CS 2022/ MA 2201 Discrete Mathematics
A term 2020

Solutions for Homework 1

1. Exercise 16 on page 14.

   Solution:

   (a) $r \wedge \neg q$

   (b) $p \wedge q \wedge r$

   (c) $r \to p$

   (d) $p \wedge \neg q \wedge r$

   (e) $(p \wedge q) \to r$

   (f) $r \leftrightarrow (q \vee p)$

   (20 points)

2. Construct truth tables for each of the following compound propositions.

   (a) $(p \wedge q) \vee (p \wedge r)$

   (b) $(q \wedge p) \leftrightarrow (q \oplus p)$

   Solution:

(a)

p	q	r	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	F

(b)

p	q	$(q \wedge p) \leftrightarrow (q \oplus p)$
T	T	F
T	F	F
F	T	F
F	F	T

(20 points)

3. Are the following compound propositions tautologies?

   (a) $((p \to q) \wedge (q \to r)) \to (p \to r)$

   (b) $((p \wedge q) \wedge (q \wedge r)) \to (p \wedge r)$

   (c) $((p \oplus q) \wedge (q \oplus r)) \to (p \oplus r)$

   In other words are the logical operators implication, conjunction and exclusive-or transitive?

   Solution:

   (a) $((p \to q) \wedge (q \to r)) \to (p \to r)$ is a tautology because

p	q	r	$((p \to q) \wedge (q \to r)) \to (p \to r)$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	T

   (b) $((p \wedge q) \wedge (q \wedge r)) \to (p \wedge r)$ is also a tautology because

p	q	r	$((p \wedge q) \wedge (q \wedge r)) \to (p \wedge r)$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	T

(c) $((p \oplus q) \wedge (q \oplus r)) \rightarrow (p \oplus r)$ is not a tautology as seen by the case where p and r are T and q is F, or the case where p and r are F and q is T.

Thus the operators $\rightarrow$ and $\wedge$ are transitive but $\oplus$ is not transitive. (20 points)

4. Exercise 28 on page 38.

Solution: They are logically equivalent since they have the same columns in the truth table:

p	q	r	$(p \rightarrow q) \vee (p \rightarrow r)$	$p \rightarrow (q \vee r)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	T	T
F	T	F	T	T
F	F	T	T	T
F	F	F	T	T

(20 points)

5. Exercise 16 on page 70.

Solution: There are more than one correct solutions. Here is one. Let $P(s, c, m)$ be the statement that student s has class standing c and is majoring in m. The variable s ranges over students in the class, the variable c ranges over the four class standings, and the variable m ranges over all possible majors.Then:

(a) $\exists s \exists m P(s, junior, m)$. This is T.

(b) $\forall s \exists c P(s, c, computerscience)$. This is F, since there are some mathematics majors.

(c) $\exists s \exists c \exists m\, (P(s, c, m) \wedge (c \neq junior) \wedge (m \neq mathematics))$. This is true, since there is a sophomore majoring in computer science.

(d) $\forall s (\exists c P(s, c, computerscience) \vee \exists m P(s, sophomore, m))$. This is false, since there is a freshman mathematics major.

(e) $\exists m \forall c \exists s P(s, c, m)$. This is false. It cannot be that m is mathematics, since there is no senior mathematics major, and it cannot be that m is computer science, since there is no freshman computer science major. Nor, of course, can m be any other major.

(20 points)