



WPI

Department of
Physics

Significant Figures and Uncertainty

Lab Objectives

- Formatting using Significant Figures
- Reporting a measurement
- Propagation of Uncertainty
- Accuracy versus Precision

Lab Equipment

- Short Ruler

Making a measurement

1. These numbered gray boxes on this Lab Instruction sheet denote that there is a question that you need to answer. There is a companion worksheet in Microsoft Word format. You should answer the questions in the text box corresponding with the question number. Once you have completed the lab and answer worksheet, you should save the file as a PDF and upload the PDF to the Canvas assignment associated with this lab. If you have any trouble converting to a PDF, please ask your Lab Instructor or Lab Assistant.

You should have a small ruler at your station. Use this ruler to measure the width (the shorter side) of your lab computer keyboard, not your laptop keyboard. Once you and your partner have agreed on a number, estimate the uncertainty of your measurement. There are many ways to determine the uncertainty of a measurement in physics, and we will look at a few of them this semester.

For this estimate, we want you to pay attention to the precision of your ruler. The ruler has division markings on it that define the distance between known units of measurements. Your estimate should be based on how accurately you are able to discern where the object you are measuring falls on the ruler. For example, on the scale in Figure 1, the gradations are in tenths of a centimeter (not to scale!). With these gradations, you can certainly measure to within a tenth of a centimeter, but what about objects that fall between the gradations? Is it reasonable that you would be able to tell whether the object was closer to one gradation than another? It is this uncertainty that you are to estimate.

Your estimate should be in the form of a range of possible values expressed as a plus-or-minus value around your measurement. For instance, if you measure an object that appears to be 2.5 cm, but the ruler makes it hard

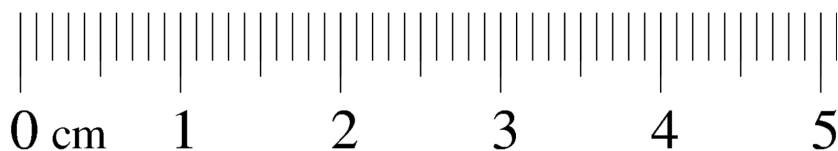


Figure 1: An example of a ruler (not to scale).

to tell if the object is longer or shorter than that up to a tenth of a centimeter, you would report a measurement of 2.5 cm with an uncertainty of ± 0.1 cm. This indicates that you are reasonably certain that the actual length of the object falls between 2.4 and 2.6 cm.

The format for your measurement should be “measured value” \pm “uncertainty” ($x \pm \delta x$). Once you have these values, enter them below, then go to the board and write down your measurement and uncertainty. **Once everyone has reported a measurement, the Lab Instructor will talk about the results, so please wait before you continue with this sheet.**

1. Width of your keyboard = _____ \pm _____

Reporting a measurement

A properly formatted measurement should have the following attributes:

- The correct number of significant figures for the uncertainty: One significant figure unless the leading digit is a 1 or 2, in which case you should use two significant figures.
- The correct number of significant figures for the measurement: At the same order of magnitude (the same decimal position) as the uncertainty. (Numbers with the same order of magnitude, when written in scientific notation, have the same factor of ten. 10 and 20 are the same order of magnitude, 100 is an order of magnitude higher than 10).
- All measurements must have units attached to them.

Update your entry on the board as well as here on the worksheet with all the proper attributes of a measurement. If your response in question 1 follows all of the rules above, your answer for this question will be the same.

2. Width of your keyboard = _____ \pm _____

From now on, all measurements that you report in physics labs will be required to have these attributes, or an explanation for why it does not. This is part of the scientific language and is common across many disciplines. Familiarity with this format is a useful skill wherever you need to make and report accurate measurements.

Types of errors

- **Systematic errors** are due to identifiable causes and can, in principle, be eliminated. They are due to biases in the equipment or experimental procedure that cause measurements to be consistently higher (or consistently lower) than the true value.
- **Random errors** are the positive and negative fluctuations that cause about half the measurements to be too high and half to be too low. Sources of random errors cannot always be identified, and are usually inherent in the limitations of the equipment.

- **Mistakes** are human errors. A person may record a wrong value, misread a scale, forget a digit when recording a measurement, or make another of the many possible mistakes. These mistakes should stick out like a sore thumb if you make multiple measurements or if one person checks the work of another. *Mistakes should not be included in the analysis of data.*

Propagation of uncertainties

Using your ruler, measure the long side of your keyboard. Because your ruler is not long enough to measure the length in one measurement, you will need to take *multiple* measurements, each with their own uncertainties.

3. Length of keyboard = (_____ \pm _____) + (_____ \pm _____) + ...

Adding the measurements is trivial, but what about adding the uncertainties? In the case of addition and subtraction, the equation for combining uncertainties is

$$\delta x = \delta x_1 + \delta x_2 + \delta x_3 , \quad (1)$$

where δx is the total uncertainty in your length calculation and δx_1 , δx_2 , and δx_3 are the uncertainties of your individual measurements.

