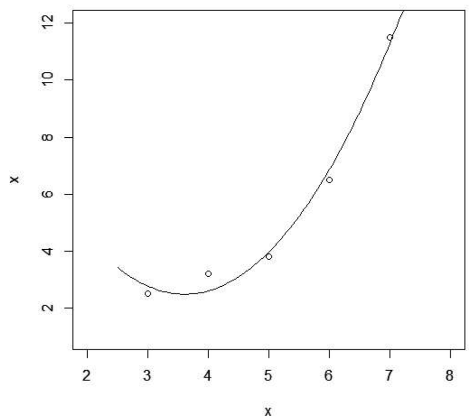

Figure 7.3: Plot of quadratic polynomial model

Solution

Let the quadratic regression model be

$$y = \alpha_0 + \alpha_1 x + \alpha_2 x^2.$$

The values of α_0, α_1 and α_2 which minimises the sum of squares of errors are a_0, a_1 and a_2 which satisfy the following system of equations:

$$\begin{aligned}\sum y_i &= n a_0 + a_1(\sum x_i) + a_2(\sum x_i^2)\\ \sum y_i x_i &= a_0(\sum x_i) + a_1(\sum x_i^2) + a_2(\sum x_i^3)\\ \sum y_i x_i^2 &= a_0(\sum x_i^2) + a_1(\sum x_i^3) + a_2(\sum x_i^4)\end{aligned}$$

Using the given data we have

$$\begin{aligned}27.5 &= 5a_0 + 25a_1 + 135a_2\\ 158.8 &= 25a_0 + 135a_1 + 775a_2\\ 966.2 &= 135a_0 + 775a_1 + 4659a_2\end{aligned}$$

Solving this system of equations we get

$$\begin{aligned}a_0 &= 12.4285714\\ a_1 &= -5.5128571\\ a_2 &= 0.7642857\end{aligned}$$

The required quadratic polynomial model is

$$y = 12.4285714 - 5.5128571x + 0.7642857x^2.$$

Figure 7.3 shows plots of the data and the quadratic polynomial model.

7.5 Multiple linear regression

We assume that there are N independent variables x_1, x_2, $\cdots$, x_N. Let the dependent variable be y. Let there also be n observed values of these variables:

Variables (features)	Values (examples)			
	Example 1	Example 2	$\cdots$	Example n
x_1	x_{11}	x_{12}	$\cdots$	x_{1n}
x_2	x_{21}	x_{22}	$\cdots$	x_{2n}
$\cdots$				
x_N	x_{N1}	x_{N2}	$\cdots$	x_{Nn}
y (outcomes)	y_1	y_2	$\cdots$	y_n

Table 7.3: Data for multiple linear regression

The multiple linear regression model defines the relationship between the N independent variables and the dependent variable by an equation of the following form:

$$y = \beta_0 + \beta_1 x_1 + \cdots + \beta_N x_N$$

As in simple linear regression, here also we use the ordinary least squares method to obtain the optimal estimates of $\beta_0, \beta_1, \cdots, \beta_N$. The method yields the following procedure for the computation of these optimal estimates. Let

$$X = \begin{bmatrix} 1 & x_{11} & x_{21} & \cdots & x_{N1} \\ 1 & x_{12} & x_{22} & \cdots & x_{N2} \\ \vdots & & & & \\ 1 & x_{1n} & x_{2n} & \cdots & x_{Nn} \end{bmatrix}, \quad Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}, \quad B = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_N \end{bmatrix}$$

Then it can be shown that the regression coefficients are given by

$$B = (X^T X)^{-1} X^T Y$$

7.5.1 Example

Example

Fit a multiple linear regression model to the following data:

x_1	1	1	2	0
x_2	1	2	2	1
y	3.25	6.5	3.5	5.0

Table 7.4: Example data for multi-linear regression

Solution

In this problem, there are two independent variables andfour sets of values of the variables. Thus, in the notations used above, we have $n = 2$ and $N = 4$. The multiple linear regression model for this problem has the form

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2.$$

The computations are shown below.

$$X = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & 2 & 2 \\ 1 & 0 & 1 \end{bmatrix}, \quad Y = \begin{bmatrix} 3.25 \\ 6.5 \\ 3.5 \\ 5.0 \end{bmatrix}, \quad B = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix}$$

$$X^TX = \begin{bmatrix} 4 & 4 & 6 \\ 4 & 6 & 7 \\ 6 & 7 & 10 \end{bmatrix}$$
$$(X^TX)^{-1} = \begin{bmatrix} \frac{11}{4} & \frac{1}{2} & -2 \\ \frac{1}{2} & 1 & -1 \\ -2 & -1 & 2 \end{bmatrix}$$
$$B = (X^TX)^{-1}X^TY$$
$$= \begin{bmatrix} 2.0625 \\ -2.3750 \\ 3.2500 \end{bmatrix}$$

The required model is

$$y = 2.0625 - 2.3750x_1 + 3.2500x_2.$$

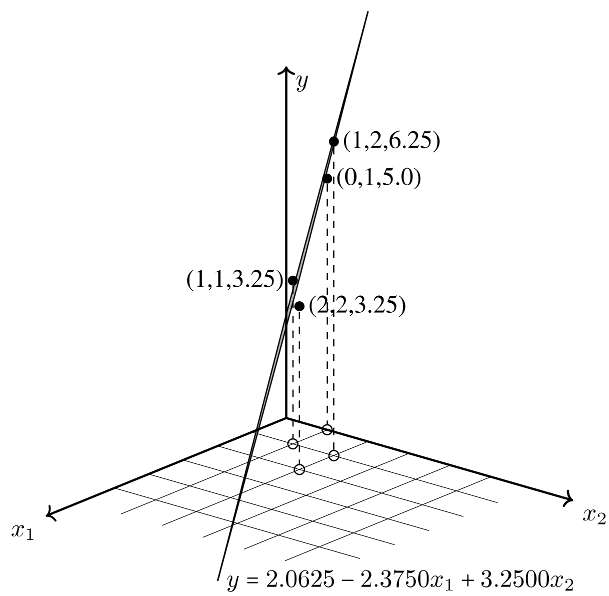

Figure 7.4: The regression plane for the data in Table 7.4

7.6 Sample questions

(a) Short answer questions

1. What are the different types of regression.
2. Is regression a supervised learning? Why?
3. Explain the ordinary least squares method for regression.
4. What are linear, multinomial and polynomial regressions.
5. If model used for regression is
$$y = a + b(x - 1)^2,$$
is it a multinomial regression? If not, what type of regression is it?
6. What does the line of regression tell you?