Study of Portfolio Optimization using Monte Carlo Simulation and Optimization Techniques in Python

Authors:

Ananya Chakraborty(M14307054), Purvi Jhala(M14568182), and Sourav Karwa(M14306203)

MS Business Analytics Program

University of Cincinnati Carl H. Lindner College of Business, Fall 2021

NOTE –

"I understand that this project report is optional for extra credit, that the amount of points awarded by this submission is solely determined by the instructor assessment, and that no requests for regrading or complaints about the number of extra points awarded will be admitted."

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INTRODUCTION

The stock market portfolio optimization problem deals to maximize return on investment within the acceptable risk. It is difficult to decide which stock to be picked and how much investment should be done to get a better return in the future because of the volatility of the stock market and other unfortunate events like the deep-water horizon oil spill (Deepwater-Horizon-oil-spill) can spook the stock market causing heavy loss in your investment.

Thus, the main objective in a portfolio selection problem is to find optimal proportions of the stock for creating a portfolio to strike a balance between maximizing the return and minimizing the risk of their investment.

MOTIVATION

Calculating the risk and returns of an investment portfolio in dynamic environments motivates us to compare the results of adjusting asset weights using the Monte Carlo Simulation and optimization algorithm. We have tried to answer the questions below through this study

- Calculate expected return for a portfolio of stocks by fetching its adjusted closing prices from yahoo finance over some time.
- Calculate standard deviation (volatility) for daily returns.
- The concept of the Sharpe Ratio is to determine what proportion of assets to buy for maximum returns and minimum risk in a portfolio.

A. APPROACHES FOLLOWED:

a. Monte Carlo Simulation – Through Monte Carlo Simulation we create different simulated portfolios with the metrics of expected returns, expected risk and Sharpe Ratio. The various efficient portfolios are then compared to find the best values for Minimum Volatility and Maximum Sharpe Ratio problems.

b. Optimization of volatility – A minimization function is applied to reduce the volatility and the Negative Sharpe Ratio that provides an optimal return versus volatility trade-off portfolio using a package from SciPy for optimization called “**scipy.optimize**”.

B. DATA SOURCE:

A portfolio containing 9 stocks are used in this analysis – ['AMZN', 'UNH','BA','NKE', 'MSFT','NFLX','TSLA','AAPL','GOOGL']. Adjusted Closing prices for all stocks for the period Jan 1, 2015 – to present are used (downloaded from yfinance package in python). Risk-Free Rate of 0.015 for 3 months is used (as sourced from Department of Treasury)

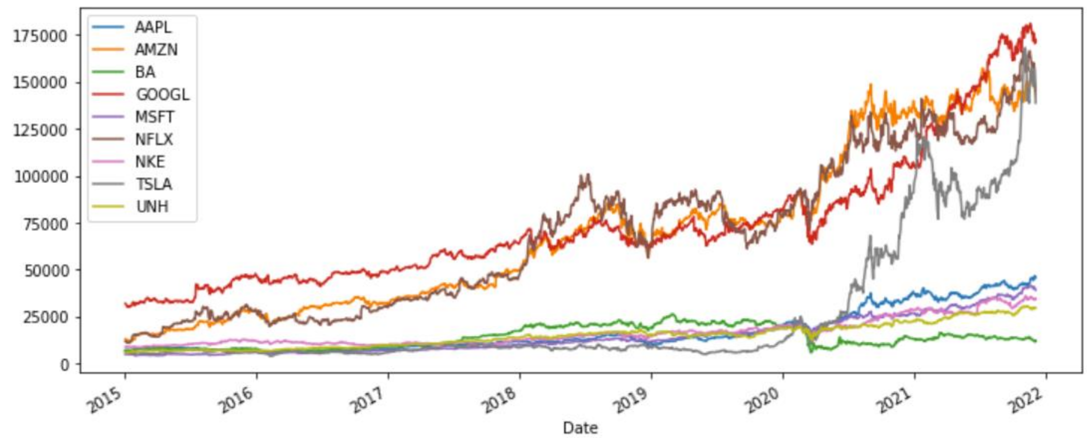

Figure 1.1: Plot of historical data of 9 stocks from 2015 to present day.