This is called error propagation. Notice that the error in your final measurement is much larger than the error of any of your individual measurements. Report your completed measurement below and on the board.

4. Length of keyboard = _____ \pm _____

5. If you could change anything besides the length of the keyboard, what might you be able to change to reduce the error in this measurement?
6. Are there any additional sources of error that you did not take into account when reporting this measurement?

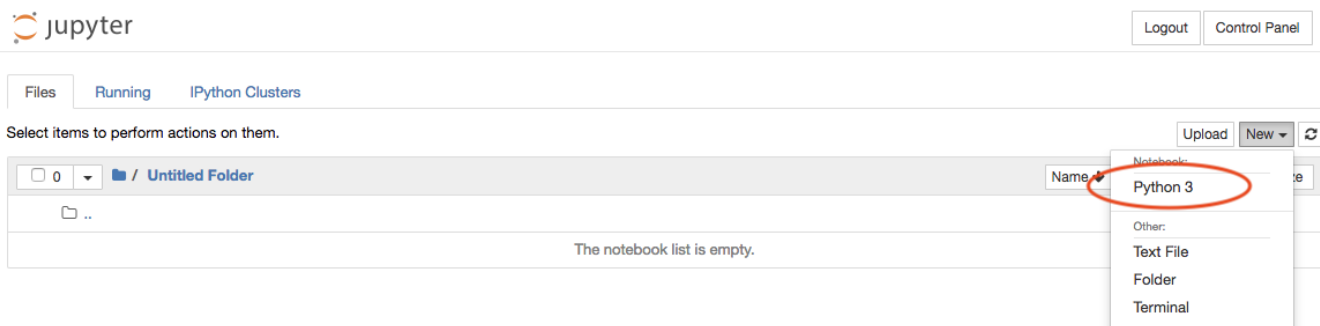


Figure 2: Navigate to `jupyterhub.wpi.edu` and sign in with your WPI email address and password, choose an instance to spawn (either is fine) and create a new Python 3 file as shown here

Python

Python is a high level programming language that is regularly used by the scientific community for general and research computation. We will be introducing it and using it in a very limited way in this lab course although we encourage you to explore it more on your own. We will primarily make use of the Python to calculate statistical values and propagate uncertainties. We will be using an interface for Python called Jupyter and WPI has set up a JupyterHub server that you can access from any browser (Figure 2). The files you create on the server will stay for at least the term, probably much longer.

Jupyter uses a cell based system and evaluated variables carry over to the next cell. There are a few different types of cells, Figure 4 shows 2 kinds, the code cell, which we will be using most of the time, and the markdown cell, which you can use to add nicely formatted notes to you file.

Statistical uncertainty

Instead of taking a single measurement and estimating the uncertainty, it is often desirable and easier to take multiple measurements and calculate the mean value (often denoted by \bar{x}) and the uncertainty (often denoted by δx). The formulas for calculating these two values are

$$\bar{x} = \frac{1}{N} \sum_{i=1}^N x_i \quad (2)$$

and

$$\delta x = \sqrt{\frac{1}{N-1} \sum_{i=1}^N (x_i - \bar{x})^2} \quad (3)$$

where N is equal to the number of measurements that you have taken and x_i are the individual measurements.¹

While the formula to calculate the mean is fairly simple, calculating the standard deviation becomes more complicated as you add more measurements. This is a perfect opportunity to use a computational tool in our case Python. Using the measurements that your peers recorded on the board for the length and width of the keyboards, calculate the standard deviation of the length and widths of the keyboards using Python. Don't forget to use the proper significant figures and units.

7. Length = _____ \pm _____, Width = _____ \pm _____

¹You may be wondering why the formula for standard deviation contains $\frac{1}{N-1}$ instead of $\frac{1}{N}$. This is more properly called the *sample* standard deviation, and this correction, called Bessel's correction, fixes a systematic error that occurs when taking a finite number of measurements. If you try to put $\frac{1}{N}$ in the above formula, you will consistently underestimate the true standard deviation.

Jupyter WPI Physics Last Checkpoint: 12 minutes ago (autosaved) Logout Control Panel

File Edit View Insert Cell Kernel Widgets Help Trusted Python 3

```
In [1]: 1+1 #use the run button above or shift + enter to evaluate the cell
Out[1]: 2
```

Markdown Cells

You can use latex for $\frac{math}{math}$ and markdown for forming text in a markdown cell.

