~~HW #7 — p.1020 - 18, 19, 21, 23, 25, 26, 37-39~~

18. $z = -x$

inside cylinder $= x^2+y^2=4$

$\vec{r}(s,t) = \langle s\cos t, s\sin t, -s\cos t\rangle$

$\frac{\partial r}{\partial t} = s(-\sin t)\hat{i} + s(\cos t)\hat{j} + s(\sin t)\hat{k}$

$\frac{\partial \vec{r}}{\partial s} = \langle \cos t, \sin t, -\cos t \rangle$

$$\frac{\partial r}{\partial t} \times \frac{\partial r}{\partial s} = \begin{vmatrix} i & j & k \\ \cos t & \sin t & -\cos t \\ -s\sin t & s\cos t & s\sin t \end{vmatrix} = (s\sin^2 t + s\cos^2 t)i - (s\cos t\sin t - s\cos t \sin t)j + (s\cos^2 t + s\sin^2 t)\hat{k}$$

$s\hat{i} - 0\hat{j} + s\hat{k}$

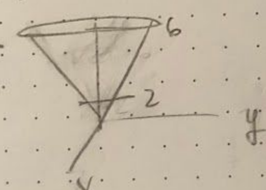
$\sqrt{s^2+s^2} = s\sqrt{2}$

$$\int_0^{2\pi}\int_0^{2} (s\sqrt{2}\, ds)\, dt$$

19. $z = 2\sqrt{x^2+y^2}$

$z=2 \quad z=6$

~~$z^2 = 4x^2+4y^2$~~



$\vec{r}(s,t) = \langle s\cos t, s\sin t, 2s\rangle$

$\frac{\partial r}{\partial t} = \langle -s\sin t, s\cos t, 0\rangle$

$\frac{\partial r}{\partial s} = \langle \cos t, \sin t, 2\rangle$

$$\begin{vmatrix} i & j & k \\ -s\sin t & s\cos t & 0 \\ \cos t & \sin t & 2 \end{vmatrix} = -2s\cos t_i + 2s\sin t_j + (s\cos^2 t + s\sin^2 t)_k$$

$\sqrt{4s^2\cos^2 + 4s^2\sin^2 t + s^2}$

$= \sqrt{5 s^2} = s\sqrt{5}$

$$\int_0^{2\pi}\int_1^{3} (s\sqrt{5})\, ds\, dt$$

21. $x^2+y^2=1$

$z=1 \quad z=4$

$s^2=1$

$s=1$

$\vec{r}(s,t)\ \langle 1\cos t, 1\sin t, s\rangle$

$\frac{\partial r}{\partial s} = 0\hat{i} + 0\hat{j} + 1\hat{k}$

$\frac{\partial r}{\partial t} = -\sin t\,\hat{i} + \cos t\,\hat{j} + 0\,\hat{k}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -\sin t & \cos t & 0 \\ 0 & 0 & 1 \end{vmatrix} = \cos t\,\hat{i} + \sin t\,\hat{j}$$

$\sqrt{\cos^2 t + \sin^2 t} = 1$

$$\int_0^{2\pi}\int_1^{4} 1\, ds\, dt$$

HW #7

23. $z = 2 - x^2 - y^2 \quad z = \sqrt{x^2+y^2}$

$z = 2 - z^2$

$\vec{r}(s,t) = (s\cos t, s\sin t, 2 - s^2)$

$\frac{\partial r}{\partial s} = \cos t\,\hat{i} + \sin\,\hat{j} - 2s\,\hat{k}$

$\frac{\partial r}{\partial t} = s\cos\,i + s\sin\theta\, j$

$\begin{vmatrix} i & j & k \\ \cos t & \sin t & -2s \\ -r\sin t & s\cos t & 0 \end{vmatrix}$

