Exam #1 Solutions

Tuesday, November 12, 2019 5:12 PM

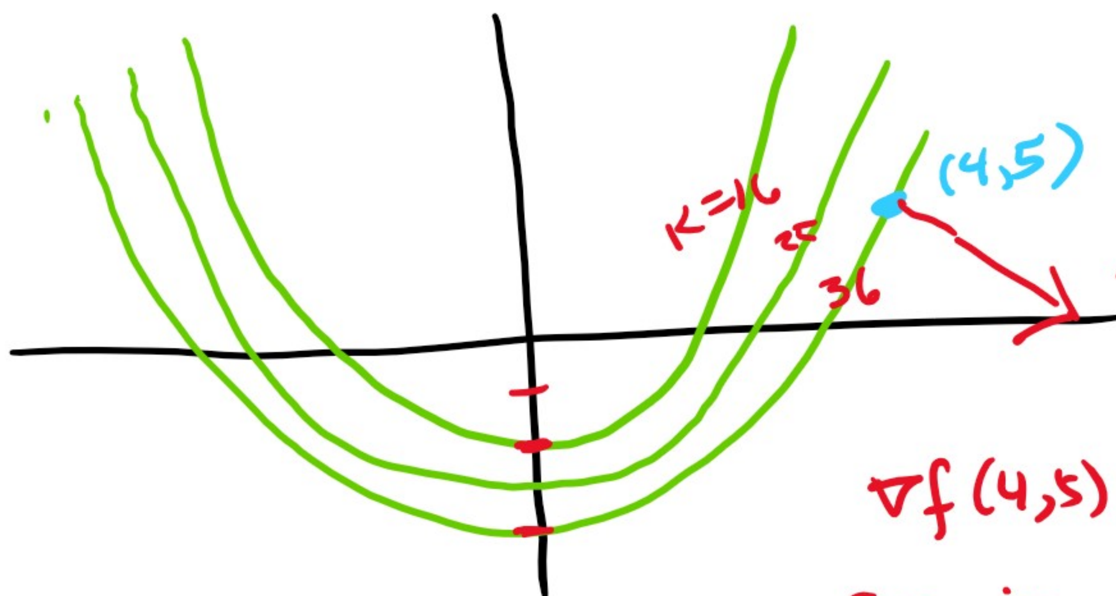
10 pts 2 pts each question

1) $f(x,y) = (x^2 - 2y)^2$

a.) $(x^2-2y)^2 = 16, 25, 36$

$(x^2 - 2y) = \pm 4, \pm 5, \pm 6$ parabolas

$y = \frac{1}{2}x^2 - \frac{\pm\sqrt{k}}{2} \leftarrow$ vertex



$-k$ not shown

$\nabla f(4,5) = \langle 96, -24 \rangle$

so in $\langle 4, -1 \rangle$ direction

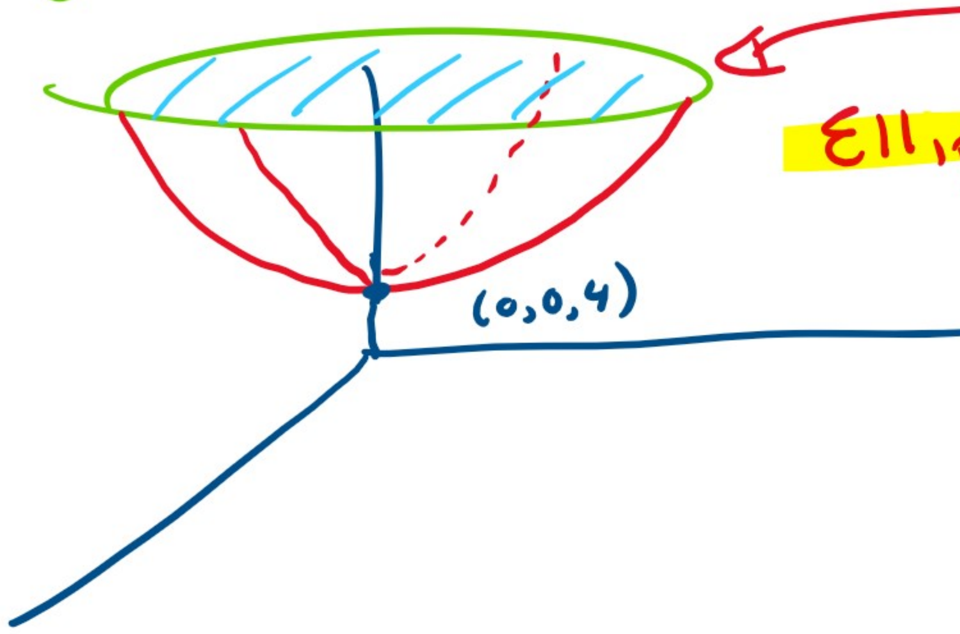
2) $z = 144x^2 + 25y^2 + 4$

a) graph 1.0 pts

$x = 0 \quad z = 25y^2 + 4$ parabola ∪?

$y = 0 \quad z = 144x^2 + 4$ parabola ∪?

$z = k \quad k = 144x^2 + 25y^2 + 4$ ellipses



Elliptic Paraboloid

Tangent Plane 1.0 pts

$F(x,y,z) = z - 144x^2 - 25y^2 = 4$ on the paraboloid,

$F(x,y,z) = z - 144x^2 - 25y^2 = 4$ on the paraboloid,

so it is a constant surface.

$\therefore$ ∇F is normal

$\nabla F = \langle -288x, -50y, 1 \rangle$ (or its negative)

0.5 $\nabla F(0,1,29) = \langle 0, -50, 1 \rangle = \vec{N}$

Plane $\langle 0, -50, 1 \rangle \cdot \langle x-0, y-1, z-29 \rangle = 0$

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3) a.) $\nabla f = \langle -\frac{1}{2}, \frac{5}{8} \rangle$ $\hat{u} = \langle \frac{-4}{5}, \frac{-3}{5} \rangle$

unit vector

2.0

$D_{\hat{u}} f = \nabla f \cdot \hat{u} = \frac{1}{40}$

b) $\langle \frac{1}{2}, \frac{-5}{8} \rangle = -\nabla f$

0.5 each part

c) $|\nabla f| = \sqrt{\frac{1}{4} + \frac{25}{64}} = \sqrt{\frac{41}{64}}$ or $\frac{\sqrt{41}}{8}$

d) zero constant on level curves

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4) 0.5 each deriv. 2.0

yes, satisfied. (everyone got it)

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5) 2.0 pts 0.5 each part

a) $6°F$ decrease for every unit (mile) in the $+x$ direction

$8°F$ incr. for each mile in the $+y$ direction.

so $\nabla T = \langle -6, 8 \rangle$ at $(18, 22)$

b) probably you want to get warmer*,

b.) probably you want to get warmer, so go in the ∇T direction, gain $10^\circ F$ per mile in that direction

$|\nabla T|$

c) has $\langle 8,6 \rangle$ as tangent
has $T = 40^\circ$ everywhere on it

d.) increase by $5(+6) = 30^\circ$, approx.

* other assumptions possible.