# Theory of Estimation

# Theory of Estimation

If no information is given about the characteristics of the population(values of parameters) then it is required to obtain estimates of these parameters.
This is the **problem of Estimation.**

**Example:** An example of estimation would be determining how many candies of a given size are in a glass jar. Because the distribution of candies inside the jar may vary.
The observer can count the number of candies visible through the glass, consider the size of the jar, and presume that a similar distribution can be found in the parts that can not be seen, thereby making an estimate of the total number of candies that could be in the jar if that presumption were true.

# Theory of Estimation

**1) Point Estimation:-**

When the estimated value is given by a single quantity, it is called point of estimation.

suppose that I estimated 100 candies are there in the jar.

**2) Interval Estimation:-**

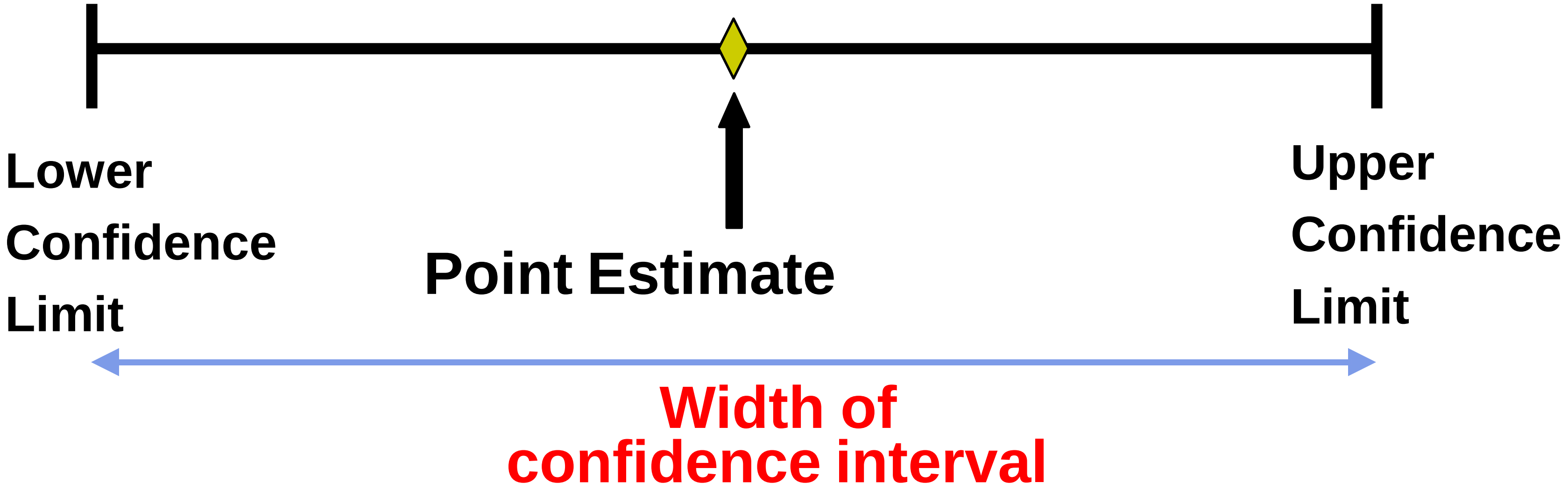
An interval within which the parameters expected to lie is given by using two quantities, is known as **Confidence Interval.**

Suppose, if I estimate 100 to 150 candies are there in the jar.

