Problem 1 - 10 points total
Express the numbers given, in binary form (using the ordinary, and NOT 2's complement, positive only binary positional number system)

(a) $(3D)_{16}$

0011 1101

(b) $(63)_8$

110 011

(c) $(AC)_{16}$

1010 1100

(d) $(24)_5$

$2(5)^1 + 4(5)^0 = 10 + 4 = (14)_{10}$

1110

(e) $(2345)_{16}$

0010 0011 0100 0101

(f) $(3255)_8$

011 010 101 101

(g) $(ACFE)_{16}$

1010 1100 1111 1110

(h) $(29)_{10}$

2	29
2	14 R1
2	7 R0
2	3 R1
2	1 R1

11101

(i) $(C4D278)_{16}$

1100 0100 1101 0010 0111 1000

(j) $(4156)_8$

100 001 101 110

Problem 2 -20 points total

Express the numbers given, in binary form (using the ordinary, and NOT 2's complement, positive only binary positional number system). Perform the conversion using the method of repeated division or repeated multiplication as appropriate, showing all steps.

(a) $(3.703125)_{10}$

2 | 3
2 | 1 R1 ⇒ 11

	Number	# after decimal	# before decimal
.703125(2)	1.40625	.40625	1
.40625(2)	.8125	.8125	0
.8125(2)	1.625	.625	1
.625(2)	1.25	.25	1
.25(2)	.5	.5	0
.5(2)	1.0	0	1

$(11.101101)_2$

(b) $(12.5078125)_{10}$

2 | 12
2 | 6 R0
2 | 3 R0 ⇒ 1100
2 | 1 R1

	Number	# after decimal	# before decimal
.5078125(2)	1.01563	.01563	1
.01563(2)	.03126	.03126	0
.03126(2)	.06252	.06252	0
.06252(2)	.12504	.12504	0
.12504(2)	.25008	.25008	0
.25008(2)	.50016	.50016	0
.50016(2)	1.00032	.00032	1
.00032(2)	.00064	.00064	0
.00064(2)	.000128	.000128	0

$(1100.1000001)_2$

Problem 3 -20 points total
Convert the following numbers (which are given in ordinary "unsigned" positional number system and not 2's complement format) to decimal values:

(a) $(C.82)_{16}$

12.507

$12(16)^0 + 8(16^{-1}) + 2(16^{-2})$

$(12.82)_{10}$

(b) $(14.404)_8$

14
$8^1\,8^0$

$4(8^{-1}) + 4(8^{-3}) =$

$= \cancel{4(8^2)+0(8)+4}$

$8(1)+4=12$

$= \cancel{4(64)+4}$

$= (4(65))$

260

12.507

$(12.260)_{10}$

(c) $(11101.110011\ldots)_2$

$1(2^4)+1(2^3)+1(2^2)+0+2^0+2^{-1}+2^{-2}+2^{-5}+2^{-6}$

$= 16 + 8 + 4 + 2^0 + .5 + .25 + .03125 + .015625$

$= 29.7969$

$(29.7969)_{10}$

(d) $(11001.0001)_2$

$2^4 + 2^3 + 2^0 + 2^{-4} =$

$16 + 8 + 1 + .0625$

$= 25.0625$

$(25.0625)_{10}$

Problem 4 – 40 points total

(a) Complete the table below: [5 pts]

Range: n-bit unsigned and signed (2's complement) numbers

	2-bit	3-bit	4-bit	8-bit	16-bit
Unsigned	0-3	0-7	0-15	0-255	0 - 65,535
2's Complement	-2^{2-1} to $2^{2-1}-1$ = -2 to 1	-4 to 3	-8 to 7	-128 to 127	-2^{16-1} to $2^{16-1}-1$ = -32768 to 32767

(b) Add -118 and -32 firstly using 8-bit 2's complement arithmetic and then using 16-bit 2's complement arithmetic, Comment on the results. **[20 pts]**

$(-118)_{10} + (-32)_{10}$

-118 → 01110110 → 10001001 + 1 = 10001010

-32 → 00100000 → 11011111 + 1 = 11100000

-118_{10} : 10001010
+ -32_{10} : 11100000
= 101101010

01101010

2 | 118
2 | 59 R0
2 | 29 R1
2 | 14 R0
2 | 7 R1
2 | 3 R1
2 | 1 R1

2 | 32
2 | 16 R0
2 | 8 R0
2 | 4 R0
2 | 2 R0
2 | 1 R1

16-bit

-118 → 00000000 01110110 → 11111111 10001001 + 1 = 1111111110001010

-32 → 00000000 00100000 → 11111111 11011111 + 1 = 11111111 11100000

1111111110001010
+ 1111111111100000
= 1111111101101010

11111111 01101010

(c) Perform the indicated arithmetic on the following **signed two's complement** binary numbers. **[15 pts]**

Your answer should show the two's complement binary result; also indicate if each result is positive or negative or overflowed (writing out **"positive", "negative" or "overflow" next to the binary answer**) and then follow with the decimal value of the result if there was no overflow. Do not extend (add any additional length) to these representations, but rather use a binary number word size equal to those found in each problem statement.

(i) 01011011 + 11100101

```
  01011011
+ 11100101
----------
 101000000
```

~~negative~~

01000000

positive

(ii) 01001 - 01110

```
  01001
- 10010
-------
  11011
```

negative

(iii) 1111 + 1100

$$\begin{array}{r} 1\,1\,1\,1\,1 \\ +\ 1\,1\,0\,0 \\ \hline 1\,1\,0\,1\,1 \end{array}$$

negative

Overflow

Problem 5 – 10 points total

(a) Perform the following decimal arithmetic problems by first converting the numbers to binary, 2's complement form (using a 7 bit word in every case). Then perform the 2's compliment addition. Show the result in binary indicating as in the problem above whether each result is positive or negative or overflowed (writing out "positive", "negative" or "overflow" next to the binary answer) and then follow with the decimal value of the result if there was no overflow.

(i) 13 – 9

2|13
2|6 R1
2|3 R0
2|1 R1

2|9
2|4 R1
2|2 R0
2|1 R0

13: 1101

$-9: 1001 \xrightarrow{inv} 0110 \xrightarrow{+1} 0111$

$$\begin{array}{r} 0001101 \\ 1110111 \\ \hline 10000100 \end{array}$$

0000100

Positive

-2^{7-1} to $2^{7-1}-1$

-2^6 to 2^6-1

(ii) –13 – 13

$-13: 1101 \xrightarrow{inv} 0010 \xrightarrow{+1} 0011$

$$\begin{array}{r} 1110011 \\ 1110011 \\ \hline 11100110 \end{array}$$

11100110 → 1100110

negative

Problem 6 – 10 points total

Fill the table below:

4-bit signed binary number comparison

Decimal	Signed Magnitude	Signed 1's Complement	Signed 2's Complement
+7	0111	0111	0111
+6	0110	0110	0110
+5	0101	0101	0101
+4	0100	0100	0100
+3	0011	0011	0011
+2	0010	0010	0010
+1	0001	0001	0001
+0	0000	0000	0000
-0	1000	1111	—
-1	1001	1110	1111
-2	1010	1101	1110
-3	1011	1100	1101
-4	1100	1011	1100
-5	1101	1010	1011
-6	1110	1001	1010
-7	1111	1000	1001