1. a) Every set contains @ least one subset because an empty set is a subset of every set. The power set therefore contains @ least one element. That means the power set cannot be empty NO

b) $\{\emptyset, \{a\}\}$ is the power set of $\{a\}$ because the empty set is a subset of a, and since $\{a\}$ only contains one element, $\emptyset$ and $\{a\}$ are included in the power set

$\emptyset \subseteq \{a\}$

$\{a\} \subseteq \{a\}$ yes

$P(\{a\}) = \{\emptyset, \{a\}\}$

c) Subsets of $\{\emptyset, a\}$ are $\{\emptyset\}, \emptyset, \{a\}$. $\{\emptyset\}$ is not an element of $\{\emptyset, \{a\}, \{\emptyset, a\}\}$ so it is not a power set NO

d) $\emptyset \subseteq \{a, b\}$

$\{a\} \subseteq \{a, b\}$ therefore $P(\{a, b\}) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$

$\{b\} \subseteq \{a, b\}$ yes

$\{a, b\} \subseteq \{a, b\}$

2. $(A-B)-C = (A-C)-(B-C)$

$(A-C)-(B-C) = \{x \mid x \in (A-C)-(B-C)\}$

$\{x \mid x \in (A-C) \land \neg(x \in B-C)\}$

$\{x \mid (x \in A \land \neg(x \in C)) \land \neg(x \in B \land \neg(x \in C))\}$

$\{x \mid (x \in A \land \neg(x \in C)) \land \neg(x \in B \lor \neg(\neg(x \in C))\}$

$\{x \mid (x \in A \land \neg(x \in C)) \land (\neg(x \in B) \lor x \in C))\}$

$\{x \mid x \in A \land (\neg(x \in C) \land (\neg(x \in B) \lor x \in C)))\}$

$\{x \mid x \in A \land (\neg(x \in C) \land (\neg(x \in B) \lor (\neg(x \in C) \land x \in C)))\}$

$\{x \mid x \in A \land \neg(x \in B) \land \neg(x \in C)$

$\{x \mid (x \in A \land \neg(x \in B)) \land \neg(x \in C)\}$

$\{x \mid (x \in (A-B) \land \neg(x \in C)\}$

$\{x \mid \in (A-B)-C\}$ ✓

Check:

A	B	C	A-B	A-C	B-C	(A-B)-C	(A-C)-(B-C)
1	1	1	0	0	0	0	0
1	1	0	0	1	1	0	0
1	0	1	1	0	0	0	0
1	0	0	1	1	0	1	1
0	1	1	0	0	0	0	0
0	1	0	0	0	1	0	0
0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0

↑ ↑

Both columns are the same,

$\therefore (A-B)-C = (A-C)-(B-C)$

3. $f: A \to B$

one to one: $f(a) = f(b)$ implies $a = b$ for all a and b

onto: for every $b \in B$ there exists an element $a \in A$ such that $f(a) = b$

a) $f(n) = 2n$

$f(a) = f(b) \to 2a = 2b \therefore a = b$ one to one ✓

$f(c) = 3$ does not exist onto X

3. b) $f(n) = [n/2]$

$f(a) = f(b) \rightarrow [a/2] = [b/2]$

If $a = 1$ and $b = 2$

$[1/2] = [2/2] \rightarrow [.5] = [1] \rightarrow 1 = 1$

$f(a) = f(b)$ but $a \neq b$ — one to one ✗

$f(2n) = [2n/2] = [n] = n$ onto ✓

c) $f(n) = \begin{cases} n+1 & \text{if } n \text{ is even} \\ n-1 & \text{if } n \text{ is odd} \end{cases}$

If a & b are even $f(a) + f(b) \rightarrow a+1 = b+1 \rightarrow a = b$

If a & b are odd $f(a) + f(b) \rightarrow a-1 + b-1 \rightarrow a = b$

one to one ✓

If a is even $f(a) = a+1$

If a is odd $f(a) = a-1$

onto ✓

$a-1$ and $a+1 \in A = N$

d) $f(n) = 0$

Not one to one because every $n \in N$ has the same integer

Not onto because every $f(n)$ is not the image of any N

4. a) $$\sum_{j=0}^{8} (1 + (-1)^j) = \sum_{j=0}^{8} 1 + \sum_{j=0}^{8} (-1)^j = a_0(n+1) + a_0\left[\frac{r^{n-1} - 1}{r-1}\right]$$

$$= 1(8+1) + 1\left[\frac{(-1)^9 - 1}{-1-1}\right]$$

$$= 9 + 1$$

$$= 10$$

b) $$\sum_{j=0}^{8} (3^j - 2^j) = \sum_{j=0}^{8} 3^j - \sum_{j=0}^{8} 2^j$$

$$= a_0\left[\frac{r^{n+1} - 1}{r-1}\right] - a_0\left[\frac{r^{n+1}}{r-1}\right]$$

$$= 1\left[\frac{3^9 - 1}{3-1}\right] - 1\left[\frac{2^9 - 1}{2-1}\right]$$

$$= 9841 - 511$$

$$= 9330$$

4 c) $\sum_{j=0}^{8} (2\cdot 3^j + 3\cdot 2^j) = \sum_{j=0}^{8} 2\cdot 3^j + \sum_{j=0}^{8} 3\cdot 2^j$

$$= a_0\left[\frac{r^{n+1}-1}{r-1}\right] - a_0\left[\frac{r^{n+1}}{r-1}\right]$$

$$= 2\left[\frac{3^9-1}{3-1}\right] + 3\left[\frac{2^9-1}{3-1}\right]$$

$$= 2(9841) + 3(511)$$

$$= 21215$$

d) $\sum_{j=0}^{8} 2^{j+1} - 2^j = \sum_{j=0}^{8} 2^{j+1} - \sum_{j=0}^{8} 2^j = \sum_{j=0}^{8} 2^1\cdot 2^j - \sum_{j=0}^{8} 2^j$

$$= a_0\left[\frac{r^{n+1}-1}{r-1}\right] - a_0\left[\frac{r^{n+1}+1}{r-1}\right]$$

$$= 2\left[\frac{2^9-1}{2-1}\right] - \left[\frac{2^9-1}{2-1}\right]$$

$$= \frac{2^9-1}{2-1}$$

$$= 511$$

5. set of $\mathbb{Z}$ is countably infinite

$\mathbb{Z} = \{\ldots, -1, 0, 1, 2, \ldots\}$

Let S = set of integers that are not multiples of 3 that are countably infinite

$$S = \{\underset{1}{-2}, \underset{2}{-1}, \underset{3}{1}, \underset{4}{2}, \underset{5}{4}, \underset{6}{5}, \underset{7}{7}, \ldots\}$$

$$f: \mathbb{Z}^+ \to S, (f(n) = \begin{cases} 3(n/4 - 1) + 1 & \text{if } n \text{ is divisible by } 4 \\ 3(n-1)/4 + 2 & \text{if } n-1 \text{ is divisible by } 4 \text{ and } n \geq 1 \\ -(3(n-2)/4 + 1) & \text{if } n-2 \text{ is divisible by } 4 \text{ and } n \geq 2 \\ -(3(n-3)/4 + 2) & \text{if } n-3 \text{ is divisible by } 4 \text{ and } n \geq 3 \end{cases}$$

f is a one to one correspondence between the positive integers and S