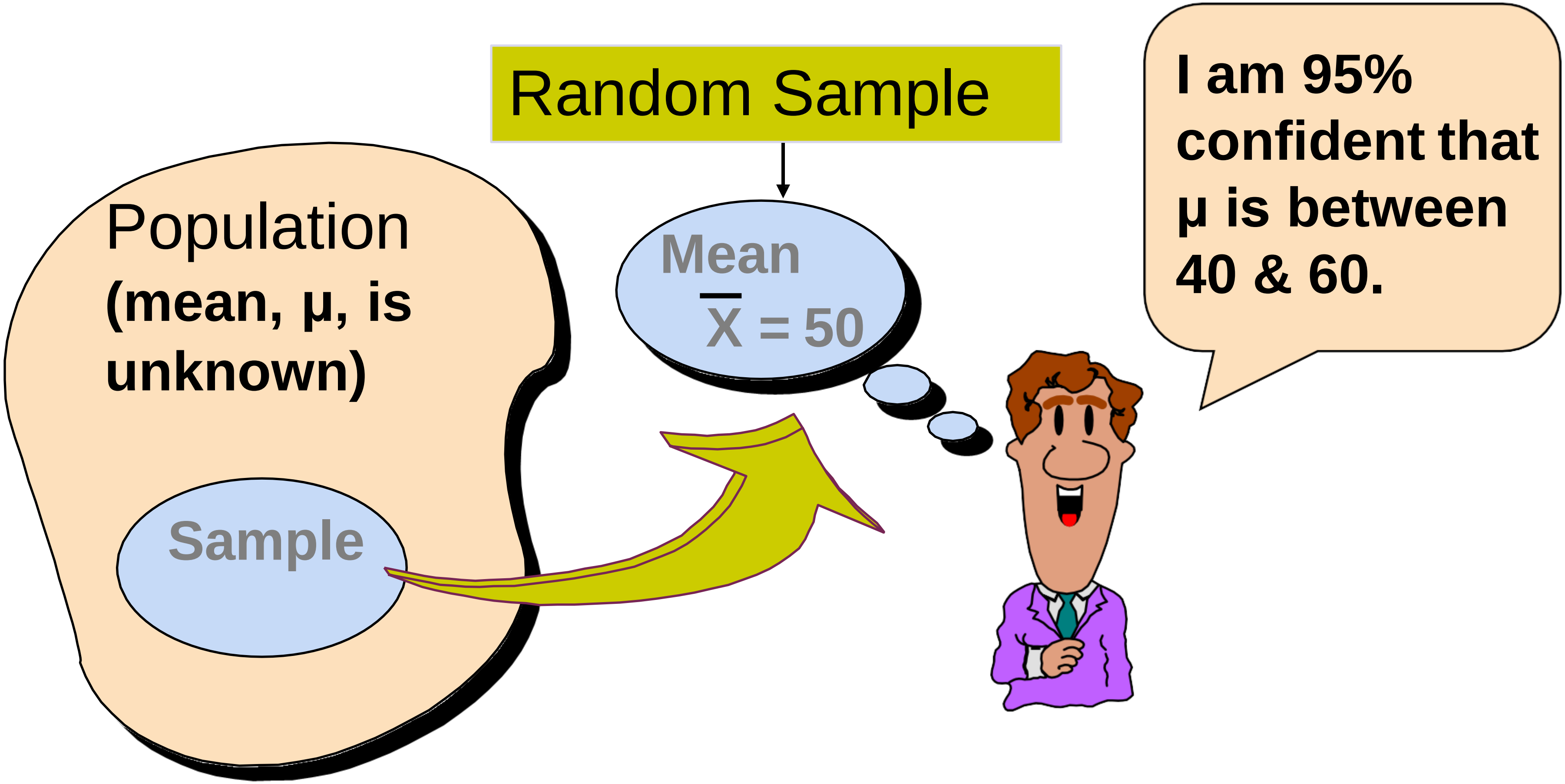
Two quantities which are used to specify the interval, are known as Confidence Limits.

# Point and Interval Estimates

- A point estimate is a single number
- a confidence interval provides additional information about the variability of the estimate



# Estimation Process



# General Formula

- The general formula for all confidence intervals is:

**Point Estimate ± (Critical Value)(Standard Error)**

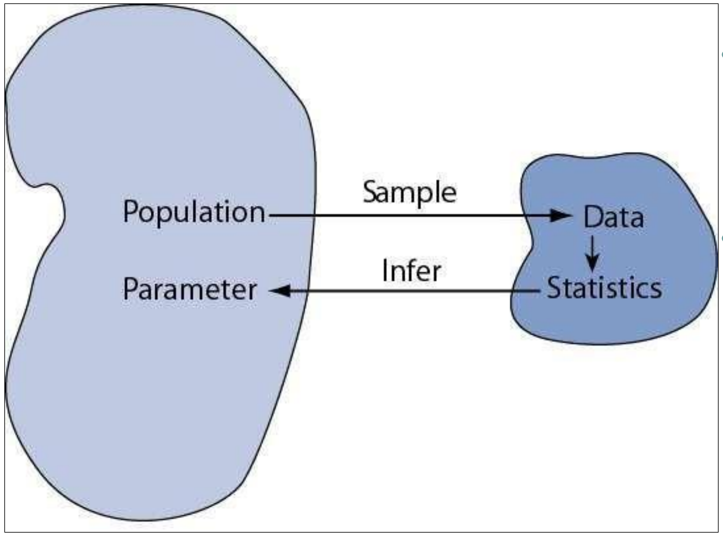
Where:

- Point Estimate is the sample statistic estimating the population parameter of interest
- Critical Value is a table value based on the sampling distribution of the point estimate and the desired confidence level
- Standard Error is the standard deviation of the point estimate

# Test of Hypothesis

# Statistical Inference

- Inferences about a population are made on the basis of results obtained from a sample drawn from that population




- A hypothesis test is a process that uses sample statistics to test a claim about the value of a population parameter.
- A verbal statement, or claim, about a population parameter is called a statistical hypothesis.

