



ADITYA DEGREE COLLEGES

ANDHRA PRADESH

IV –SEMESTER, MID-I - EXAMINATIONS FEB 2025

BSC-MATHEMATICS - REAL ANALYSIS

Date : 14.02.2025

TIME: 2HRS

MAX MARKS: 50M

SECTION-A

I. Answer all the following questions.

3 x 10 = 30M

1. (a) Prove that a monotone Sequence is convergent if and only if it is bounded.

(or)

(b) State and Prove Cauchy's first Theorem on limits

Prove that $\lim_{n \rightarrow \infty} \frac{1}{n} [1+2^{1/2}+3^{1/3}+ \dots + n^{1/n}] = 1$

2. (a) State and Prove Cauchy's n^{th} root Test.

(or)

(b) Test for Convergence $\sum_{n=1}^{\infty} (-1)^{n+1} \sqrt{n+1} - \sqrt{n}$

3. (a) If f is continuous on $[a, b]$ then f is bounded on $[a, b]$

(or)

(b) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be such that

$$\begin{aligned} F(x) &= \frac{\sin(a+1)x + \sin x}{x} \quad \text{for } x < 0 \\ &= C \quad \text{for } x = 0 \\ &= \frac{(x+bx^2)^{\frac{1}{2}} - x^{1/2}}{bx^{3/2}} \quad \text{for } x > 0 \end{aligned}$$

SECTION-B

II. Answer any four of the following questions.

4 x 5 = 20M

4. Prove that the Sequence $\{S_n\}$ where $S_n = \frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{n+n}$ is Convergent

5. Prove that every Convergent Sequence is bounded.

6. Test for Convergence of $\sum_{n=1}^{\infty} \frac{1}{2^n + 3^n}$

7. Test for Convergence of $\sum_{n=1}^{\infty} \frac{n^4}{n!}$

8. Examine the Continuity of the function f defined by $f(x) = |x| + |x - 1|$ at $x = 0, 1$

9. Show that $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = x$ if $x \in \mathbb{R} - \mathbb{Q}$ and $f(x) = -x$ if $x \in \mathbb{Q}$ is Continuous only at '0'