$= (2r^2\cos t)\hat{i} + (2r^2\sin t)j + rk$

$\sqrt{4s^4\cos^2 t + 4s^4\sin^2 t + s^2}$

$= s\sqrt{4s^2+1}$

$\int_0^{2\pi}\int_0^1 s\sqrt{4s^2+1}\,ds\,dt$

25. $x^2+y^2+z^2 = 2 \quad z = \sqrt{x^2+y^2}$

$x = \rho\sin\phi\cos\theta, \; y = \rho\sin\phi\sin\theta$

$z = \rho\cos\phi \Rightarrow \rho = \sqrt{x^2+y^2+z^2} = \sqrt{2}$

$\vec{r}(\phi,\theta) = (\sqrt{2}\sin\phi\cos\theta)i + (\sqrt{2}\sin\phi\sin\theta)j + (\sqrt{2}\cos\phi)k$

$\frac{\partial r}{\partial \phi} = (\sqrt{2}\sin\phi(\cos\theta)i + (\sqrt{2}\cos\phi\sin\theta)j - (\sqrt{2}\sin\phi)k$

$\frac{\partial r}{\partial \theta} = (-\sqrt{2}\sin\phi\sin\theta)i + (\sqrt{2}\sin\phi\cos\theta)j$

$\begin{vmatrix} i & j & k \\ \sqrt{2}\cos\phi\sin\theta & \sqrt{2}\cos\phi\sin\theta & -\sqrt{2}\sin\phi \\ -\sqrt{2}\sin\phi\sin\theta & \sqrt{2}\sin\phi\cos\theta & 0 \end{vmatrix} = (2\sin^2\phi\cos\theta)\hat{i} + (2\sin^2\phi\sin\theta)\hat{j} + (2\sin\phi\cos\phi)\hat{k}$

$-\sqrt{4\sin^4\phi\cos^2\theta + 4\sin^4\phi\sin^2\theta + 4\sin^2\phi\cos^2\phi} - \sqrt{4\sin^2\phi} - 2\sin\phi$

$-2\sin\phi$

$\int_0^{2\pi}\int_{\pi/4}^{\pi} 2\sin\phi\,d\phi\,d\theta$

26. $x^2+y^2+z^2 = 4, \; z = 4, \; z = \sqrt{3}$

$r(\phi,\theta) = (2\sin\phi\cos\theta)i + (2\sin\phi\sin\theta)j + (2\cos\phi)k$

$\frac{\partial r}{\partial \phi} = (2\cos\phi\cos\theta)i + (2\cos\phi\sin\theta)j - (2\sin\phi)k$

$\frac{\partial r}{\partial \theta} = (-2\sin\phi\sin\theta)i + (2\sin\phi\cos\theta)j$

$\begin{vmatrix} i & j & k \\ 2\cos\phi\cos\theta & 2\cos\phi\sin\theta & -2\sin\phi \\ -2\sin\phi\sin\theta & 2\sin\phi\cos\theta & 0 \end{vmatrix} = (4\sin^2\phi\cos\theta)i + (4\sin^2\phi\sin\theta)j + (4\sin\phi\cos\phi)k$

$\sqrt{16\sin^4\phi\cos^2\theta + 16\sin^4\phi\sin^2\theta + 16\sin^2\phi\cos^2\phi}$

$\int_0^{2\pi}\int_{\pi/6}^{2\pi/3} 4\sin\phi\,d\phi\,d\theta$

HW #7

37. $x^2+y^2-z=0$ $\quad z=0 \quad z=2$

$\nabla f = 2x\hat{i} + 2y\hat{j} - \hat{k}$ $\quad p=k$

$\sqrt{(2x)^2+(2y)^2+(-1)^2} = \sqrt{4x^2+4y^2+1}$

$\iint_R \sqrt{4x^2+4y^2+1}\,dx\,dy = \iint_R \sqrt{4r^2\cos^2\theta+4r^2\sin^2\theta+1}\, r\,dr\,d\theta$

$\int_0^{2\pi}\int_0^{\sqrt{2}} \sqrt{4r^2+1}\, r\,dr\,d\theta$

38. $x^2+y^2-z=0$ $\quad z=2, \cancel{z=6}$

$\nabla f = 2x\hat{i} + 2y\hat{j} - \hat{k}$ $\quad \sqrt{4x^2+4y^2+1}$

$\iint_R \sqrt{4x^2+4y^2+1} = \iint_R \sqrt{4r^2+1}\, r\,dr\,d\theta = \int_0^{2\pi}\int_{\sqrt{2}}^{\sqrt{6}} \sqrt{4r^2+1}\, r\,dr\,d\theta$

