Last modification: February 3, 2020

RBE 1001: Introduction to Robotics
C-Term 2019-20
Worked problem: Putting the fun back in Funicular

Figure 1: Fløibanen funicular in Bergen, Norway.

1. You've been hired to design the drive system for a funicular, a rail car that is pulled up steep hills via a cable system, an example of which is shown in Figure 1. The drive train will consist of a motor, a gear box, a drive pulley, and a cable to pull a car up a whopping 40% grade.

   Figure 2 shows the layout of the drive pulley and the cable.

   Figure 3 shows the torque and power curves of the motor, and the data used to make the curves is shown in Figure 4.

   You wish to minimize the inefficiency of the overall system, including the motor. You estimate each gear stage will have an efficiency of $\eta = 0.95$, and, due to other constraints, no gear stage can have more than a 5:1 gear ratio.

   The funicular is expected to weigh 7 tons (14,000 lbs.) and the design speed is 10 ft./s. The pulley that will pull the cable has a diameter of 3 ft.

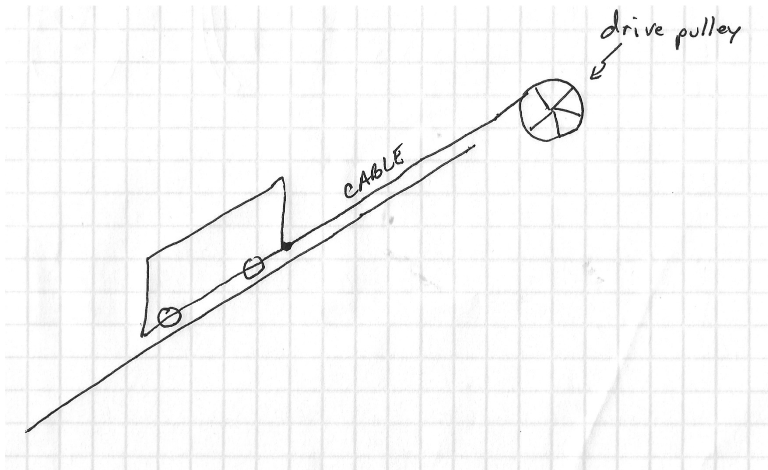


Figure 2: Basic layout of the pulley/cable system.

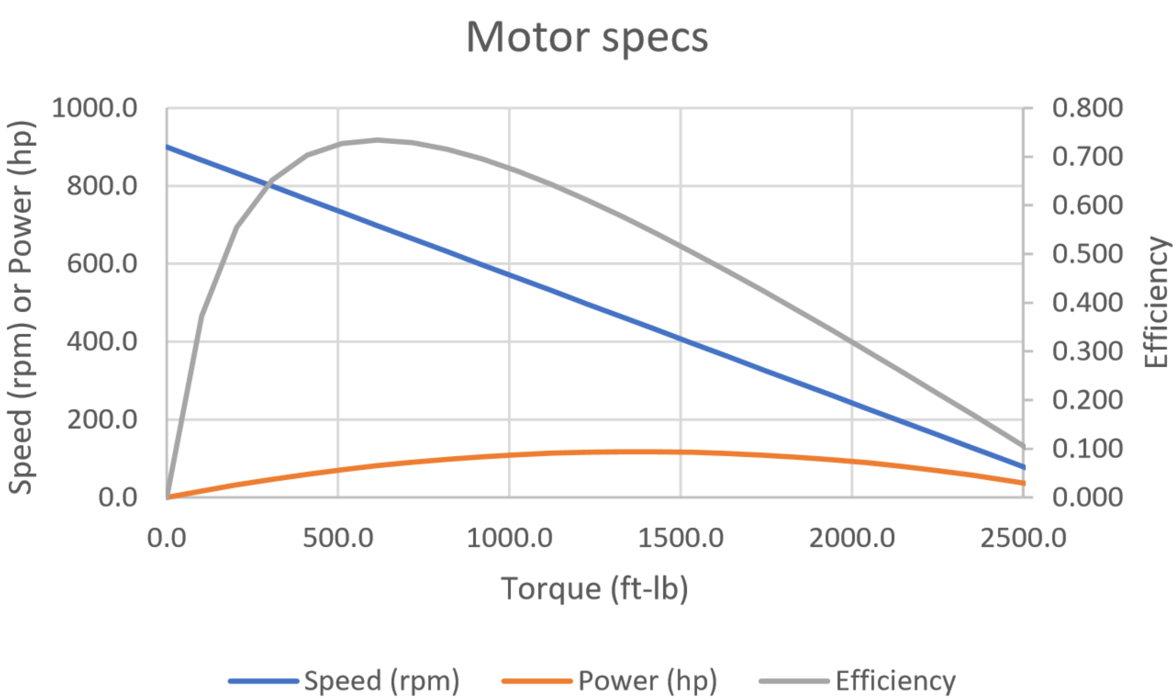


Figure 3: Motor performance curves.