# Test of Hypothesis

Suppose, we assume that the average income of Kolkata's population is 30k/month.
Now the validity of a hypothesis will be tested analyzing the sample. The procedure which enables us to decide whether a certain hypothesis is true or not, is called **Test of Hypothesis or Test of Significance.**

**Null Hypothesis:-**
The Hypothesis made about population characteristic, is called **Null Hypothesis($H_0$).**

**Alternative Hypothesis:-**
Any Hypothesis which differs from null hypothesis is called **Alternative Hypothesis($H_1$).**

Here, the **Null Hypothesis = Avg income is 30k/month**
**Alternative Hypothesis = Avg income is not 30k/month**

# What are Decision errors?

**Type I error**. A Type I error occurs when the researcher rejects a null hypothesis when it is true. The probability of committing a Type I error is called the **significance level**. This probability is also called **alpha**, and is often denoted by α.

**Type II error**. A Type II error occurs when the researcher fails to reject a null hypothesis that is false. The probability of committing a Type II error is called **Beta**, and is often denoted by β. The probability of *not* committing a Type II error is called the **Power** of the test.

For example:

**Null Hypothesis =** No wolf present

**Alternative Hypothesis =** Wolf present

**Type I Error -** When no wolf was there, but villagers came to save shepherd, believing that there was a wolf ($H_0$ true but rejected)

**Type II Error -** When wolf was truly there, but no one came as they believed no wolf was there ($H_0$ false but accepted).

# Level of significance

- The maximum probability of Type I error is called **level of significance**
- If 5% level of significance is chosen, then there are about 5 chances in 100 that we would reject the hypothesis when it should be accepted.
- In other words, we are about 95% confident that we have made the right decision.
- At 0.05 significance level, the hypothesis has a 0.05 probability of being wrong.
- **Example -** When no wolf was there, but villagers came to save shepherd 5 out of 100 times, believing that there was a wolf, then level of significance of their hypothesis is 0.05

# Another Example

- Suppose, I've applied for a typing job and I've stated in my resume that my typing speed is 60 words per minute on an average. My recruiter may want to test my claim. If he finds my claim to be acceptable, he will hire me otherwise reject my candidature. So he asked me to type a sample letter and found that my speed is 54 words a minute. Now, he can decide on whether to hire me or not [assuming that I meet all other eligibility criteria].

# Another Example

**In statistical terms:**

- **Hypotheses:**
- "my typing speed is 60 words per minute on an average" is a hypothesis to be tested, called **null hypothesis.**

- The **alternating hypothesis** is "my typing speed is not 60 words per minute on an average"

- **Population & Sample:** My average typing speed is population parameter and my sample typing speed is sample statistics.

# Another Example

**Level of significance:** The criteria of accepting /rejecting my claim is to be decided by the recruiter. For example, he may decide that an error of 5 words is ok to him so he would accept my claim between 55 to 65 words/minutes.