39. $x+2y+2z=5$ $\quad x=2-y^2$ $\quad p=k$

$|\nabla f\cdot p| = 2$

$\nabla f = \hat{i}+2\hat{j}+2\hat{k}$

$\sqrt{1^2+2^2+2^2} = 3$

~~$x=y^2$ and $x=2-y^2$~~ intersect @ $(1,1)$ and $(1,-1)$

$\iint_R \frac{|\nabla f|}{|\nabla f\cdot p|}\,dA = \iint_R \frac{3}{2}\,dx\,dy = \int_{-1}^{1}\int_{y^2}^{2-y^2} \frac{3}{2}\,dx\,dy$

HW # 7 - 1030, # ~~14~~ - ~~22~~

19. $F = z^2 i + x j - 3z k \quad z = 4 - y^2$

$\bar{r}(x,y) = \langle x, y, 4-y^2 \rangle$

$\frac{\partial r}{\partial x} = 1, 0, 0$

$\frac{\partial r}{\partial y} = 0, 1, -2y$

$$\begin{vmatrix} i & j & k \\ 1 & 0 & 0 \\ 0 & 1 & -2y \end{vmatrix} = -2y j + k$$

$F = \langle z^2, x, -3z \rangle$

$\langle (4-y^2)^2, x, -3(4-y^2) \rangle$

$\langle y^4 - 8y^2 + 16, x, 3y^2 - 12 \rangle$

$F \cdot N = \langle y^4 - 8y^2 + 16, x, 3y^2 + 12 \rangle \cdot \langle 0, 2y, 1 \rangle$

$0 + 2yx + 3y^2 - 12$

$= 2xy + 2y^2 - 12$

$$\int_0^1 \int_{-2}^{2} (2xy + 3y^2 - 12)\, dy\, dx$$

20. $F = x^2 \hat{j} - xz \hat{k} \qquad y = x^2 \qquad y = x^2 \quad -1 \le x \le 1$

$z = 0 \quad z = 2$

$f(x,z) = x \hat{i} + x^2 \hat{j} + z \hat{k}$

$\frac{\partial f}{\partial x} = \hat{i} + 2x \hat{j} + 0 \hat{k}$

$\frac{\partial f}{\partial z} = 0 \hat{i} + 0 \hat{j} + \hat{k}$

$$\begin{vmatrix} i & j & k \\ 1 & 2x & 0 \\ 0 & 0 & 1 \end{vmatrix} = (2x - 0)\hat{i} - (1-0)\hat{j} + (0-0)\hat{k}$$

$= 2x \hat{i} - \hat{j}$

$(0 \hat{i} + x^2 \hat{j} + (-xz)\hat{k}) \cdot (2x \hat{i} - \hat{j} + 0 \hat{k})$

$(2 \cdot 2x) + (x^2 \cdot -1) + (-xz \cdot 0) = -x^2$

$$\int_0^2 \int_{-1}^{1} -x^2\, dx\, dz$$

21. $F = zk \quad x^2 + y^2 + z^2 = a^2$

$r(\phi, \theta) = (a \sin\phi \cos\theta) i + (a \sin\phi \sin\theta) j + (a \cos\phi) k$

$\frac{\partial r}{\partial \phi} = (a \cos\phi \cos\theta) i + (a \cos\phi \sin\theta) j - (a \sin\phi) k$

$\frac{\partial r}{\partial \theta} = (-a \sin\phi \sin\theta) i + (a \sin\phi \cos\theta) j$

$$\begin{vmatrix} i & j & k \\ a\cos\phi\cos\theta & a\cos\phi\sin\theta & -a\sin\phi \\ -a\sin\phi\sin\theta & a\sin\phi\cos\theta & 0 \end{vmatrix} = (a^2 \sin^2\phi \cos\theta) i + (a^2 \sin^2\phi \sin\theta) j + (a^2 \sin\phi \cos\phi) k$$