LITERATURE REVIEW

The Capital Asset Pricing Model (CAPM) is a widely used pricing model in the investing and finance industries for pricing hazardous stocks or assets with significant volatility. It essentially displays a link between expected risk and expected return for stocks/assets.

Harry Markowitz invented the CAPM, which requires an investor to choose one portfolio at time t-1 that yields a stochastic return at time t. The presented model assumes that investors are risk averse, and that when selecting a portfolio from a set of options, they assess profit only in terms of the mean and variance of their investment return, ignoring the risk associated with it. When given an expected return E(R), the portfolios developed by this approach minimize the variance of the returns and maximize E(R) when given the variance. This model is referred as the Markowitz approach which signifies a basic "mean-variance model".

Therefore, this Markowitz approach is called a "mean-variance model". (Cited)

To find a portfolio that must be mean-variance efficient, Sharpe (1964) and Lintner (1965) add two crucial assumptions to the Markowitz model. The first assumption is total agreement: investors agree on the joint distribution of asset returns from t-1 to t given market clearing asset prices at t-1. And this is the genuine distribution—that is, the one from which we derive the returns we use to test the model. The second assumption is that there is risk-free borrowing and lending, which is the same for all investors and is unaffected by the quantity borrowed or lent. (Cited)

Modern finance theory (MFT) is based on CAPM and hinges on numerous assumptions: (1) Securities markets are extremely competitive and efficient (that is, important information about firms is widely transmitted and absorbed fast); (2) these markets are dominated by rational, risk-averse investors who strive to maximize satisfaction. (3) Commissions and market taxes are not included in the computation. (4) Investors have unrestricted access to the market to borrow and lend money at a risk-free rate (https://hbr.org/1982/01/does-the-capital-asset-pricing-model-work). (Cited)

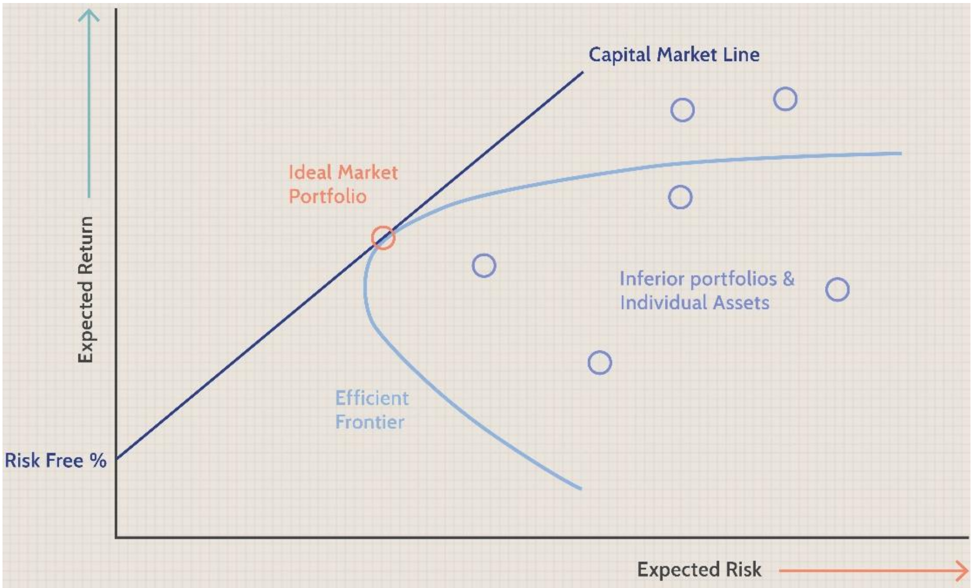

Figure 1.2: Reward v/s Risk, return of assets at varying risk with optimal asset allocation. (Source – Investopedia)

