**CS 2022/ MA 2201 Discrete Mathematics**
**A term 2020**

# Solutions for Homework 2

1. Exercise 26 on page 132.

   **Solution:**

   a) The power set of every set includes at least the empty set, so the power set cannot be empty. Thus $\emptyset$ is not the power set of any set.

   b) This is the power set of $\{a\}$.

   c) This set has three elements. Since 3 is not a power of 2, this set cannot be the power set of any set.

   d) This is the power set of $\{a, b\}$. (20 points)

2. Exercise 26 on page 145.

   **Solution:** By using a membership table:

| $A$ | $B$ | $C$ | $(A-C)-(B-C)$ | $(A-B)-C$ |
|---|---|---|---|---|
| 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |

   (20 points)

3. Exercise 20 on page 162.

   **Solution:**

   (a) $f(n) = n + 1$

(b) $f(n) = \lfloor \frac{n}{2} \rfloor$

(c) $f(n) = \begin{cases} n+1 & \text{if } n \text{ is even} \\ n-1 & \text{if } n \text{ is odd} \end{cases}$

Thus we have $f(0) = 1$, $f(1) = 0$, $f(2) = 3$, $f(3) = 2$, etc.

(d) $f(n) = 2022$

(20 points)

4. Exercise 32 on page 179.

**Solution:**

(a)

$$\sum_{j=0}^{8}(1 + (-1)^j) = \sum_{j=0}^{8} 1 + \sum_{j=0}^{8}(-1)^j = 9 + 1 = 10$$

(b)

$$\sum_{j=0}^{8}(3^j - 2^j) = \frac{3^9 - 1}{2} - (2^9 - 1) = 9330$$

(c)

$$\sum_{j=0}^{8}(2 \times 3^j + 3 \times 2^j) = 2\sum_{j=0}^{8} 3^j + 3\sum_{j=0}^{8} 2^j = 2\frac{3^9 - 1}{2} + 3(2^9 - 1) =$$

$$= 19682 + 1533 = 21215$$

(d)

$$\sum_{j=0}^{8}(2^{j+1} - 2^j) = 2^9 - 1 = 511$$

(20 points)

5. Show that the set of integers that are *not* multiples of 3 is a countable set.

**Solution:**

Here is a bijection between the set of positive integers and the set of integers that are not multiples of 3:

$$f(n) = \begin{cases} 3k+1 & \text{if } n = 4k+1 \\ -(3k+1) & \text{if } n = 4k+2 \\ 3k+2 & \text{if } n = 4k+3 \\ -(3k+2) & \text{if } n = 4k+4 \end{cases}$$

Thus $f(1) = 1, f(2) = -1, f(3) = 2, f(4) = -2$, etc. we get all the integers that are not multiples of 3 exactly once. (20 points)