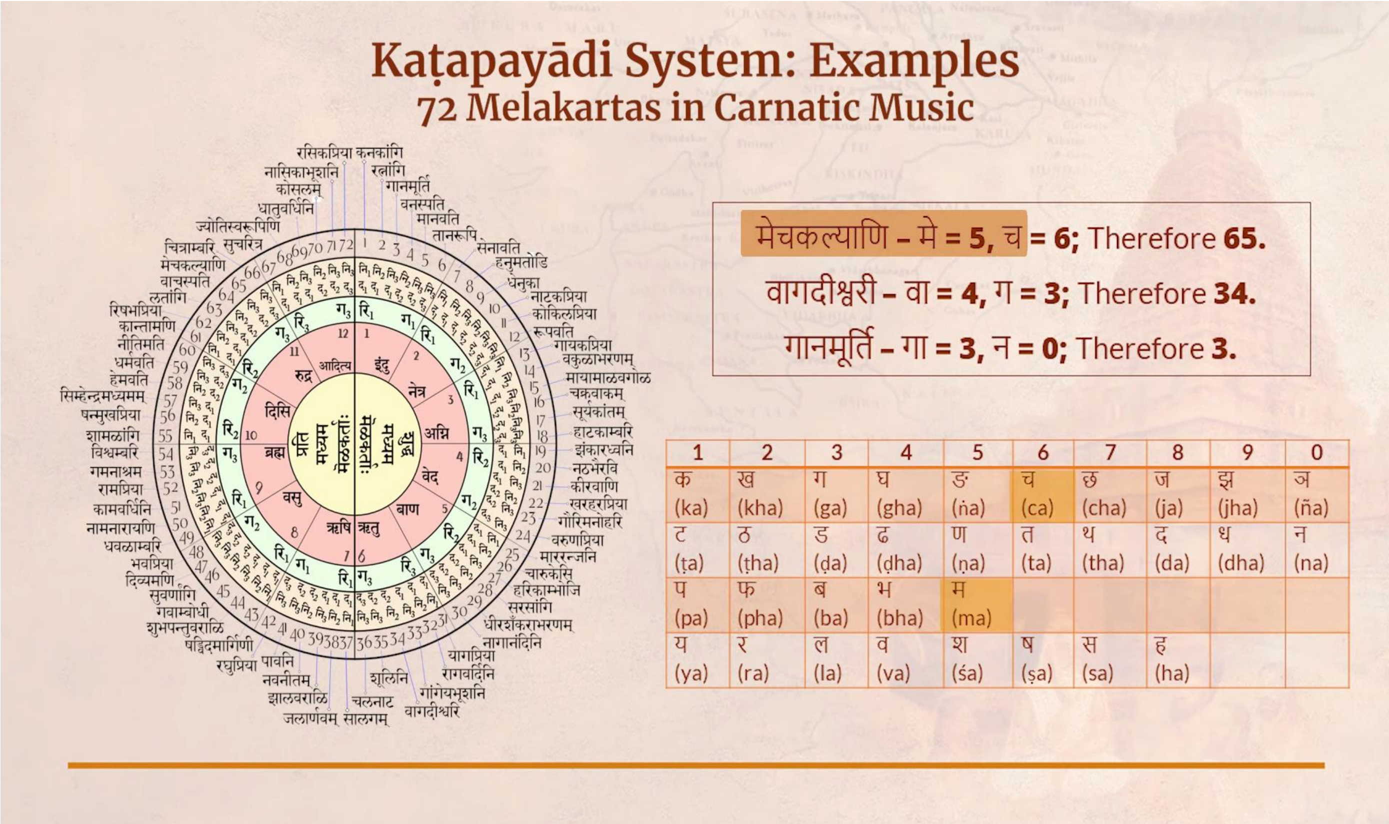


https://notes.collegehive.in/books/indian-knowledge-system/page/ashtadhyayi-a-brief#bkmrk-panini-and-a%C5%9F%C8%9B%C4%81dhy%C4%81y

The first two syllables of the name of each melakartha raga have been so ingeniously and dexterously fitted in as to make them subserve the purposes of this formula.