For a portfolio of assets that are potentially correlated to one another, the returns, the weighted average of the mean returns, and the volatility is calculated using the below formulas:

$$Return_{t,t+1} = \frac{Price_{t+1} - Price_t}{Price_t}$$

This measures the returns from stocks from t → t+1 for the given price.

$$Expected\ Return(E(R)) = w_1E(R_1) + w_2E(R_2) + \cdots + w_nE(R_n)$$

Where $w_i: n - dimensional\ weight\ vector\ \ \Sigma wi = 1$ & $R_1, \ldots\ R_n$ is the returns on different stocks.

$$Var(R) = w^T\Sigma w$$

Where Σ is the covariance matrix between the stock returns.

$$Sharpe\ Ratio = \frac{E(R) - RiskFreeRate}{\sqrt{Var(R)}}$$

Where $RiskFreeRate$ is taken as 0.015 for the current time.

As stated, - "In a single calculation, the Sharpe ratio includes both risk and return—actual or predicted, depending on the circumstances. A rising return differential or falling standard deviation—both "good" events—leads to an increase in the Sharpe ratio; on the other hand, a falling return differential or rising standard deviation—both "bad" events—leads to a decrease in the Sharpe ratio. As a result, a greater Sharpe ratio is preferable than a lower one. When deciding between two options, the Sharpe ratio criterion dictates that the one with the larger Sharpe ratio be chosen. We would choose the investment with the highest existent Sharpe ratio if we were deciding on investments before the event; if we were evaluating traders after the incident, we would give better marks to the trader with the highest ex-post Sharpe ratio " (Kevin 2000 - Cited).

PROPOSED MODEL

Under this section we have discussed about our model, and approach towards building an optimized portfolio. To achieve our goal, we have performed a study of two methods:

A. Monte Carlo Simulation
B. Optimization Algorithm

We have solved our problem computationally i.e., by using computer to crunch possible permutations of the portfolio. Optimal portfolio is considered the one with the highest return per risk portfolio.

As a first step, we pulled data from verified site: Yahoo. The dates for which we have pulled the data is from 2015-01-01 to present day in a time that covers the full business cycle, of trough, recession, expansion, and peak.

Stocks included in our portfolio are:

ID	Tickers	Stock Name
1.	AMZN	Amazon
2.	UNH	United Health
3.	BA	Boeing
4.	NKE	Nike
5.	MSFT	Microsoft
6.	NFLX	Netflix
7.	TSLA	Tesla
8.	AAPL	Apple
9.	GOOGL	Google

Table 1.1: 9 stocks selected for optimization on returns.

Snapshot of data-

Date	AAPL	AMZN	BA	GOOGL	MSFT	NFLX	NKE	TSLA	UNH
2015-01-02	24.746002	308.519989	113.657211	529.549988	41.193840	49.848572	44.185089	43.862000	90.677490
2015-01-05	24.048861	302.190002	112.870064	519.460022	40.815033	47.311428	43.473709	42.018002	89.183899
2015-01-06	24.051128	295.290009	111.540627	506.640015	40.215973	46.501431	43.217972	42.256001	89.003944
2015-01-07	24.388376	298.420013	113.272385	505.149994	40.726929	46.742859	44.110710	42.189999	89.912674
2015-01-08	25.325434	300.459991	115.275284	506.910004	41.925030	47.779999	45.128963	42.124001	94.204529
...	...	...	...	...	...	...	...	...	...
2021-11-29	160.240005	3561.570068	198.500000	2910.610107	336.630005	663.840027	169.869995	1136.989990	452.000000
2021-11-30	165.300003	3507.070068	197.850006	2837.949951	330.589996	641.900024	169.240005	1144.760010	444.220001
2021-12-01	164.770004	3443.719971	188.190002	2821.030029	330.079987	617.770020	166.699997	1095.000000	444.339996
2021-12-02	163.759995	3437.360107	202.380005	2859.320068	329.489990	616.469971	170.000000	1084.599976	446.019989
2021-12-03	161.839996	3389.790039	198.460007	2840.030029	323.010010	602.130005	170.320007	1014.969971	449.140015