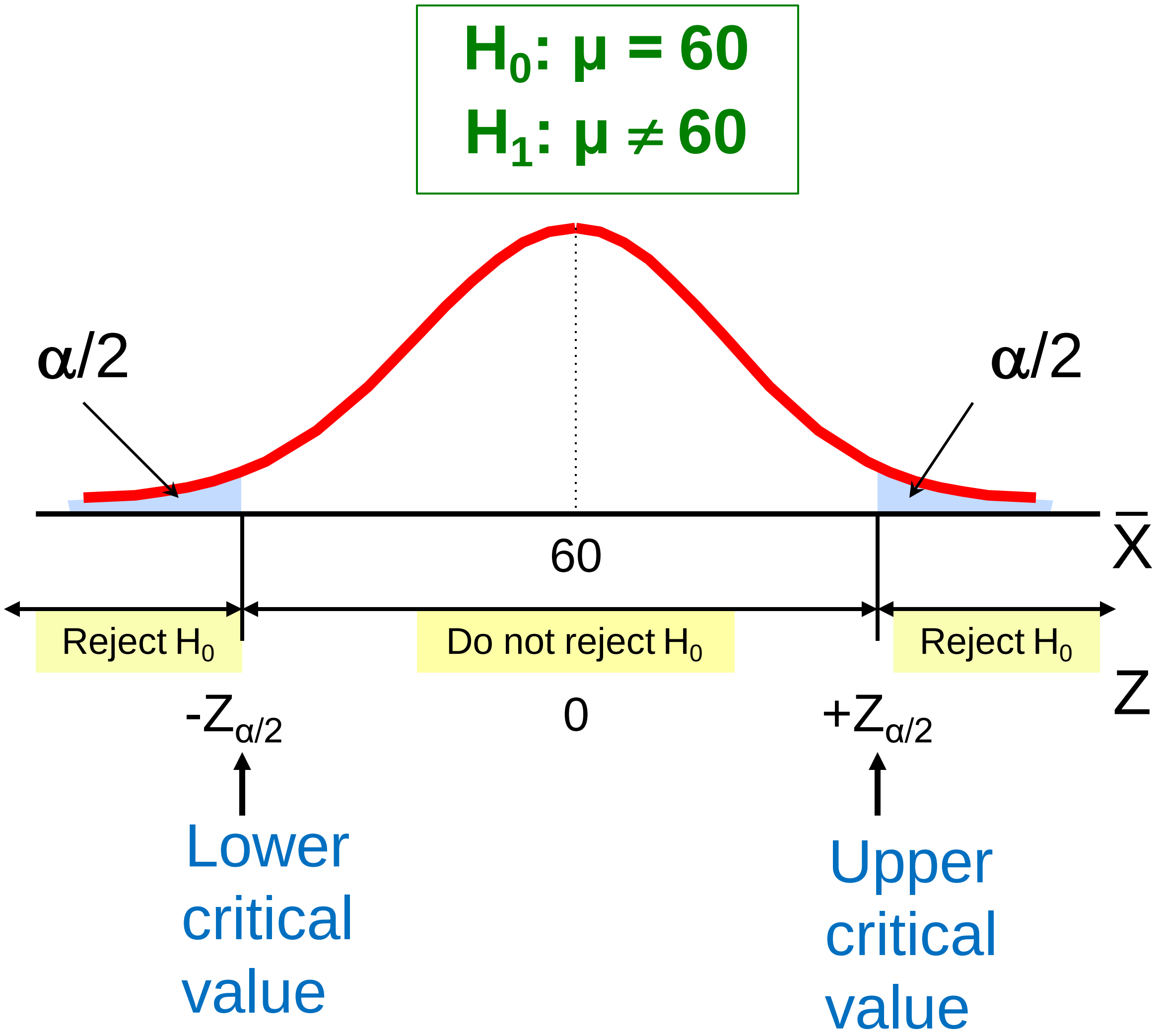
In that case, my sample speed 54 words/minute will conclude to reject my claim. And the decision will be "I was making a false claim“. **Null hypothesis is rejected for 0.05 level of significance.**

But if the recruiter extends his acceptance region to +/- 7 words [that is 53 to 67 words], he would be accepting my claim.

So, to conclude, **Hypothesis testing is a process to test claims about the population on the basis of sample.**

# Two-Tail Tests

There are two cutoff values (critical values), defining the regions of rejection



# One-Tail Tests

- In many cases, the alternative hypothesis focuses on a particular direction

$H_0$: $\mu \geq 60$

$H_1$: $\mu < 60$

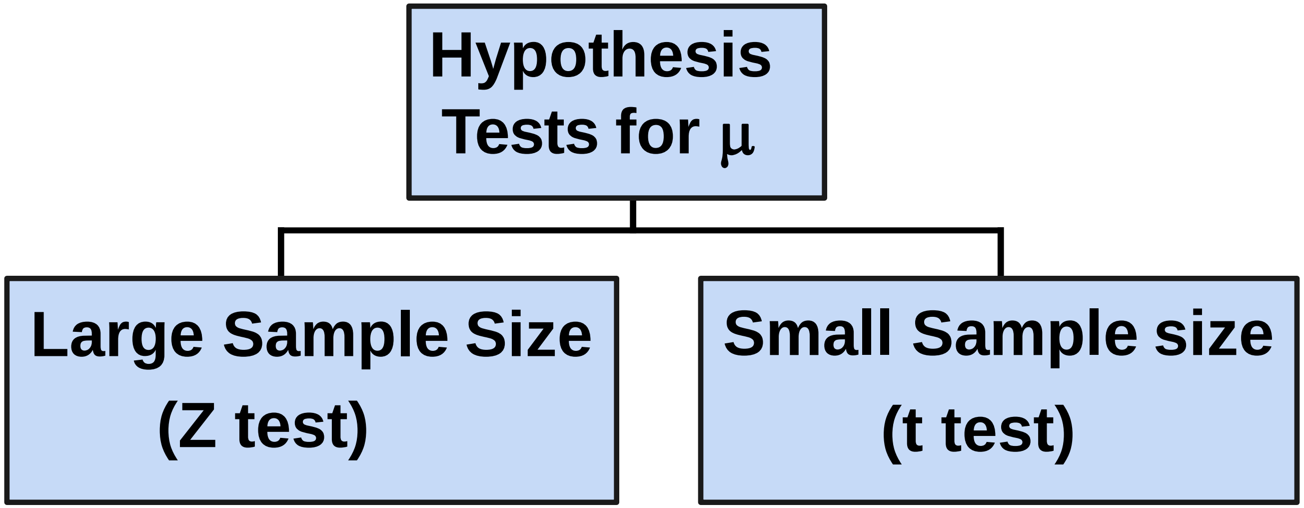
This is a lower-tail test since the alternative hypothesis is focused on the lower tail below the mean of 60