Shastriya Raga : Swarajathi | Mechakalyani Raga | Adi Tala | Sa Ni da pa pa ma pa ga ma

https://www.youtube.com/watch?v=clYNFyD7hj8&list=RDclYNFyD7hj8&start_radio=1



Āryabhaṭa System

https://sanskrit.iitk.ac.in/jnanasangraha/sankhya/aaryabhatiya

वर्गाक्षराणि वर्गेऽवर्गेऽवर्गाक्षराणि कात् ङ्मौ यः ।

खद्विनवके स्वरा नव वर्गेऽवर्गे नवान्त्यवर्गे वा ॥

It attributes a numerical value to each syllable of the form consonant + vowel possible in Sanskrit phonology, from *ka* = 1 up to *hau* = 10^{18}

Varga consonants, i.e., all alphabets from क (ka) to म (ma) each take a numerical value of 1 to 25 respectively. For the **avarga consonants**, य takes the numerical value of ङम (ṅama) (which is ङ (ṅ = 5) + म (m = 25) = 30).

9 vowels are duplicated to make varga — avarga pairs. Each vowel, has a varga and avarga form and these 9 vowels denote the places in the **place value system**

क्/k 1	ख्/kh 2	ग्/g 3	घ्/gh 4	ङ्/ṅ 5	च्/c 6	छ्/ch 7	ज्/j 8	झ्/jh 9	ञ्/ñ 10
ट्/ṭ 11	ठ्/ṭh 12	ड्/ḍ 13	ढ्/ḍh 14	ण्/ṇ 15	त्/t 16	थ्/th 17	द्/d 18	ध्/dh 19	न्/n 20
प्/p 21	फ्/ph 22	ब्/b 23	भ्/bh 24	म्/m 25					
य्/y 30	र्/r 40	ल्/l 50	व्/v 60	श्/ś 70	ष्/ṣ 80	स्/s 90	ह्/h 100		

अ / आ	इ / ई	उ / ऊ	ऋ/ /ॠ	ऌ	ए	ओ	ऐ	औ
a / ā	i / ī	u / ū	ṛ / ṝ	lṛ	e	o	ai	au
10^0	10^2	10^4	10^6	10^8	10^{10}	10^{12}	10^{14}	10^{16}

Rule	Meaning
वर्गाक्षराणि वर्गे	Varga letters (i.e., क to म) appear in the varga places (even powers of 10 like units, hundreds, etc.)
अवर्गे अवर्गाक्षराणि	Avarga letters (य to ह) appear in avarga places (odd powers of 10 like tens, thousands, etc.)
कात्	Letters are assigned values 1, 2, ... starting from क
ङ्मौ यः	ङ and म are equivalent to य (i.e., value 30)
खद्विनवके स्वरा नव वर्गे अवर्गे नव अन्त्यवर्गे वा	The nine vowels अ, इ, उ, ऋ, ऌ, ए, ऐ, ओ, औ represent powers of ten and combine with consonants. They appear in both varga and avarga places and beyond if needed.

Examples :

गुण (Guna) gu = 3 (vowel) × 10^4 (consonant) = 30000, ṇa = 15 (vowel) × 10^0 (consonant) = 15

Total: 30000 + 15 = **30015**

Decode text

गुण

Submit

Split	ग	उ	ण	अ
Numbers	3	10^4	15	10^0

Āryabhaṭīya Number
30015

मखि (Makhi) 25 + 200 = 225	**Split** म अ ख इ **Numbers** 25 10^0 2 10^2

Split	म	अ	ख	इ
Numbers	25	10^0	2	10^2

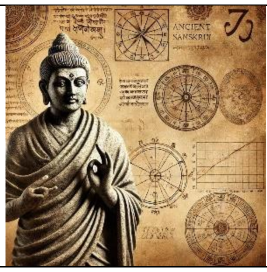
ख्यघु	Calculation
ख् = 2, य् = 30, उ = 10^4 घ् = 4, ऋ = 10^6	$(2 \times 30) \times 10^4$ $+ 4 \times 10^6$ $= 4320000$