$F \cdot N\, d\sigma = F \cdot \frac{r_\phi r_\theta}{|r_\phi r_\theta|} \cdot |r_\phi r_\theta|\, d\theta\, d\phi = a^3 \cos^2\phi \sin\phi\, d\theta\, d\phi$

$F = zk = a \cos\phi\, \hat{k}$

$$\iint_S F \cdot N\, d\sigma = \int_0^{\pi/2} \int_0^{\pi/2} a^3 \cos^2\phi \sin\phi\, d\phi\, d\theta$$

HW #7

22. $F = x\,i + y\,j + z\,k \quad x^2 + y^2 + z^2 = a^2$

$r(\phi, \theta) = (a\sin\phi\cos\theta)\,i + (a\sin\phi\sin\theta)\,j + (a\cos\phi)\,k$

$\frac{\partial r}{\partial \phi} = (a\cos\phi\cos\theta)\,i + (a\cos\phi\sin\theta)\,j - (a\sin\phi)\,k$

$\frac{\partial r}{\partial \theta} = (-a\sin\phi\sin\theta)\,i + (a\sin\phi\cos\theta)\,j$

$$\begin{vmatrix} i & j & k \\ a\cos\phi\cos\theta & a\cos\phi\sin\theta & -a\sin\phi \\ -a\sin\phi\sin\theta & a\sin\phi\cos\theta & 0 \end{vmatrix} = (a^2\sin^2\phi\cos\theta)\,i + (a^2\sin^2\phi\sin\theta)\,j + (a^2\sin\phi\cos\phi)\,k$$

$F \cdot N\,d\sigma = F \cdot \frac{r_\phi \times r_\theta}{|r_\phi \times r_\theta|}\,|r_\phi \times r_\theta|\,d\theta\,d\phi = a^3\sin^3\phi\cos^2\phi + a^3\sin^3\phi\sin^2\theta + a^3\sin\phi\,(\cos^2\phi)\,d\theta\,d\phi$

$\iint_S F \cdot N\,d\sigma = \int_0^{2\pi}\int_0^{\pi} a^3\sin\phi\,d\phi\,d\theta$

Worksheet

1. b) $r(s,t) = (-s, 4-s^2, t)$, $s: -2 \to 2 \quad t: 0 \to 5$

$\frac{\partial r}{\partial s} = (-1, -2s, 0)$

$\frac{\partial r}{\partial t} = (0, 0, 1)$

$$\frac{\partial r}{\partial s} \times \frac{\partial r}{\partial t} = \begin{vmatrix} i & j & k \\ -1 & -2s & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1(-2s\,i + j) = -2s\,i + j$$

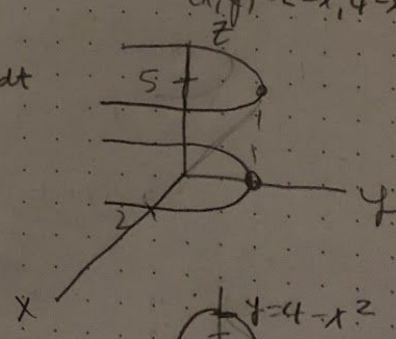
$\left\| \frac{\partial r}{\partial s} \times \frac{\partial r}{\partial t} \right\| = \sqrt{4s^2+1}$

$\text{Area} = \int_0^5 \int_{-2}^{2} \sqrt{4s^2+1}\,ds\,dt$

(a) $r(s,t) = \langle -s, 4-s^2, t \rangle$

$s = x \quad t = y$

$r(x,y) = \langle -x, 4-x^2, y \rangle$



c) $v(x,y,z) = (2, 3y\ 5z)$

$\iint F \cdot N\,ds = \iint [2(-2s) + 3y]\,ds\,dt$

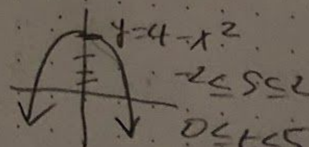
$\int_0^5\int_{-2}^{2} [-4s + 12 - 3s^2]\,ds\,dt$

