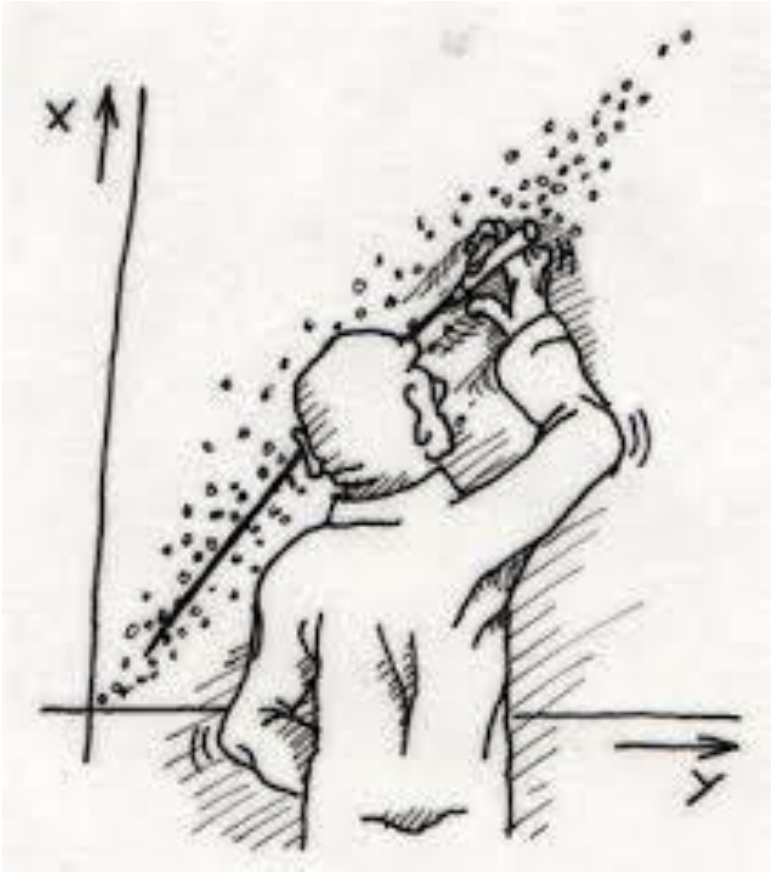
Regression Modelling



Interesting Hypothesis

- Vegetarians miss fewer flights.
- Women use camera phone more than men.
- Left handed men earn more money!
- Smokers are better sales people.
- Those who whistle at work place are more efficient.

Regression enables us to test few of these Hypotheses -

What is Regression?

Regression is a tool for finding **existence of an association** relationship between a dependent variable (Y) and one or more independent variables(X1,X2,…,Xn) in a study.

The relationship can be **linear** or **non-linear**.

- Mathematical relationship is an exact relationship.

$$Y = \beta 0 + \beta 1 X$$

- Statistical relationship is not an exact relationship.