***Yuga* cycle**

Yuga	Start (– End)	Length
Krita (Satya)	3,891,102 BCE	1,728,000 (4,800)
Treta	2,163,102 BCE	1,296,000 (3,600)
Dvapara	867,102 BCE	864,000 (2,400)
*Kali**	3102 BCE – 428,899 CE	432,000 (1,200)
	Years: 4,320,000 solar (12,000 divine)	

Piṅgala and the Binary system

Pingala, an Indian scholar from the **2nd Century BCE**, and his contribution to the **development of the binary system**. While the use of binary numbers is primarily associated with modern computing, Pingala's work on **Chandah-shastra**, a treatise on prosody, contains the fundamental principles of the binary system.



Chandah-shastra and the Hierarchical Structure of Meter

Chandah-shastra deals with the rules of prosody, which govern the meter and structure of poetic compositions. The meter of a poem has a hierarchical structure, starting with syllables, which are then grouped into padas, and finally combined to form the meter.

A syllable, the basic building block of poetry, is defined as a vowel or a vowel with one or more consonants preceding it.

Laghu and Guru: The Binary Building Blocks

Pingala classified syllables into two types: Laghu (short syllable) and Guru (long syllable).

A **Laghu** is any syllable with a **short vowel**.

A **Guru** is any syllable with a **long vowel**, a short syllable followed by a conjunct consonant, a short syllable followed by an anusvara or visarga, or the last syllable in a meter.

By assigning the number **1 to Laghu** and **0 to Guru**, we can represent the syllables in binary form.

https://notes.collegehive.in/books/indian-knowledge-system/page/ashtadhyayi-a-brief#bkmrk-panini-and-a%C5%9F%C8%9B%C4%81dhy%C4%81y

Ganas: Binary Words of Length 3

Pingala further defined groups of three syllables called Ganas, which are essentially binary numbers of length 3.

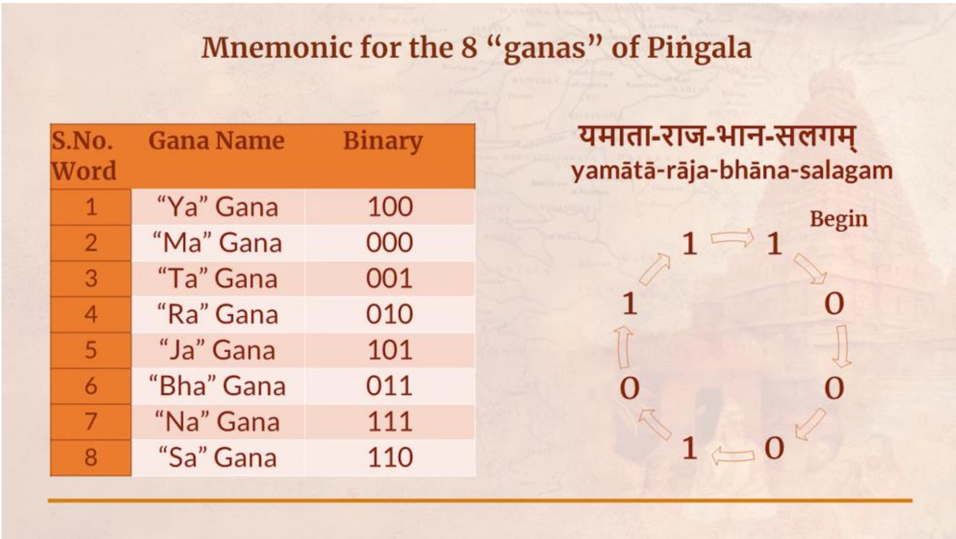
There are **8 possible Ganas**, representing all combinations of Laghu and Guru (or 0 and 1).

These Ganas were used to establish various rules for Chandah-shastra.

Mnemonic for Ganas: Yamata-raja-bhana-salagam

A mnemonic phrase, "yamata-raja-bhana-salagam," was used to remember the 8 Ganas.

Each consecutive group of three letters in the phrase represents a Gana.



