RBE 1001: Introduction to Robotics
C-Term 2019-20
HW 2.3: Drive trains

Last modification: February 4, 2020

1. A cyclist in the *George Street Bike Challenge for Major Taylor* wants to climb George Street as fast as possible. Figure 1 shows the elevation profile for the short, but intense climb.

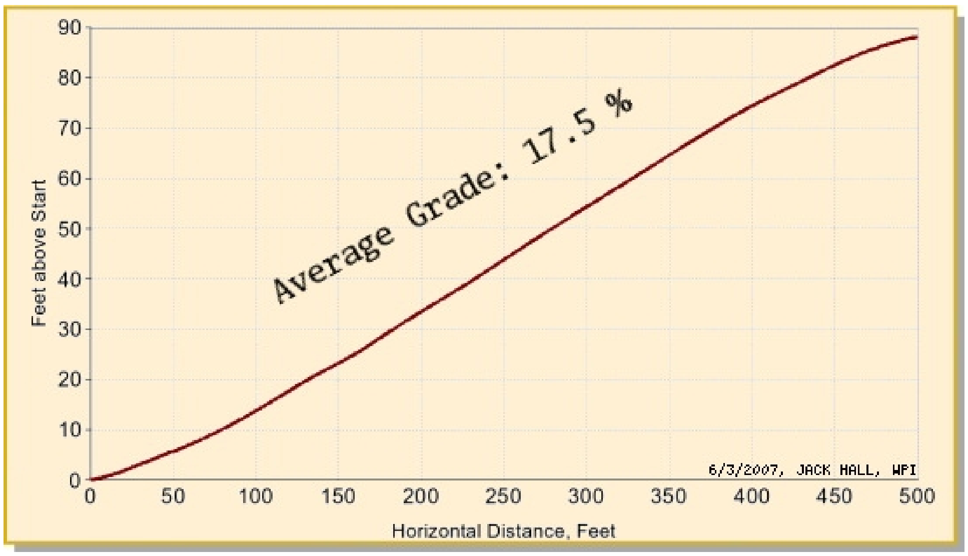

Figure 1: Elevation profile of George Street.

Let's pretend that the cyclist has done some experiments and determined that the output of his pedalling is given by the data in Figure 2. That is, you can pretend that the cyclist pedals as a motor with the output characteristics shown, and that that "motor" acts at the *bottom bracket*, which is the axle around which the pedals revolve. Assume the cyclist can maintain the output in the figure for the entire sprint up the hill – his goal is to complete the climb in under 30 seconds. Table 1 shows some key points in the output profile.

Condition	Power (W)	Torque (in-lb)
Max efficiency	650	400
Max power	775	665

Table 1: Key output characteristics for the cyclist represented in Figure 2.

The cyclist's bike is outfitted with a 34-tooth *chain ring*, which is the "sprocket" that is connected to the pedals (i.e., it's the driving gear on the virtual motor shaft). Assume a drivetrain efficiency of 0.95. The bike and rider weigh 190 lbs., combined. The diameter of the rear tire is 25 inches.

Figure 3 shows the gears and geometry of the system.

