HW 3

1. a) $f(x) = 2x^2 + \underbrace{x^3 \log x}_{O(x^4)}$

$n = 4$

b) $f(x) = \underbrace{3x^5}_{O(x^5)} + (\log x)^4$

$n = 5$

c) $f(x) = \frac{(x^4 + x^2 + 1)}{(x^4 + 1)} = 1 + \underbrace{\frac{x^2}{x^4+1}}_{O(1)}$

$n = 0$

d) $f(x) = \frac{(x^3 + 5\log x)}{(x^4 + 1)}$

$\frac{x^3}{x^4} = x^{-1}$

$O(x^{-1})$

$n = -1$

3 b) $W(x)$: I work on x

$S(x)$: It is sunny on x

$P(x)$: It is partly sunny on x

1. $W(\text{Monday}) \vee W(\text{Friday})$ premise
2. $W(x) \to (S(x) \vee P(x))$ premise
3. $\neg P(\text{Friday})$ premise
4. $W(\text{Monday}) \to (S(\text{Monday}) \vee P(\text{Monday}))$ Universal instatiation from (2)
5. $W(\text{Friday}) \to (S(\text{Friday}) \vee P(\text{Friday}))$ Universal instantiation from (2)
6. $W(\text{Friday}) \to S(\text{Friday})$ Disjunctive syllogism from (3) and (5)
7. $S(\text{Monday}) \vee P(\text{Monday}) \vee S(\text{Friday}) \vee P(\text{Friday})$ Modus ponens from (1), (4), & (6)

Conclusion: It was either sunny or partly sunny on Monday or sunny on friday

2. 1. Set firstMin = Max and secondMin = Max
2. iterate the sequence, for i=0 to size
   If a[i] < firstMin then
      1. set SecondMin = firstMin
      2. set firstMin = a[i]
   else if secondMin < a[i] and a[i] != firstMin
      1. set secondMin = a[i]
3. end of iteration
4. firstMin will store the smallest of the sequence
5. secondMin will store the second smallest of the sequence

worst case complexity : $O(n)$

3. a) h: I play hockey

s: I am sore

w: I use the whirlpool

1. $\neg w$ premise
2. $s \to w$ premise
3. $\neg s$ Modus Tollens from (1) and (2)
4. $h \to s$ premise
5. $h \to w$ hypothetical syllogism from (2) and (4)
6. $\neg h$ Modus tollens from (1) and (5)

Conclusion: I did not play hockey

HW3

3 c) $I(x)$: x is an insect

$D(x)$: x is a dragonfly

$\mathcal{L}(x)$: x has legs

$S(x)$: x is a spider

$E(x,y)$: x eats y

1. $\forall x\,[I(x) \rightarrow \mathcal{L}(x)]$ premise
2. $I(c) \rightarrow \mathcal{L}(c)$ universal instantiation from (1)
3. $\forall x\,[D(x) \rightarrow I(x)]$ premise
4. $D(c) \rightarrow I(c)$ universal instantiation from (3)
5. $D(c) \rightarrow \mathcal{L}(c)$ Hypothetical syllogism from (2) & (4)
6. $\forall x\,[D(x) \rightarrow \mathcal{L}(x)]$ universal generalization from (5)
7. $\forall x\,[S(x) \rightarrow \mathcal{L}(x)]$ premise
8. $S(c) \rightarrow \neg \mathcal{L}(c)$ universal instantiation from (7)
9. $\neg \mathcal{L}(c) \rightarrow I(c)$ contrapositive of (2)
10. $S(c) \rightarrow \neg I(c)$ Hypothetical syllogism from (8) and (9)
11. $\forall x\,[S(x) \rightarrow \neg I(x)]$ universal generalization from (10)

Conclusion:
All spiders are not insects, or Spiders are not insects

3 d) $S(x)$: x is a student

$I(x)$: x is an internet account

1. $\forall x\,(S(x) \rightarrow I(x))$ premise
2. $S(\text{Homer}) \rightarrow I(\text{Homer})$ universal instantiation from 1
3. $\neg I(\text{Homer})$ premise
4. $\neg S(\text{Homer})$ Modus Tolllens from (2) and (3)

Conclusion: Homer is not a student

HW3

3 e) $H(x)$: x is healthy to eat

$G(x)$: x tastes good

$E(x)$: You eat x

1. $\forall x\ (H(x) \to \neg G(x))$ premise
2. $H(tofu) \to \neg G(tofu)$ universal instatiation from 1
3. $H(tofu)$ premise
4. $\neg G(tofu)$ Modus ponens from (2) and (3)
5. $\forall x (E(x) \leftrightarrow G(x))$ premise
6. $E(c) \leftrightarrow G(c)$ universal instantiation from (5)
7. $H(c) \to \neg G(c)$ universal instantiation from (1)
8. $\neg E(c) \leftrightarrow \neg G(c)$ Contrapositive of 6
9. $H(c) \to \neg E(c)$ Hypothetical syllogism from (7) and (8)
10. $\forall x (H(x) \to \neg E(x))$ Universal generalization from (9)

Conclusion: you do not eat healthy foods

3 f) d: I am dreaming

h: I am hallucinating

e: I see elephants running down the road

1. $\neg d$ premise
2. $d \vee h$ premise
3. h disjunctive syllogism
4. $h \to e$ premise
5. e modus ponens from (3) and (4)

Conclusion: I see elephants running down the road

4. a) no

b) yes

5. rational (irrational) = rational

$\frac{a}{b}\ (x) = \frac{m}{n}$

$x = \frac{mb}{na}$ integer values, so rational

irrational

as a result x has to be rational, leading to a contradiction

HW 3

6. $m = 2a+1$

$n = 2b+1$

$(2a+1)(2b+1) = 4ab+2a+2b+1 = 2\underbrace{(2ab+a+b)}_{\mathcal{N} = 2ab+a+b \in \mathbb{Z}}+1 = 2\mathcal{N}+1$

Because mn is odd when n is odd and m is odd,
mn is even when n is even or m is even. } Contrapositive