ADITYA DEGREE COLLEGES

ANDHRA PRADESH

IV –SEMESTER, MID-I - EXAMINATIONS FEB 2025

BSC – MATHEMATICS - RING THEORY

Date : 13.02.2025

TIME: 2HRS MAX MARKS: 50**M**

SECTION-A

I.WRITE ALL QUESTIONS. 3x10=30M

1. A) Define Integral domain & field and prove that every finite integral domain is field.

   OR

   B) If Q($\sqrt{2}$)=$\{a + b\sqrt{2}/a, b \in Q\}$, then show that Q($\sqrt{2}$) is a field.

2. A) Define ideal and prove that If $U_1$and U_2 are two ideals of a ring R then $U_1 U U_2$ is an ideal of R if and only if $U_1 \subset U_2$ or $U_2 \subset U_1$

   OR

   B) Define a sub ring of ring R and prove that the Intersection of two subrings of a ring R is a sub ring of R.

3. A) Prove that the ring of integers Z is a principal Ideal ring.

   OR

   B) If U is an ideal type of a ring R then the set R/U=$\{x + U \ /x \in R\}$ is a ring with respect to the induced operation of addition (+) and multiplication(.) of cosets defined follows: (a+u)+(b+u) = (a+b)+u and (a+u).(b+u)=ab+u for a+u,b+u ∈ R/U.

SECTION-B

II. WRITE ANY FOUR QUESTIONS 4X5=20M.

4. Show that the field has no zero divisors.
5. If R is a Boolean ring then (i) a+a=0 $\nvdash \in R$
   ii) a+b=0$\Longrightarrow a = b$ and
   iii) R is commutative under multiplication.
6. Prove that A field has no proper non-trivial ideals.
7. If U1,U2 are two ideals of a ring R then U1+U2=$\{x + y/x \in U_1, y \in U_2\}$ is also an ideal of R.
8. A ring R has no zero divisions if and only if the cancellation laws hold in R.
9. Prove that the characteristic of an Integral domain is either a prime of Zero.