Figure 1.3: Snapshot of stock data

A. **Monte Carlo Simulation Method-**

It is difficult to determine the probability of varying because of random variable interference. Therefore, Monte Carlo simulation concentrates on repeating random samples to achieve certain results. It helps to justify the impact of risk and uncertainty in prediction and forecasting models. In MC simulation model is built which will select a variable that has uncertainty and assigns it a random value. This is repeated multiple times to assign variable different values. Once the simulation is complete results are averaged to obtain an estimate.

We are already aware of the concept that as the number of portfolios increases, we get closer to the optimum portfolio. Thus, in our study we are generating 50000 random portfolios to achieve most optimum result.

B. Optimization in python-

Suppose your portfolio consists of expected returns E(R), and a risk-free rate for the assets, then the maximum value of Sharpe Ratio can be achieve by using the below objective function, where wi s weights. Thus, the expected return of portfolio can be given by maximizing the below objective function or minimizing the negative Sharpe Ratio.

Objective Function:

$$Maximize(Sharpe\ Ratio) = Maximize(\frac{E(R) - RiskFreeRate}{\sqrt{Var(R)}})$$

Constraints:

$$subject\ to,$$

$$\Sigma wi = 1 \ldots\ldots\ldots\ldots.. Eqn\ (1)\ (For\ all\ wi)$$

$$wi\ \geq 0 \ldots\ldots\ldots\ldots.. Eqn\ (2)\ (For\ all\ wi)$$

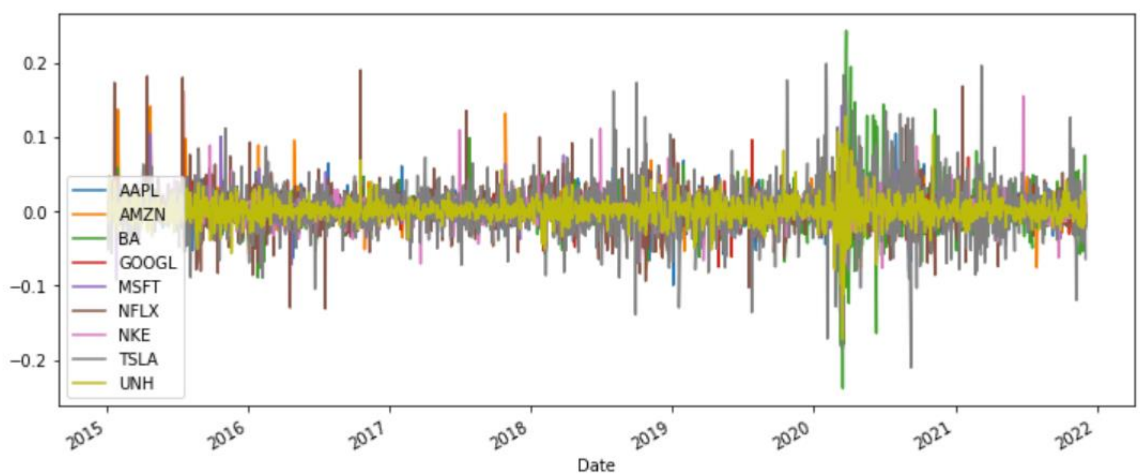

Figure 1.4: Analysis of expected stock return

MODEL VALIDATION AND COMPUTATION RESULTS

Prior to validating the model results we will describe the results and algorithm of the Monte Carlo Simulation and the Sharpe Ratio Maximization and Volatility Minimization performed using the "optimize" package in Python.