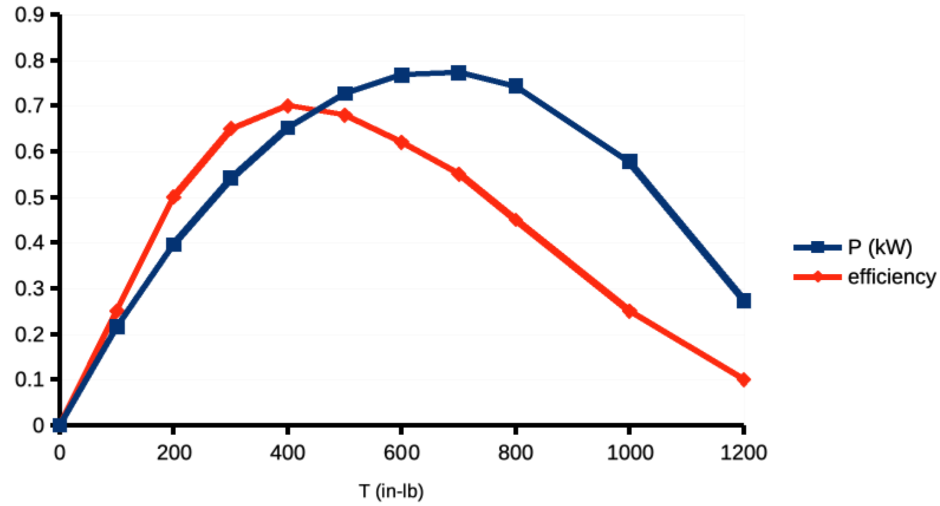

Figure 2: “Motor" (pedalling) characteristics for a half-way decent cyclist. “Efficiency" refers to the internal efficiency of the cyclist and has nothing to do with the drive train efficiency.

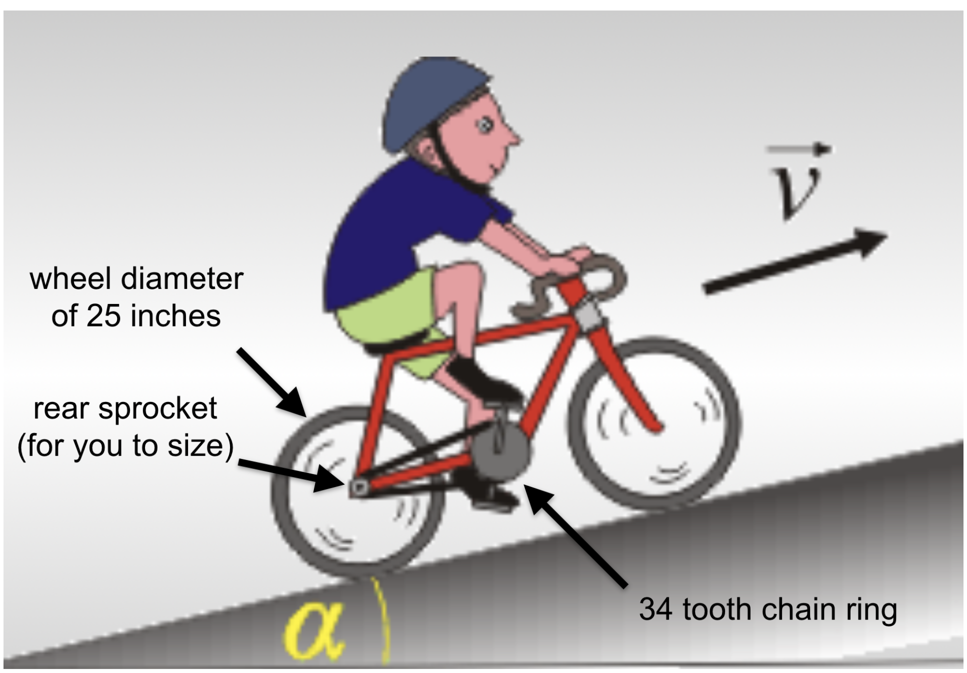

Figure 3: Parts of a bike needed for this problem.

(a) Where should the cyclist operate on the given curve to minimize his time to climb George Street? What is the pedalling torque and speed at that point? (Sanity check: A cyclist typically shoots for a *cadence* in the 90 - 120 rpm range. Don't forget the 2π to convert from $1/s$ to rot/s!)

(b) Given the power at that point and neglecting air resistance (a reasonable assumption for steep climbs), how fast could our hero expect to go up the hill? Even though you haven't yet specified any gear sizes, don't forget to include the efficiency of the drive train in your calculation. Also, since we're neglecting air resistance, you can do this from energy principles alone – no FBD needed!