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```
In [4]: #propagation of uncertainties for addition and subtraction
#anything written after the # sign is treated as a comment and will affect the execution of your code.
#For this class, we will require you to comment every line of your code for full credit.

x_1 = 3 #first measurement in cm
x_1_uncertainty = 0.01 #uncertainty of first measurement in cm
x_2 = 4 #second measurement in cm
x_2_uncertainty = 0.01 #uncertainty of second measurement in cm
x_3 = 2 #third measurement in cm
x_3_uncertainty = 0.01 #uncertainty of third measurement in cm

|
#calculation for the total of the measurements in cm
x = x_1 + x_2 + x_3

#calculation for the propagated uncertainty in x in cm
x_uncertainty = x_1_uncertainty + x_2_uncertainty + x_3_uncertainty

#print x and x_uncertainty in cm
print("x = ", x, "cm")
print("x_uncertainty = ±", x_uncertainty, "cm")

x = 9 cm
x_uncertainty = ± 0.03 cm
```

In []:

Figure 3: Above is the code that you could use to propagate uncertainty for values that are added or subtracted. Always remember to comment every line of your code in this class.

```
In [8]: import statistics as stat # import the statistics module

x = [1,2,3,4,5] # assign the numbers to the variable x
mean = stat.mean(x) #use the statistics package to assign the mean of x to the variable "mean"
stdev = stat.stdev(x) #use the statistics package to assign the standard deviation of x to the variable "stdev"

#print the values for mean and stdev
print ("mean of x =", mean)
print ("uncertainty in x =" , stdev)

mean of x = 3
uncertainty in x = 1.5811388300841898
```

Figure 4: Above is the code that you could use to calculate the mean and standard deviation. Always remember to comment every line of your code in this class.

Now is also a good time to do a sanity check on your calculations.

8. Is your mean value on the same order of magnitude as the individual measurements reported by the class?
9. How does the standard deviation compare to the estimated uncertainties?

Propagation of uncertainties

Using the calculated measurements you just found, calculate the area of the keyboard. Again, the initial calculation is trivial, but what about the uncertainties? This time we will use a slightly different method of error propagation because we are multiplying them instead of adding them. This method is valid for both multiplication and division of measurements with uncertainties. The formula is

$$\frac{\delta A}{|A|} = \frac{\delta x}{|x|} + \frac{\delta y}{|y|}, \quad (4)$$

where A is the area, x is the length, y is the width, δx and δy are the uncertainties associated with these measurements, and δA is the propagated uncertainty of the area.

Using the equation above, report the area of the keyboards in the room with the propagated uncertainty and correct units. Also paste your code that you used to calculate these quantities including comments about what each line is doing.

10. Area of the keyboard = _____ \pm _____
11. Python code with comments used to calculate the above values.

Note that this measurement is not of your individual keyboard, but represents the average of all the keyboards in the room. This may seem like a minor distinction, and in this case it probably is; however, if we wanted to know how tall Olin Hall is, we wouldn't measure all the buildings on campus and then take the average—we would make multiple measurements of Olin Hall alone.

Accuracy versus precision

Figure 5 is a cartoon illustrating the difference between accuracy and precision. There are five data points shown for each scenario, with the “actual” point represented by the bullseye. While an accurate and precise measurement is the best, an accurate measurement is most important. You can make an accurate measurement with most measurement devices by minimizing systematic errors. Precision is almost completely dependent on the type of instrument and the level of precision it is capable of.

12. The calibration of a measurement device would help ensure *what* about our measurements?
13. What is one way that we could make more precise length measurements if we were to repeat today's measurements?

You should consider saving this file as a reference sheet; many of the topics presented here will be useful throughout your physics coursework and your scientific career. For further information on uncertainties and error propagation, a good reference is John R. Taylor's *An Introduction to Error Analysis*, which is available in the physics library.

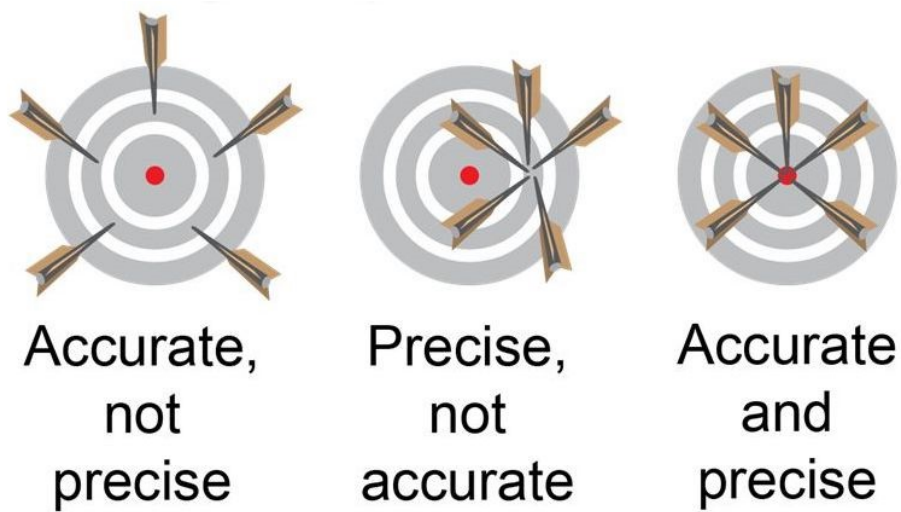


Figure 5: A cartoon explaining the difference between accuracy and precision of collections of measurements.