- **MONTE CARLO SIMULATION - ALGORITHM:**

  The algorithm for performing Monte Carlo Simulation on the stocks under study is as below:
  - We calculate the Expected Return (ER), Expected Risk/Volatility (EV), and the Variance ($Var(ER)$) for each of the stocks selected in the portfolio.
  - We assign weights (w_i) to the expected returns to calculate the weighted average of the expected returns.
  - We perform Monte Carlo Simulation for 50,000 possible weight allocations for portfolios and compute the Expected Return, Expected Volatility and Sharpe Ratio.
  - We evaluate the best portfolios generated for both Maximum Sharpe Ratio and Minimum Variance (Volatility) portfolios comparing the results in both.

- **MONTE CARLO SIMULATION - RESULTS:**

  The results from running the above algorithm over the 9 selected stocks provides us with the below results.
  - The plot of Expected Return against Expected Volatility with respect to the Sharpe Ratio is provided below.
  - An investor expecting to make a return of > 40% on the investment would get a High Sharpe Ratio of about 1.5 but would also have to account in the high volatility of 30-32% on this portfolio.

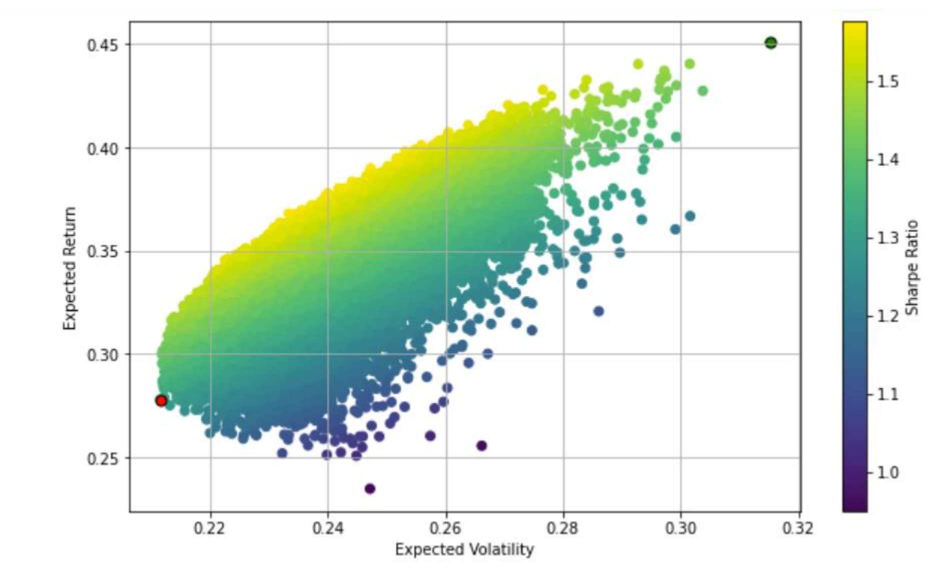

Figure 5.1 – Depicts the effect of variation of weight vector on the Expected Return and Volatility with respect to the Sharpe Ratio. The Maximum Sharpe Ratio is obtained at 1.55 (green dot) and Minimum Volatility is obtained at 23% (red dot).

- **MONTE CARLO SIMULATION – PERFORMANCE METRICS:**

Based on the below attributes it shows that the Maximum Sharpe Ratio provides us a better return but with a considerably high amount of risk as in the case of stocks like Google and Apple, where volatility is high, the risk increases many folds.
In contrast the Minimum Volatility methodology gives us lower returns but lower risks as well, which is an inherent observation in the domain of stock and investment markets.