$$Y = \beta 0 + \beta 1 X + e$$

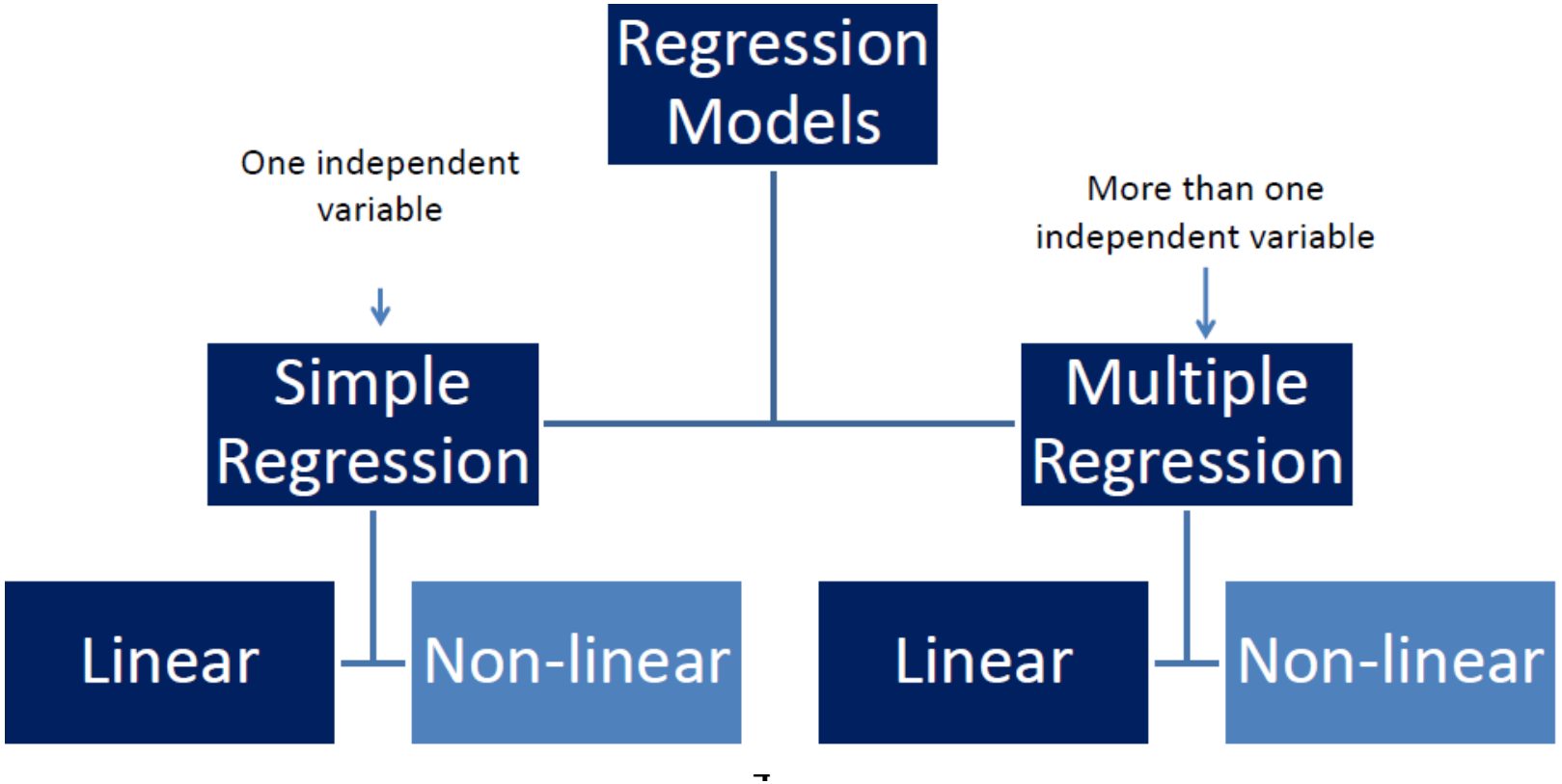
(This is the Population Regression Function)

Regression Nomenclatures

Dependent Variable	Independent Variable
Explained Variable	Explanatory variable
Regressand	Regressor
Predictand	Predictor
Endogenous Variable	Exogenous Variable
Controlled Variable	Control Variable
Target Variable	Stimulus Variable
Response Variable	

Dependent and Independent Variables

- Terms dependent and independent does not necessarily imply a causal relationship between two variables.
- Regression is **not** designed to capture **causality**.
- Purpose of regression is to **predict the value of dependent variable given the value(s) of independent variable(s).**