$H_0$: $\mu \leq 60$

$H_1$: $\mu > 60$

This is an upper-tail test since the alternative hypothesis is focused on the upper tail above the mean of 60

# Testing of Hypothesis for mean in case of large & small samples



# Z Test of Hypothesis for the Mean

- Convert sample statistic ( $\bar{X}$ ) to a $Z_{STAT}$ test statistic

**Hypothesis Tests for $\mu$**

**Large Sample size (Z test)**

**Small sample size (t test)**

The test statistic is:

$$Z_{STAT} = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

# Hypothesis Testing Example

**Test the claim that the true mean # of TV sets in US homes is equal to 3.**
**(Assume $\sigma$ = 0.8)**

1. State the appropriate null and alternative hypotheses
   - $H_0$: $\mu = 3$ $H_1$: $\mu \neq 3$ (This is a two-tail test)
2. Specify the desired level of significance and the sample size
   - Suppose that $\alpha = 0.05$ and $n = 100$ are chosen for this test

# Hypothesis Testing Example

*(continued)*

3. Determine the appropriate technique
   - σ is assumed known so this is a Z test.
4. Determine the critical values
   - For $\alpha = 0.05$ the critical Z values are ±1.96
5. Collect the data and compute the test statistic
   - Suppose the sample results are

     $n = 100$, $\bar{X} = 2.84$ ($\sigma = 0.8$ is assumed known)

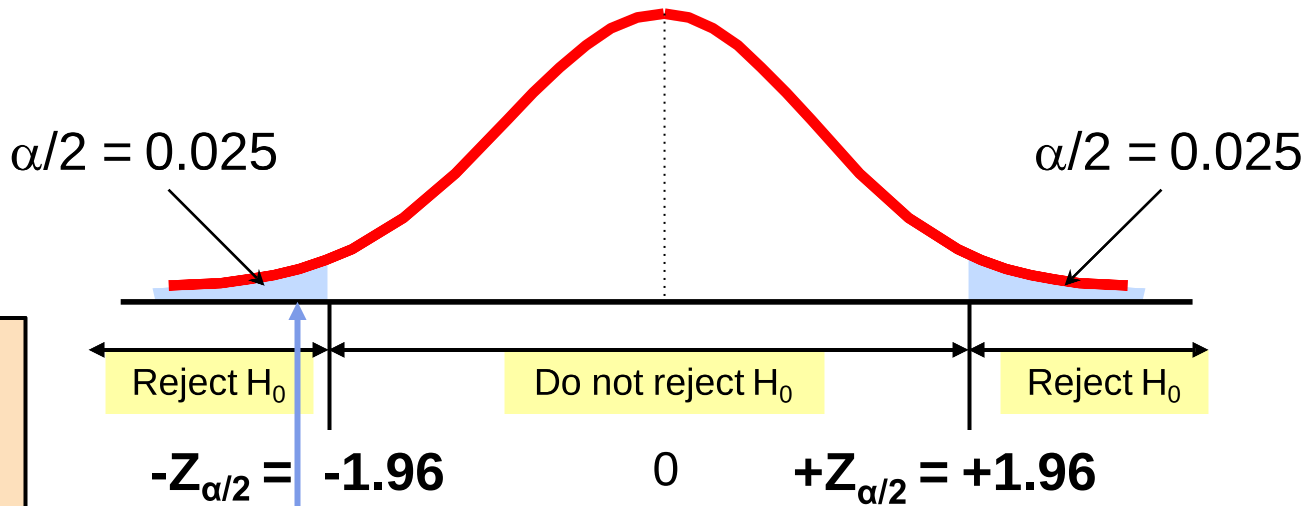
So the test statistic is:

$$Z_{STAT} = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{2.84 - 3}{\frac{0.8}{\sqrt{100}}} = \frac{-.16}{.08} = -2.0$$

# Hypothesis Testing Example

*(continued)*

- 6. Is the test statistic in the rejection region?