S.No. Word	Gana Name	Binary
1	"Ya" Gana	100
2	"Ma" Gana	000
3	"Ta" Gana	001
4	"Ra" Gana	010
5	"Ja" Gana	101
6	"Bha" Gana	011
7	"Na" Gana	111
8	"Sa" Gana	110

https://sanskrit.iitk.ac.in/jnanasangraha/chanda/text

यदा यदा ही धर्मस्य ग्लानिर्भवति भारत अभ्युत्थानमधर्मस्य तदात्मानम सृज्याहम

<table>
<tr><th>Akṣarāṇi</th><td>य</td><td>दा</td><td>य</td><td>दा</td><td>ही</td><td>ध</td><td>र्म</td><td>स्य</td><td>ग्ला</td><td>नि</td><td>र्भ</td><td>व</td><td>ति</td><td>भा</td><td>र</td><td>त</td><td>अ</td><td>भ्यु</td><td>त्था</td><td>न</td><td>म</td><td>ध</td><td>र्म</td><td>स्य</td><td>त</td><td>दा</td><td>त्मा</td><td>न</td><td>म</td><td>सृ</td><td>ज्या</td><td>ह</td><td>म</td></tr>
<tr><th>Laghu-Guru</th><td>ल</td><td>ग</td><td>ल</td><td>ग</td><td>ग</td><td>ग</td><td>ग</td><td>ग</td><td>ग</td><td>ग</td><td>ल</td><td>ल</td><td>ल</td><td>ग</td><td>ल</td><td>ल</td><td>ग</td><td>ग</td><td>ग</td><td>ल</td><td>ल</td><td>ग</td><td>ग</td><td>ल</td><td>ल</td><td>ग</td><td>ग</td><td>ल</td><td>ल</td><td>ग</td><td>ग</td><td>ल</td><td>ल</td></tr>
<tr><th>Gaṇa</th><td colspan="3">ज</td><td colspan="3">म</td><td colspan="3">म</td><td colspan="3">भ</td><td colspan="3">ज</td><td colspan="3">य</td><td colspan="3">भ</td><td colspan="3">त</td><td colspan="3">य</td><td colspan="3">स</td><td colspan="3">भ</td></tr>
<tr><th>Counts</th><td colspan="33">33 अक्षराणि, 51 मात्राः</td></tr>
</table>

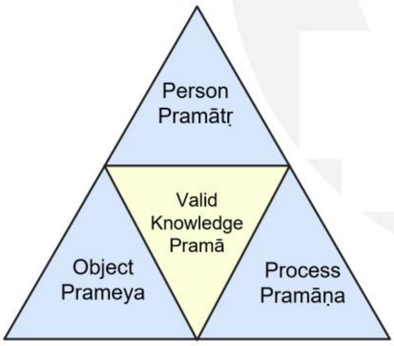
Binary Cycle and de Bruijn Sequence

The arrangement of Ganas in Pingala's work forms a binary cycle of length 3.

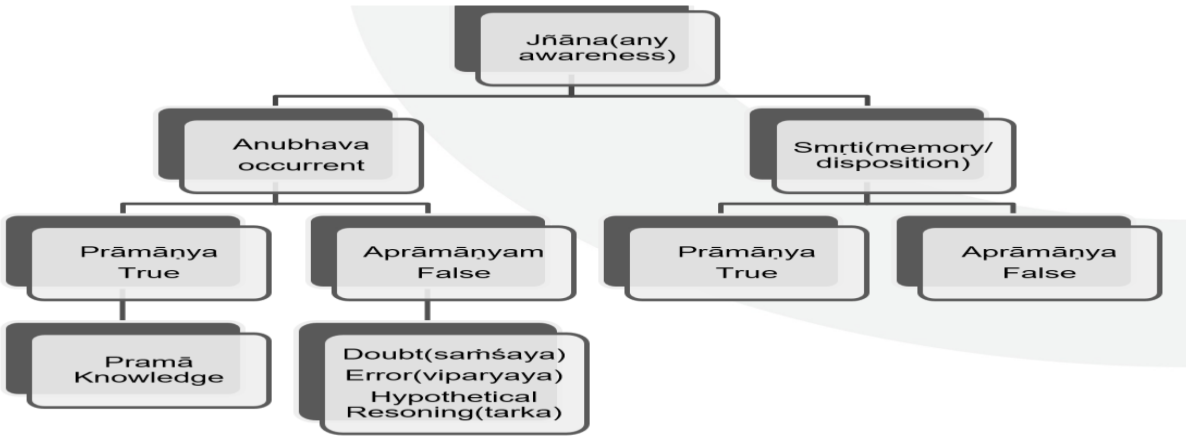
This binary cycle is known as the de Bruijn sequence in **modern computer science**.