Portfolio Metrics	Maximum Sharpe Ratio	Minimum Volatility
Expected Stock Return	45%	27.5%
Expected Volatility/Risk	31%	23%
Sharpe's Ratio	1.6	1.09

Table 5.1 – Performance metrics for Monte Carlo Simulation

- **OPTIMIZATION - ALGORITHM:**

The algorithm for performing Optimization on the stocks under study using a "**SciPy**" package in Python is as below:

- We calculate the Expected Return (ER), Expected Risk/Volatility (EV), and the Variance ($Var(ER)$) for each of the stocks selected in the portfolio.
- We assign weights (w_i) to the expected returns to calculate the weighted average of the expected returns.
- We perform minimization on the Negative Sharpe Ratio and on Portfolio Volatility individually using the Sequential Least Squares Programming method.
- We evaluate the best portfolios generated for both Maximum Sharpe Ratio and Minimum Variance (Volatility) portfolios comparing the results in both.

- **OPTIMIZATION - RESULTS:**

The results from running minimization on the Volatility and Negative Sharpe Ratio functions for the 9 selected stocks provides us with the below results.

- The plot of Expected Return against Expected Volatility with respect to the Sharpe Ratio is provided below.
- An investor expecting to make a return of 40% on the investment would get a High Sharpe Ratio of about 1.4 but would also have to account in the high volatility of 25.1-37.5% on this portfolio.
- The Maximum Sharpe Ratio is obtained at 1.3 (green dot) and Minimum Volatility is obtained at 21% (red dot) which is a lesser volatility than achieved from the Monte Carlo Simulations.

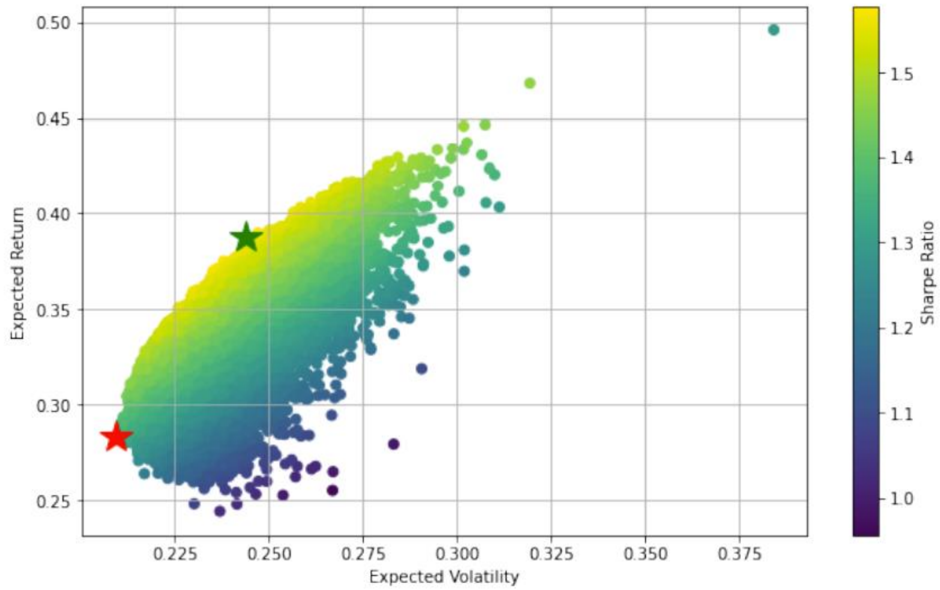

Figure 5.2 – Depicts the effect of variation of weight vector on the Expected Return and Volatility with respect to the Sharpe Ratio after minimization. The Maximum Sharpe Ratio is obtained at 1.3 (green dot) and Minimum Volatility is obtained at 21% (red dot).

- **OPTIMIZATION – PERFORMANCE METRICS:**
  Based on the below attributes it shows that the Maximum Sharpe Ratio provides us a better return but with a considerably high amount of risk which is the same even when applying a minimization optimization on these problems. The minimization causes reduction in the in the volatility in both cases.

