# Exam 2 Solutions

Sunday, December 15, 2019 6:54 AM

**Points:**

Question #1: 4 points total
, per section (a, b, c, d)

Question #2: 2 pts
, per section

Question #3: 2 pts
, per section

Question 4: 2 points

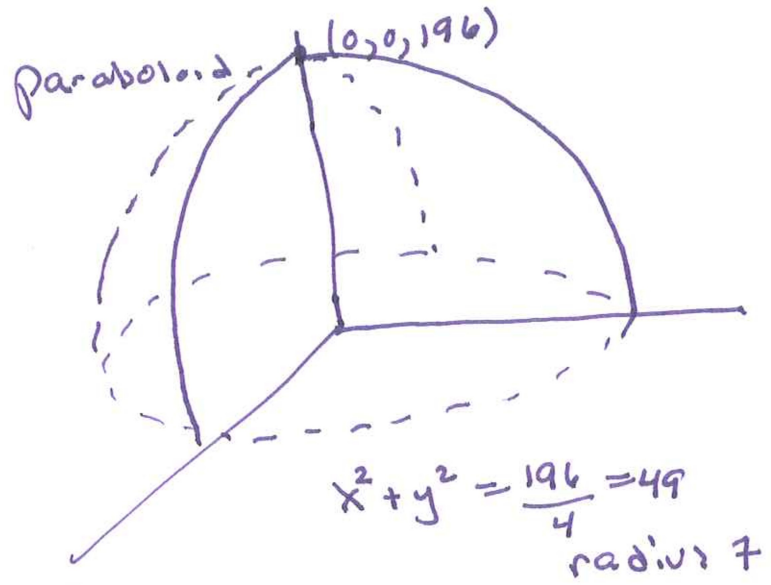
10 pts

Exam 2 Solutions

# Solutions

1.) $z = 196 - 4x^2 - 4y^2$

a.)




b.) $$\text{Volume} = \int_{-7}^{+7} \int_{-\sqrt{49-x^2}}^{\sqrt{49-x^2}} (196 - 4x^2 - 4y^2)\, dy\, dx$$

$$= \int_0^{2\pi} \int_0^{7} (196 - 4r^2)\, r\, dr\, d\theta$$

$$= \int_0^{2\pi} \int_0^{7} \int_0^{196-4r^2} r\, dz\, dr\, d\theta$$

etc.

1 d) $\vec{F}$ on surface:

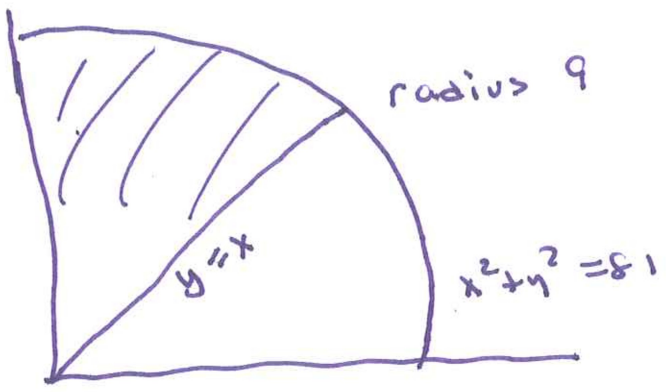
$$\vec{F}(\vec{r}(s,t)) = \vec{F}\left(s\cos t,\ s\sin t,\ \sqrt{196-4s^2}\right)$$

$$= \langle 5s\cos t,\ 7,\ 0\rangle$$

$$\vec{F}\cdot\vec{N} = \frac{20s^3\cos t}{\sqrt{196-4s^2}} + \frac{28s^2\sin t}{\sqrt{196-4s^2}}$$

$$\text{Flux} = \int_0^{2\pi}\int_0^{7}\left(\quad\right)\,ds\,dt$$

2.)




(1 pt)

$$= \int_{\pi/4}^{\pi/2} \int_{0}^{9} \frac{1}{\sqrt{9+3r^2}}\, r\, dr\, d\theta$$

$$\int_0^9 (9+3r^2)^{-1/2}\, r\, dr \qquad \frac{1}{6}\int u^{-1/2}\, du$$

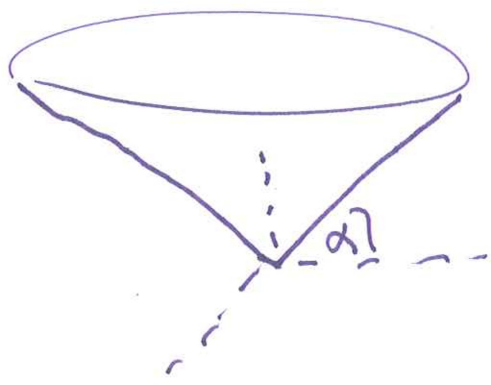
$$= \frac{1}{3}(9+3r^2)^{1/2}\Big|_0^9 \qquad = \frac{1}{6}\,\frac{u^{1/2}}{\frac{1}{2}} = \frac{1}{3}u^{1/2}$$

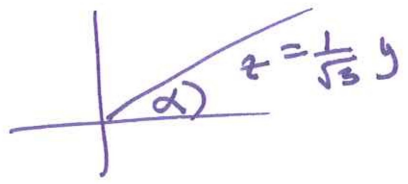
$$= \frac{1}{3}\left[\sqrt{252} - \sqrt{9}\right] \qquad (1\text{ pt})$$

Final integral $\int_0^{2\pi} \quad d\theta$

$$= \frac{2}{3}\pi\left[\sqrt{252} - 3\right] \quad \underline{\text{ans}}$$

3.)




$\alpha = 30^\circ$

Cartesian: $V = \int_{-20\sqrt{3}}^{20\sqrt{3}} \int_{-\sqrt{}}^{+\sqrt{}} \int_{\sqrt{\frac{x^2+y^2}{3}}}^{20} dz\,dy\,dx$

Circle on top:
$20^2 = \frac{x^2+y^2}{3}$
$x^2+y^2 = 3 \cdot 20^2$
so radius $20\sqrt{3}$

Polar: $\int_0^{2\pi} \int_0^{20\sqrt{3}} \left(20 - \frac{r}{\sqrt{3}}\right) r\,dr\,d\theta$

Cylindrical: $\int_0^{2\pi} \int_0^{20\sqrt{3}} \int_{r/\sqrt{3}}^{20} r\,dz\,dr\,d\theta$

Spherical: $\int_0^{2\pi} \int_0^{\pi/3} \int_0^{\frac{20}{\cos\phi}} \rho^2 \sin\phi\,d\rho\,d\phi\,d\theta$

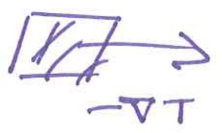



$\cos\phi = \frac{20}{\rho}$ $\rho = \frac{20}{\cos\phi}$

4.) a.) Heat Flow

Fourier's Law of Heat Conduction

rate of heat flow = $-K \nabla T \cdot \vec{N}$




Total thru surface: $\iint_R -K \nabla T \cdot \vec{N} \, dA$

K = thermal conductivity coeff etc.

b.) Ion or Gas flow across membrane

$-K \nabla C \cdot \vec{N}$

Fick's Law (s) of Diffusion

C = concentration

K = permeability

c.) Electric Flux

$\iint \vec{E} \cdot \vec{N} \, ds$ $\quad \vec{E}$ = electric field

d.) Fluid Flow

$\vec{V}$ = velocity at $(x, y, t)$

$\iint_S \vec{V} \cdot \vec{N} \, dA$ $\quad ft^3/min$

fluid flow thru S per second