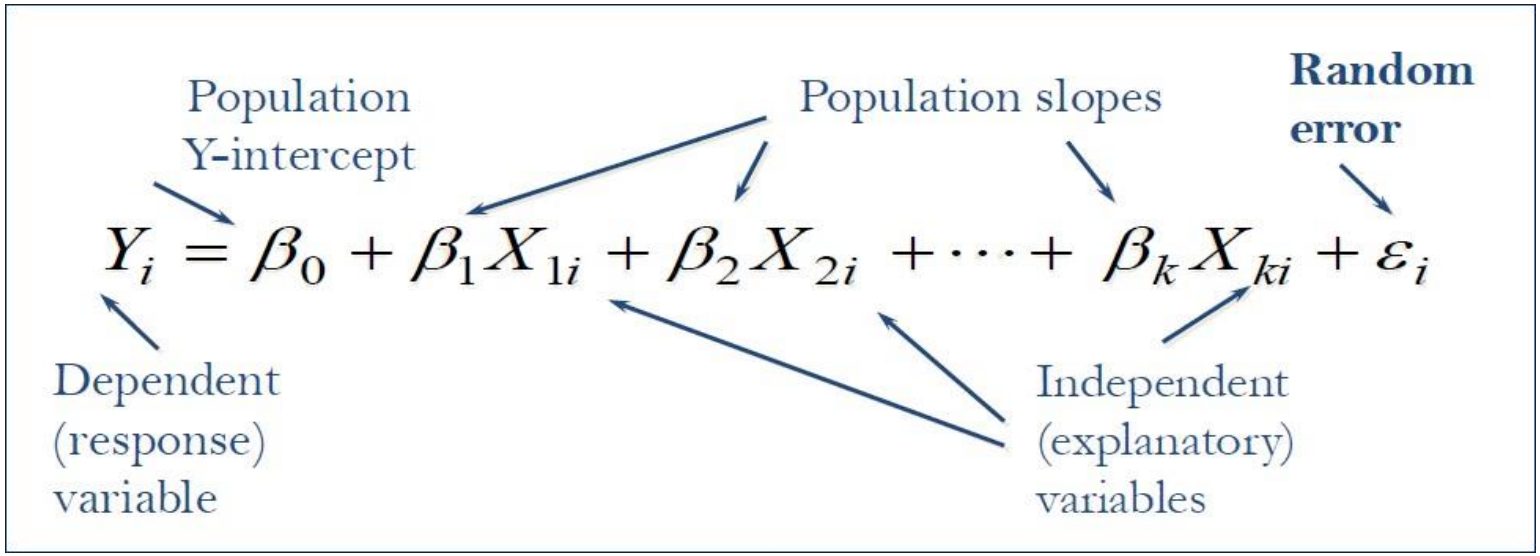


Types of Linear Regression

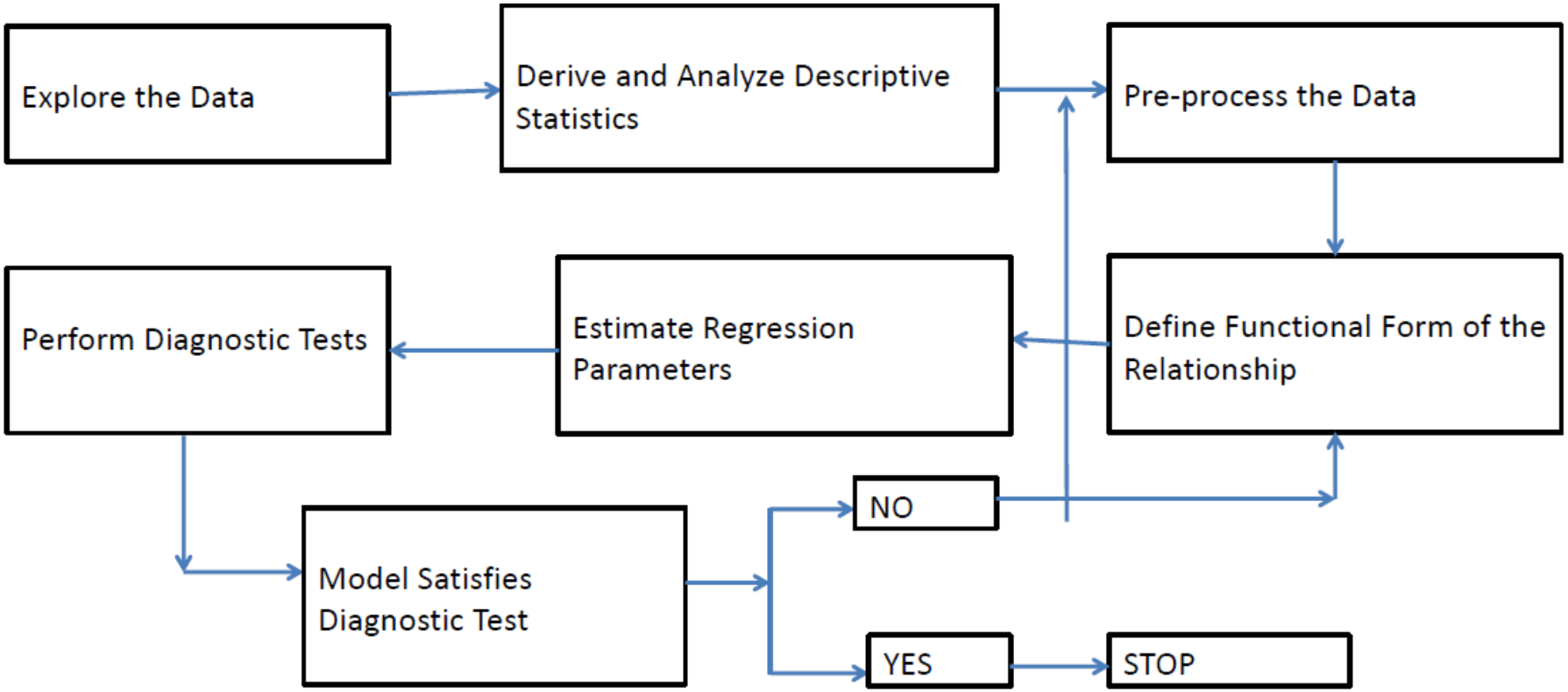
- Simple Linear Regression: $Y=B_0+B_1 X+e$
- Multiple Linear Regression: $Y=B_0+B_1 X+B_2 X_1+B_3 X_2+\ldots\ldots+B_k X_k+e$

Multiple Linear Regression Model (MLRM)

