ADITYA DEGREE COLLEGES :: ANDHRA PRADESH

MID-I Semester –IV

Mathematics-Ring Theory

Time:2 hrs. Marks: 60 M

SECTION-A **4 x 10 = 40 M**

I. Answer ALL questions:

1.(a)Prove that Every finite integral domain is a field.

(or)

(b)Prove that the set Z [i]= {a+bi/a,b$\in$z, i^2=-1} of Gaussian integers is an integral domain with respect to addition and multiplication of numbers. Is it a field?

2. (a)Prove that the characteristic of an integral domain is either a prime or zero.

(or)

(b) Prove that Q [$\sqrt{2}$]= {a+b $\sqrt{2}$ /a,b$\in$Q} is a field with respect to ordinary addition and multiplication of numbers.

3. (a) S is a non-empty subset of a ring R then S is a subring of R if and only if a-b$\in$S and ab$\in$S for all a,b$\in$S.

(or)

(b)i) Prove that the intersection of two subrings of a ring R is a subring of R.

(ii)Let R be a ring and a$\in$R be a fixed element. Then prove that S= { x$\in$R/ax=0} is a subring of R.

4. (a)If U_1 and U_1 are two ideals of a ring R then U_1 U U_2 is an ideal of R if and only if $U_1 \subset U_2$ or $U_2 \subset U_1$.

(or)

(b) Define an ideal of R. If U_1 ,U_2 are two ideals of a ring R then U_1+U_2= {x+y/x$\in U_1$, y$\in U_2$ } is also an ideal of R.

SECTION-B **5 x 4 = 20 M**

II.Answer any “FIVE” of the following:

5. Prove that A filed has no zero- divisors.

6. Prove that Every Boolean ring is abelian .

7.Prove that $\pm$1, $\pm$i are the only four units in the domain of Gaussian integers.

8. Prove that A ring R has no zero divisors if and only if the cancelation laws hold in R.

9. Show that the set of matrices $\begin{pmatrix} a & b \\ o & c \end{pmatrix}$ is a subring of the ring of 2x2 matrices whose elements are integers.

10.Prove that A field has no proper non- trivial ideals.

11) Prove that A commutative ring R with unity element is a field if R have no proper ideals.

12. Prove that the intersection of two ideals of a ring R is an ideal of R.