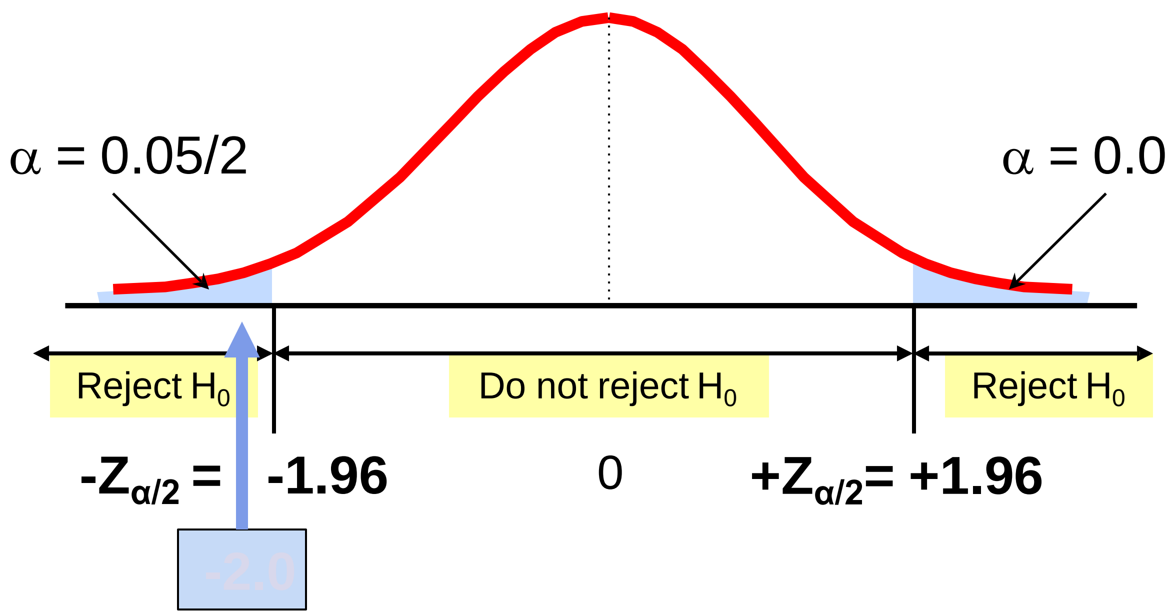

Reject $H_0$ if $Z_{STAT} < -1.96$ or $Z_{STAT} > 1.96$; otherwise do not reject $H_0$

Here, $Z_{STAT} = -2.0 < -1.96$, so the test statistic is in the rejection region

# Hypothesis Testing Example

*(continued)*

6 (continued). Reach a decision and interpret the result




Since $Z_{STAT} = -2.0 < -1.96$, reject the null hypothesis and conclude there is sufficient evidence that the mean number of TVs in US homes is not equal to 3

# Hypothesis Testing: p-Value Approach

**All hypothesis tests ultimately use a p-value to weigh the strength of the evidence (what the data are telling you about the population).**

**P value is the probability of observing the observed data given the null hypothesis is true.**

**Higher the p value, more likely it is to observe the data we have just observed. If the probability is very low, it raises questions on whether the null hypothesis is true or not. In which case we reject the null hypothesis.**

**P low → Null Go**

**The p-value is a number between 0 and 1 and interpreted in the following way:**

**A small p-value (typically ≤ 0.05) indicates strong evidence against the null hypothesis, so you reject the null hypothesis.**

**A large p-value (> 0.05) indicates weak evidence against the null hypothesis, so you fail to reject the null hypothesis.**

# p-value Hypothesis Testing Example

**Test the claim that the true mean # of TV sets in US homes is equal to 3.**
**(Assume $\sigma = 0.8$)**

1. State the appropriate null and alternative hypotheses
   - $H_0$: $\mu = 3$ $H_1$: $\mu \neq 3$ (This is a two-tail test)
2. Specify the desired level of significance and the sample size
   - Suppose that $\alpha = 0.05$ and $n = 100$ are chosen for this test