$= \int_0^5 (-2s^2 + 12s - s^3)\Big|_{-2}^{2}\,dt$

$(-2(0) + 12 \times 4 - 16)\,t\Big|_0^5$

5×32

160



HW #7 - worksheet

2. a) $\vec{r}(s,k) = 5\sin(s)\cos t\,\hat{i} + 5\sin(s)\sin(t)\,\hat{j} + 5\cos(s)\,\hat{k}$

$(5\sin s\cos t)^2 + (5\sin s\sin t)^2 + (5\cos s)^2$

$x^2+y^2+z^2 = r^2$

$25[\sin^2 s\cos^2 t + \sin^2 s\sin^2 t + \cos^2 s)$

$25[\sin^2 s[\cos^2 t + \sin^2 t] + \cos^2 s]$

$25[\sin^2 s + \cos^2 s] = 25$

b) $\iint dA = $ Surface Area

$x = r\sin\theta\cos\phi \quad y = r\sin\theta\sin\phi \quad z = r\cos\theta$

~~$dA = r^2\sin\theta d\phi$~~

$dA = r^2\sin\phi\, dr\, d\theta$

$$\int_0^{\pi/2}\int_0^{\pi/2} 5^2\sin s\, dt\, ds$$

$$\int_0^{\pi/2} 5^2 \times \frac{\pi}{2}\sin s = \frac{5^2\pi}{2}\left[-\cos s\right]\Big|_0^{\pi/2} = \frac{25\pi}{2}$$

c) $\bar{v} = y^2\hat{j} + z\hat{k}$

$d\bar{A} = \hat{r}\,dA = \hat{r}[r^2\sin(s)\,dt\,ds]$

where $\hat{r} = \frac{\bar{r}}{|r|} = \frac{x\hat{i}+y\hat{j}+z\hat{k}}{r}$

$\bar{v}\cdot d\bar{A} = (y^2\hat{i} + z\hat{k})\cdot\frac{(x\hat{i}+y\hat{j}+z\hat{k})}{\cancel{r}} \times (\cancel{r^2}^{\,r}\sin s\, dt\, ds)$

$(y^3+z^2)[r\sin(s)\,dt\,ds]$

$(5^3\sin^3(s)\sin^3 k + 5^2\cos^2(s)) \times 5\sin(s)\,dt\,ds$

$$\int_0^{\pi/2}\int_0^{\pi/2}[5^3\sin^3(s)\sin^3 t + 5^2\cos^2(s)] \times 5\sin(s)\,dt\,ds$$

$$\frac{5^4}{4}\int_0^{\pi/2}\int_0^{\pi/2}\sin^2(s)[3\sin t - \sin(3t)]\,dt\,d\theta - \frac{\pi}{2}\int_0^{\pi/2} 5^3\cos^2(s)\,d(\cos s)$$

$$\frac{5^4}{4}\int_0^{\pi/2}\sin^4(s)\left[3+\frac{1}{3}\right]ds - \frac{\pi}{2}\times 5^3\left[\frac{\cos^3(s)}{3}\right]\Big|_0^{\pi/2}$$

$$\frac{5^4\times 10}{3\times 4}\int_0^{\pi/2}\frac{(3\sin s - \sin 3s)}{4}\sin s\, ds + \frac{\pi}{6}5^2$$

$$\frac{5^4}{2^4}\cdot\frac{3}{2}\cdot\frac{\pi}{2} - 0 + 0 - 0 + \frac{\pi}{6}\times 5^3$$

$$= \frac{3\pi\times 5^4}{96} + \frac{\pi}{6}\times 5^3$$

12/9 - Flu

$\iint_S \vec{F}\cdot\vec{N}$

Applications

1) $\vec{F}$

2) $\vec{F}$

3) $\vec{F}$

4) $\vec{F}$

Problem 1: T(x,y

surfac

What

Heat = $\iint_S$ -

1) $\nabla T = \langle 2x, 4y$

2)

3) Heat Flow

$\vec{N} = \begin{vmatrix} i \\ 1 \\ 0 \end{vmatrix}$

$\int_{-5}^{5}\int_{-2}^{2} -K\langle 2$

$\int_{-5}^{5}\int_{-2}^{2} -K[$