Portfolio Metrics	Maximum Sharpe Ratio	Minimum Volatility
Expected Stock Return	38%	27.5%
Expected Volatility/Risk	24%	21%
Sharpe's Ratio	1.7	1.07

Table 5.2 – Performance metrics for Optimization

CONCLUSION

In recent years there has been an increase in involvement of investing and trading in the financial market. For researcher's optimization and study of stock market is a major attraction. Therefore, we have also tried to optimize investment in stocks by using monte Carlo simulation and mean variance optimization.

A. SHARPE RATIO COMPARISON OF PORTFOLIO

i. Monte Carlo Simulation:

Portfolio Metrics	Maximum Sharpe Ratio	Minimum Volatility
Sharpe's Ratio	1.6	1.09

ii. Optimization of Volatility and Sharpe Ratios:

Portfolio Metrics	Maximum Sharpe Ratio	Minimum Volatility
Sharpe's Ratio	1.7	1.07

B. ANALYSIS OF WEIGHT ALLOCATION FOR STOCKS:

Based on the results of the stock weight distribution assigned to the different approaches for stock investment optimization, we decided to plot the allocations on a stacked histogram, to get a better understanding of the investment distribution in the portfolio. Investing on multiple stocks increases the returns, and this case we see the percentage allotments for Maximum Sharpe's Ratio, with shows us an optimal portfolio assignment for the 9 selected stocks with the highest investment to United Heath stocks, Google, and Apple stocks. The portfolio shows 0% allotment for Boeing and Nike due to high volatility.

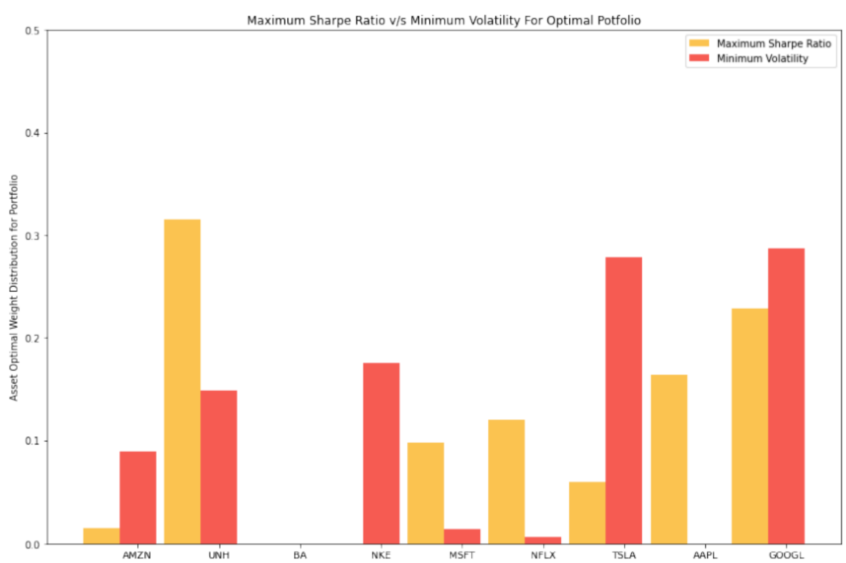

Figure 6.1 – Depicts the weight allocation for the 9 selected stocks in both Max Sharpe Ratio and Min Risk

	Stocks	Optimal_Weights
0	AMZN	1.486%
1	UNH	31.509%
2	BA	0.000%
3	NKE	0.000%
4	MSFT	9.782%
5	NFLX	12.078%
6	TSLA	5.888%
7	AAPL	16.365%
8	GOOGL	22.891%

Figure 6.1 – Depicts Optimal Portfolio Weights

Although, we have certain limitations in both the methodologies of stock portfolio optimization,

1. We have only selected 9 stocks of our choices. Research could have been complete and more sophisticated if a more selective and greater number of stocks were used.
2. We have considered only previous year's data. However, in the real-world factors like market risk (interest rate risk, foreign exchange risk, and political risk) also affect the returns.

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