# p-value Hypothesis Testing Example

*(continued)*

3. Determine the appropriate technique
   - σ is assumed known so this is a Z test.
4. Collect the data, compute the test statistic and the p-value
   - Suppose the sample results are

     n = 100, $\bar{X}$ = 2.84 (σ = 0.8 is assumed known)

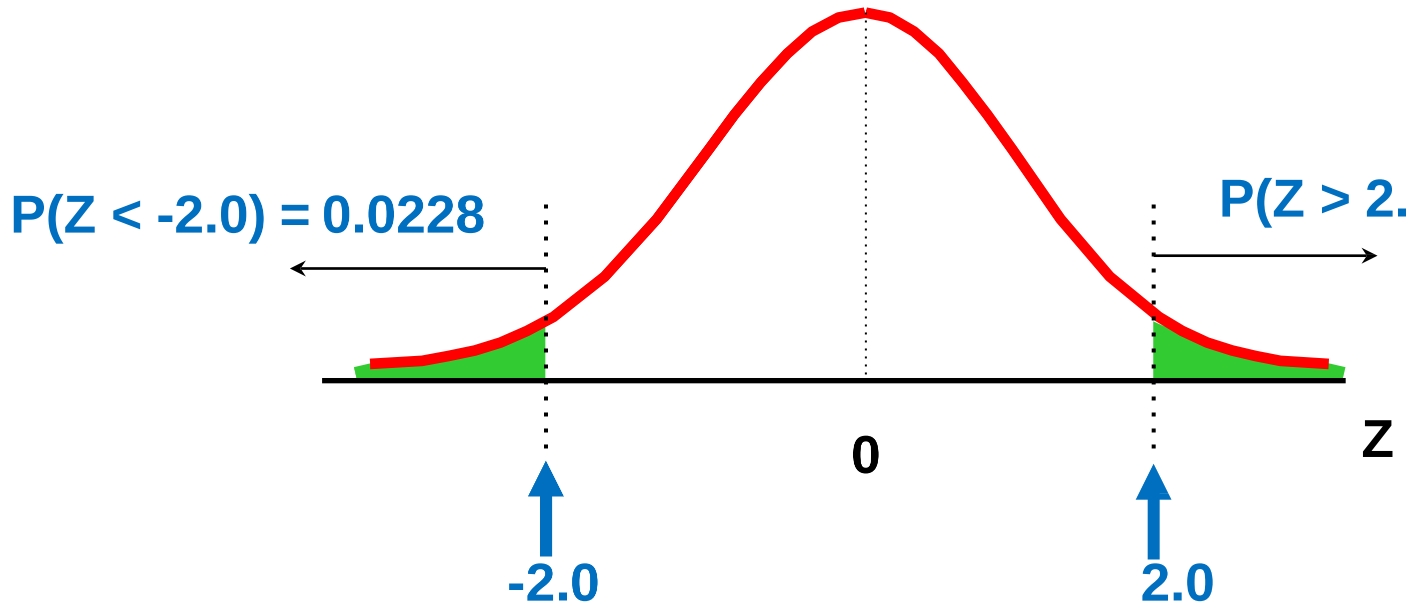
So the test statistic is:

$$Z_{STAT} = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{2.84 - 3}{\frac{0.8}{\sqrt{100}}} = \frac{-.16}{.08} = -2.0$$

# p-Value Hypothesis Testing Example: Calculating the p-value

4. (continued) Calculate the p-value.

- How likely is it to get a $Z_{STAT}$ of -2 (or something further from the mean (0), in either direction) if $H_0$ is true?




**p-value = 0.0228 + 0.0228 = 0.0456**

# p-value Hypothesis Testing Example

*(continued)*

- 5. Is the p-value < α?
  - Since p-value = 0.0456 < α = 0.05 Reject $H_0$
- 5. (continued) State the managerial conclusion in the context of the situation.
  - There is sufficient evidence to conclude the average number of TVs in US homes is not equal to 3.

# t Test of Hypothesis for the Mean (Small Sample size)

- Convert sample statistic ( $\bar{X}$ ) to a $t_{STAT}$ test statistic

**Hypothesis Tests for μ**

**Large Sample size (Z test)**

**test)**

**Small sample size (t test)**

**test)**

The test statistic is:

$$t_{STAT} = \frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}}$$

# T test requires
# Degrees of Freedom (df)

# Degrees of Freedom (df)

Degrees of freedom is the number of values that are free to vary when the value of some statistic, like $\bar{X}$ or $\hat{\sigma}^2$, is known.

Idea: Number of observations that are free to vary after sample mean has been calculated

**Example:** Suppose the mean of 3 numbers is 8.0

Let $X_1 = 7$

Let $X_2 = 8$

What is $X_3$?