Relationship between 1 dependent & 2 or more independent variables is a **linear function**.

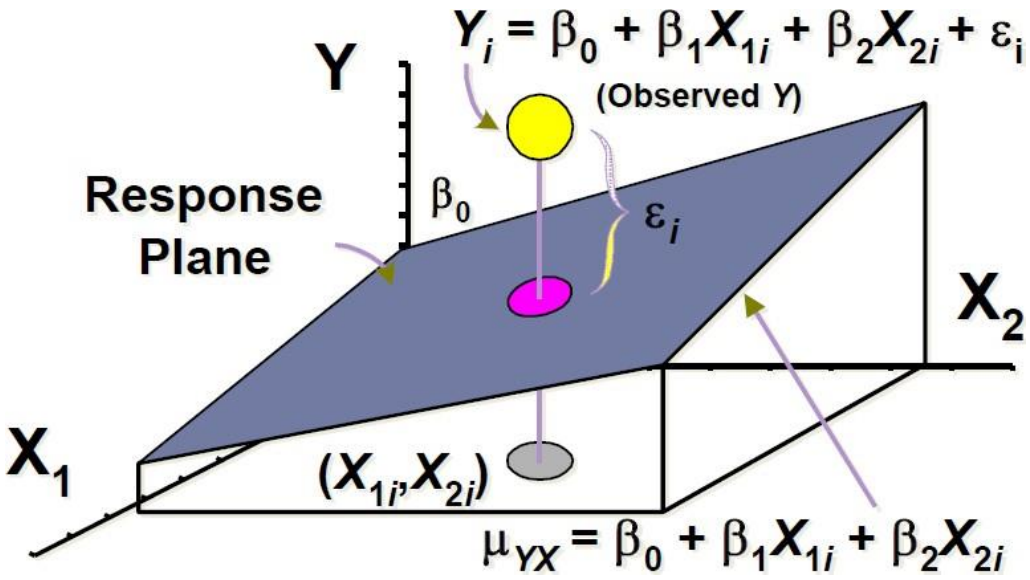


MLRM Development Prcoess



MLRM Prediction Equation

$$\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X_{1i} + \hat{\beta}_2 X_{2i} + \cdots + \hat{\beta}_k X_{ki}$$



