ADITYA DEGREE COLLEGES: AU REGION

(Affiliated to Andhra University)

IV SEM, MID-I EXAMINATIONS, FEB 2025

Dt: 8-2-2025 Course: B.Sc. Max. Marks: 60;

Time: 3 Hrs

MATHEMATICS(MINOR) Ring Theory

SECTION - A

Answer any five questions **5 x 4= 20 M**

1. The intersection of two sub rings is again a subring
2. A field has no zero divisions
3. If R is a ring with unity element then R has characteristic p>0 iff p is the least positive integer such p.1=0
4. If the characteristic of a ring is 2 then show that $(a+b)^2 = a^2 + b^2 =(a-b)^2$
5. An integral domain has no nilpotnet elements other than zero
6. A field has no proper nontrivial ideals
7. If R is a non zero ring so that $a^2 =a \ \forall a \in R$. Prove that characteristic of R=2
8. Find the characteristic of the rings i. 2 Z ii. ZxZ

SECTION - B

Answer any three questions **4 x 10= 40 M**

9. (a) The characteristic of field is either prime (or) zero ?

(or)

(b) If Z is the set of all integers and addition (+) , multiplication(X) are defined as $a + b = a+b -1$, $a \times b = a+b -ab \ \forall a, b \in Z$ then prove that (Z, +, X)) is a commutative ring?

10. (a) Let S be an non empty subset of ' R' then 'S' is said to be subring of R iff for $a, b \in S$
i. $a - b \in S$ ii. $a \cdot b \in S$

(or)

(b) Every finite integral domain is a field ?

11. (a) If U_1 , U_2 are two ideals then $U_1 \cup U_2$ is ideal iff $U_1 \subseteq U_2 (or) U_2 \subseteq U_1$

(or)

(b) i) Define ideal ii) The intersection of two ideal is again ideal

12. (a) Prove that the set $Z[i] = \{a + ib \mid a, b \in Z, i^2 = -1\}$ of Gaussian integers is an integral domain w. r. t addition and multiplication of numbers? is it a field?

(or)

(b) i. every field is integral domain
ii. A commutative ring R with unity element is a field if |R have proper ideals