If the mean of these three values is 8.0,
then $X_3$ must be 9
(i.e., $X_3$ is not free to vary)

Here, n = 3, so degrees of freedom = n – 1 = 3 – 1 = 2

(2 values can be any numbers, but the third is not free to vary for a given mean)

# Example: Two-Tail Test (Small sample size)

The average cost of a hotel room in New York is said to be \$168 per night. To determine if this is true, a random sample of 25 hotels is taken and resulted in an $\bar{X}$ of \$172.50 and an S of \$15.40. Test the appropriate hypotheses at $\alpha = 0.05$.

(Assume the population distribution is normal)

$H_0: \mu = 168$

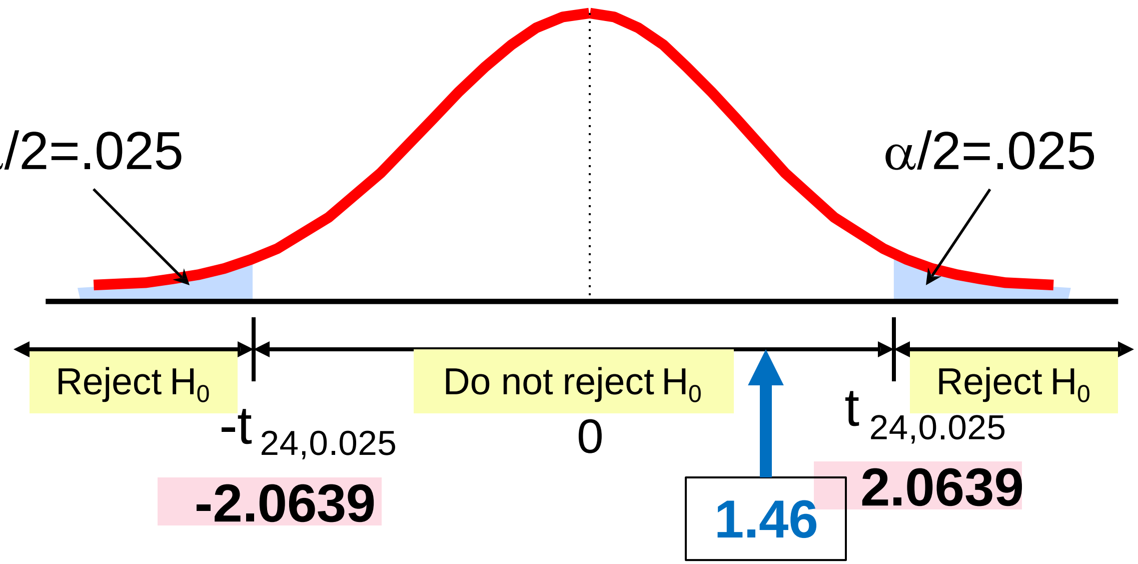
$H_1: \mu \neq 168$

# Example Solution: Two-Tail t Test

$H_0$: $\mu = 168$
$H_1$: $\mu \neq 168$

- $\alpha = 0.05$
- $n = 25$, $df = 25-1=24$
- $\sigma$ is unknown, so use a t statistic
- Critical Value:

$\pm t_{24,0.025} = \pm\ 2.0639$




$$t_{STAT} = \frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}} = \frac{172.50 - 168}{\frac{15.40}{\sqrt{25}}} = 1.46$$

**Do not reject $H_0$:** insufficient evidence that true mean cost is different than $168

**All done till now was One-sample Test**

# Two Sample Hypothesis Testing

In a two-sample hypothesis test, two parameters from two populations are compared.

- For a two-sample hypothesis test,

1. the **null hypothesis** $\boldsymbol{H_0}$ is a statistical hypothesis that usually states there is no difference between the parameters of two populations. The null hypothesis always contains the symbol $\leq$, $=$, or $\geq$.
2. the **alternative hypothesis** $\boldsymbol{H_1}$ is a statistical hypothesis that is true when $H_0$ is false. The alternative hypothesis always contains the symbol $>$, $\neq$, or $<$.

# Two Sample Hypothesis Testing

To write a null and alternative hypothesis for a two-sample hypothesis test,

$$\begin{cases} H_0: \mu_1 = \mu_2 \\ H_1: \mu_1 \neq \mu_2 \end{cases}$$

If independent samples (comparing class scores of two classes)

Large Sample set : Use z – test

Small Sample size: Use t –test (Pooled estimate t -test)

If dependent samples (comparing scores of a class after conducting a special course )

Use t –test (Paired t - test)