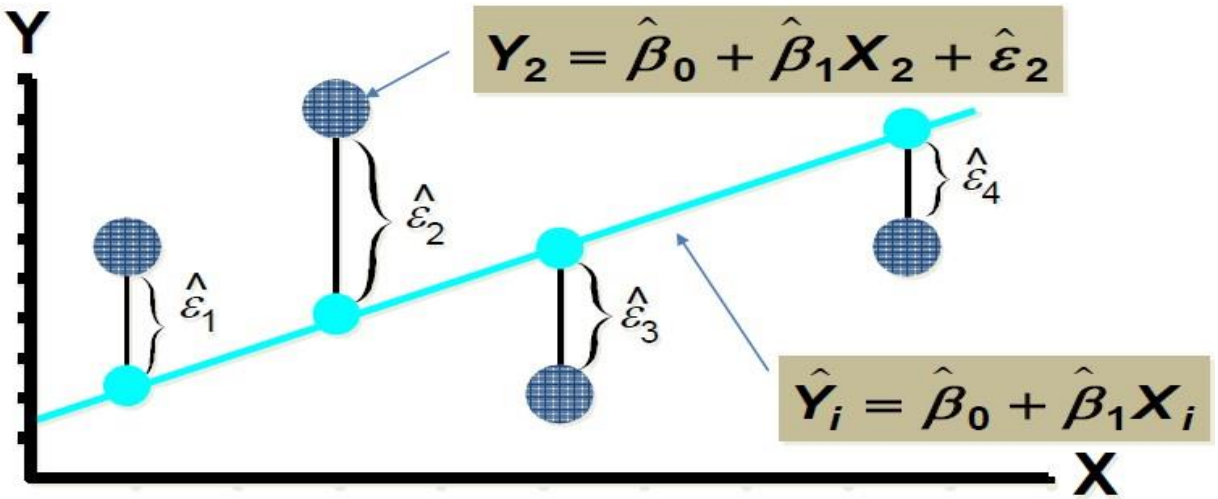
MLRM Model Assumptions

- The error term, e_i, follows a **normal distribution**.
- For different values of X, the variance of e_i is constant(**Homoscedasticity**).
- There is **no Multi-collinearity** (no perfect linear relationship among explanatory variables).
- There is **no autocorrelation** between two e_i values.

MLRM Methodology

Ordinary Least Square (OLS) Methodology, minimizes the sum of square of errors.

$$\text{LS minimizes} \sum_{i=1}^{n} \hat{\varepsilon}_i^2 = \hat{\varepsilon}_1^2 + \hat{\varepsilon}_2^2 + \hat{\varepsilon}_3^2 + ... + \hat{\varepsilon}_n^2$$



Interpretation of MLRM Coefficients

- The intercept,0, is the mean value of the dependent variable Y, when the independent variable X=0
- The slope, Bi, is the change in the value of the dependent variable, Y, for unit change in the independent variable Xi, keeping all other Xs constant (controlled).