Torque (ft-lb)	Speed (rpm)	Power (hp)	Efficiency
0.0	900.0	0	0.000
102.3	866.4	16.9	0.372
204.6	832.8	32.4	0.555
306.9	799.2	46.7	0.652
409.2	765.6	59.6	0.703
511.5	732.0	71.3	0.727
613.7	698.4	81.6	0.734
716.0	664.8	90.6	0.729
818.3	631.2	98.3	0.715
920.6	597.6	104.7	0.695
1022.9	564.0	109.8	0.671
1125.2	530.4	113.6	0.642
1227.5	496.8	116.1	0.611
1329.8	463.2	117.3	0.577
1432.1	429.6	117.1	0.541
1534.4	396.0	115.7	0.504
1636.7	362.4	112.9	0.465
1739.0	328.8	108.9	0.425
1841.2	295.2	103.5	0.385
1943.5	261.6	96.8	0.343
2045.8	228.0	88.8	0.301
2148.1	194.4	79.5	0.258
2250.4	160.8	68.9	0.214
2352.7	127.2	57.0	0.170
2455.0	93.5	43.7	0.126
2557.3	59.9	29.2	0.081

94.4 hp with perfect efficency

with two stages at $\eta = 0.95$ each

Figure 4: Motor performance specifications.

(a) Calculate the speed of the drive pulley needed to meet the design conditions.

Method 1:

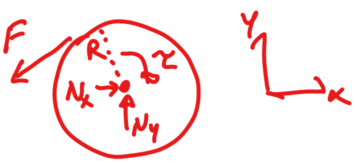
$$\text{ang. speed} = \frac{\text{speed of car}}{\text{circumference of pulley}} = \frac{10\ ft/s}{3\ ft \cdot \pi} = 1.061\ \frac{rot}{s} = \boxed{63.66\ \frac{rot}{min}}$$

method 2:

$$v = \omega \cdot r \Rightarrow \omega = \frac{v}{r} = \frac{10\ ft/s}{1.5\ ft} = 6.667\ \frac{rad}{s} = 6.667\ \frac{rad}{s} \times \frac{rot}{2\pi\ rad} \times \frac{60\ s}{min} = \boxed{63.66\ rpm}$$

(b) Calculate the torque needed to drive the drive pulley at the design conditions.

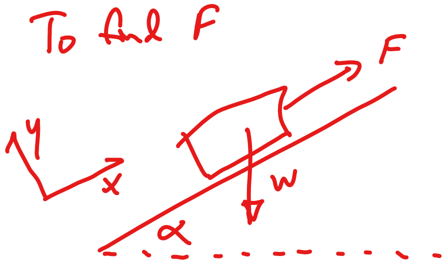
FBD of pulley



$$\Sigma M_{\bullet} = 0 = F \cdot R - \tau$$

$$\Rightarrow \tau = F \cdot R$$

To find F



$$\Sigma F_x = 0 = F - W\sin\alpha$$

$$\Rightarrow F = W\sin\alpha$$

$$\therefore \tau = W\sin\alpha \cdot R$$

$$= 14000\ lb_f \cdot \sin\left[\tan^{-1}(.4)\right] \cdot 1.5\ ft$$

$$= \boxed{7799\ lb \cdot ft}$$

(c) Assuming no gear train losses **for just this part**, determine the approximate speed ratio of the gear train (i.e., you can do a rough interpolation). How many gear stages will you need? (Hint: What is the most important factor in motor design?)

First, find POWER

$$P = \omega \cdot \tau = 63.66\ \frac{rot}{min} \cdot 7799\ ft \cdot lbs \times \frac{2\pi\ rad}{1\ rot} \times \frac{1\ min}{60\ sec} \times \frac{1\ hp}{550\ \frac{ft\ lbs}{s}} = \underline{94.4\ hp}$$

From the chart, the motor would turn @ ~650 rpm, so $\underline{e \approx 0.1}$

Each stage has at least 5:1 ratio, or $e = 0.2$, so we'll need $\boxed{2\ \text{stages}}$

[Note that ratios are multiplied, so we could go as high as 25:1 w/ 2 stages]

(d) Using the number of gear stages you found in the previous part, what is the torque, speed, and power of the *motor*? Circle the line in the chart that is closest to the operating point of your motor.

While η doesn't directly affect e, it does effect power.

$$P_{pulley} = \text{efficiency} \cdot P_{motor} = (0.95)^2 P_{motor}$$

$$\therefore P_{motor} = \frac{94.4 \text{ hp}}{(0.95)^2} = \boxed{104.7 \text{ hp}}$$

From the chart, $\boxed{\begin{array}{l} \omega = 597.4 \text{ rpm} \\ \hline \tau = 920.6 \text{ ft-lbs} \end{array}}$

(e) What is the speed ratio with inefficiency taken into account? Does that affect the number of gear stages found above?

$$e = \frac{63.66 \text{ rpm}}{597.6 \text{ rpm}} = \boxed{0.107} \quad (\text{or } 9.39:1)$$

(f) What is the efficiency of the *overall system*?

$$\eta = \eta_{motor} \cdot (0.95)^2 = 0.695 \cdot (0.95)^2$$

$$= \boxed{0.627}$$

(g) Your colleague says that you should operate the motor at peak efficiency to maximize the overall efficiency of the system. Can you do that and still meet the other design requirements? Explain.

No. Not enough power.