Framework & Classifications: Prameya, Pramana, Samsaya.

- All knowledge systems are founded on some philosophical (metaphysical + epistemological + ethical) assumptions
- Indian Knowledge Systems (IKS) are no different
- "kāṇādaṃpāṇinīyaṃca sarvaśāstropakārakam"

Pramāṇa Śāstra (Epistemology)	
	**What is knowledge?** As per Indian Philosophy Knowledge is a bank of true awareness acquired by Mode of inquiry

Nyaya theory of Cognition: The conception of knowledge



Pramāṇas : Means to Valid Cognition

Pramāṇas are the instruments (strongest means) of acquiring true knowledge.

- Pratyakṣa – Perception
- Anumāna – Inference
- Upamāna – Analogy
- Śabda – Verbal Testimony

Pratyakṣa (Perception) Process

It is knowing an object by direct contact through our 5 external senses such as vision (cakṣu), hearing (śrotra), taste (rasana), smell (ghrāṇa), and touch (tvak) and 6th sense Manas(Mind). This allows us to perceive inner states like happiness (sukha), sadness (duḥkha), desire (icchā), or even cognition itself (buddhi/jñāna).	आत्मा मनसा संयुज्यते । मनः इन्द्रियेण, इन्द्रियम् अर्थेन । ततः प्रत्यक्षम् । Ātmā manasā saṁyujyate । Manaḥ indriyeṇa, indriyam arthena । Tataḥ pratyakṣam ।

Ātman → Manaḥ → Indriya → Artha

Anumāna (Inference)

Anumāna is the process by which we arrive at a cognition based on another cognition. It is knowledge that follows from prior understanding.

An inference has four logical elements:

1. Hetu (The sign/reason: the property which is known)
2. Sādhya (The unknown: the property we want to infer)
3. Vyāpti (The invariable concomitance relating the hetu and the sādhya)
4. Pakṣa (Locus: The place where one wants to prove the sādhya)

Pañca-Avayava Anumāna (The Five-Step Reasoning Process for logical coherence**)**

Anumāna(Inference)

Example : Force has been imparted to this ball. Because it is moving.

Sādhya= having the property of being subjected to force

Pakṣa= the ball

Hetu= having motion

Vyāpti= Wherever there is motion, there is the property of being subjected to force

Pratijñā: The ball has the property of being subjected to force

Hetu: Because it has motion

Udāharaṇa: Wherever there is motion, there is the property of being subjected to force, like a cart.

Upanaya: This ball is also so.

Nigamana: Hence, the ball has the property of being subjected to force

Śabda (verbal testimony)- Knowledge through language

āptopadeśaḥśabdaḥ : The assertion of a reliable person (āpta) is word (NS 1.1.7)

Śabda, *is the knowledge derived from the utterances (oral or written) of a reliable source (āpta), which applies both Empirical Reality and Transcendental Realms.*	***āpta :*** *Having right knowledge, Willingness to share and No intention to deceive,* *Free from attachment and aversion*

Upamāna: Analogy

Upamāna is the means of knowing the relationship between a word and its meaning through similarity or by comparison.

	• A person unfamiliar with the word **Gavaya (a wild animal)** is told that it resembles a cow. • Later, upon seeing an animal (e.g., a Neelgai) that resembles a cow, they recognize it as a Gavaya. • The key to upamāna is the recognition of similarity, which links the unknown term (gavaya) with the familiar concept (cow).