MLRM Model Diagnostics

- Test for overall model fitness (R-Square and Adjusted R-Square)
- Test for overall model statistical significance (F test test)
- Test for statistical significance of individual explanatory variables (t test)
- Test for Normality and Homoscedasticity of residuals
- Test for Multi-collinearity and Auto Correlation

Test for overall Model Fitness:
Coefficient of Multiple Determination R^2

- R Square ranges from 0 to 1

$$R^2 = \frac{SSR}{SST} = 1 - \frac{SSE}{SST} = \frac{\sum_i (\hat{y}_i - \bar{y})^2}{\sum_i (y_i - \bar{y})^2}$$

R^2 = Multiple coefficient of multiple determination

R^2 is the percentage of the variation of y explained by $x_1, x_2, \ldots, x_k$

Test for overall Model Fitness: Adjusted R^2

- Inclusion of additional explanatory variables will increase the R^2 value
- By introducing an additional explanatory variable, we increase the numerator of the expression for R2 while the denominator remains the same.
- To correct this defect, we adjust the R2 by taking into account the degrees of freedom

$$R_A^2 = 1 - \frac{SSE/(n-k-1)}{SST/(n-1)}$$

$R_A^2 = \text{Adjusted R - Square}$

$n = \text{number of observations}$

$k = \text{number of explanatory variables}$

Test for overall Model Significance: F-Test

$$H_0 : \beta_1 = \beta_2 = \ldots\ldots = \beta_k = 0$$

$$H_A : \text{Not all } \beta \text{ values are zero}$$

Test for overall significance of multiple regression model.

Checks if there is a statistically significant relationship between Y and any of the explanatory variables ($X_1, X_2, \ldots, X_k$).

Analysis of Variance					
Source	**DF**	**Sum of Squares**	**Mean Square**	**F Value**	**Pr > F**
Model	5	7E+14	1.48E+14	55.2	**<.0001**
Error	52	1.40E+14	2.69E+12		
Corrected Total	57	8.80E+14			

Test for individual explanatory variables (t-test)

$$H_0 : \beta_i = 0$$
$$H_A : \beta_i \neq 0$$

$$t = \frac{\hat{\beta}_i}{S_e(\hat{\beta}_i)}$$

T- test:

By rejecting the null hypothesis, we can claim that there is a statistically significant relationship between the response variable Y and explanatory variable X_i.

Parameter Estimates					
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	493778	1520007	0.32	0.7466
Ratings	1	651375	90775	7.18	<.0001
Price	1	-1833.67081	127.24677	-14.41	<.0001
Num_new_features	1	547167	85234	6.42	<.0001
Stock_market_ind	1	-105.38867	106.01603	-0.99	0.3248
Market_promo_budget	1	100.92891	8.38744	12.03	<.0001

Test for Normality and Homoscedasticity of Residuals

- **Normality:** The errors should be normally distributed, we test for normality of residuals and the p-value should be more than 0.05 (we accept Ho)
  - **Test:** Anderson-Darling (AD) Test, Shapiro-Wilk normality test

- **Homoscedasticity:** At each level of the predictor variables, the variance of the residual term should be constant and the p-value should be more than 0.05 (we accept Ho)
  - **Test:** Breusch-Pagan Test, Cook-Weisberg Test

Test for Multicollinearity

- **Multicollinearity:** High correlation between explanatory variables is called Multi-collinearity
  - **Test:** Variance Inflation Factor (VIF), is a relative increase in variance in S.E. of beta because of collinearity , VIF should be less than 1.7. To reduce the Multicollinearity, one of the ideal measures is to drop the variable causing multi-collinearity

Test for Auto-Correlation

- **Auto Correlation:** For any two observations of Yi, error terms should be not correlated

  - **Test:** Durbin Watson Test, test statistic ranges from 0-4. When DW=2, no auto-correlation exists. Values away from 2, show that auto-correlation exists.

Mean Absolute Percentage Error (MAPE)

The mean absolute percentage error (**MAPE**), also known as mean absolute percentage deviation (MAPD), is a measure of prediction accuracy of a forecasting method in statistics. For example, if the MAPE is 5, on average, the forecast is off by 5%. The equation is:

$$\frac{\sum \left| (y_t - \hat{y}_t) \ / \ y_t \right|}{n} \times 100, \ (y_t \neq 0)$$

where y_t equals the actual value, $\hat{y}_t$ equals the fitted value, and n equals the number of observations.

Actual Vs Predicted