(c) Calculate the speed of the rear wheel and the torque needed at the wheel axle to produce the expected time. Use the average grade in your calculations and assume the speed is constant (that is, ignore the time needed to get up to full speed).

(d) Still using the average grade, what size sprocket (gear) should be used on the rear wheel? (Sanity check: The hill climb can be done with a typical set of sprockets, which can range anywhere from, say, 11 - 32 teeth.)

(e) The cyclist realizes that the sprocket set on his bike doesn't include the specified size, so he plans to climb using a rear gear with two *fewer* teeth. Will he go faster or slower than expected? Will his cadence be higher or lower than it would with the ideal sprocket?

(f) Let's say the cyclist could find the *exact* gear ratio needed to be at maximum power on the *average* grade. If he didn't change gears during the climb, how would the fact that actual grade deviates both above and below the average grade affect his time? Explain.

2. Figure 4 shows an arm that is driven via a complex multi-stage gear transmission. When lifting the weight as shown, the arm motor (at shaft A) is turning 100 rpm. Recall that the speed ratio of a gear train equals the product of the number of teeth on all driving gears divided by the product of the number of teeth on all driven gears. Drivers are defined as gears that convert applied shaft torque to force at the pitch diameter. Likewise, driven gears convert this applied force to output shaft torque.

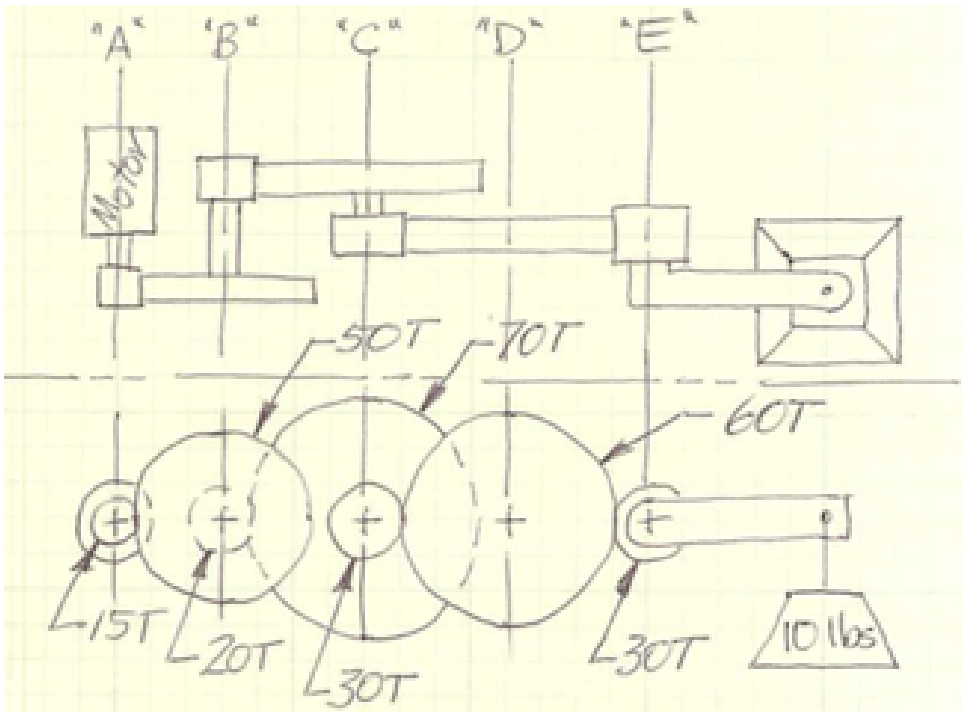

Figure 4: Top and side views of a geartrain. The 30T output gear that is at shaft E is connected to an arm that, in the horizontal position shown, has a moment arm of 8 inches.

(a) What is the rotation speed of the arm (shaft E)?

(b) What is the effect of the gear at shaft D on the overall speed ratio? What function does it serve?

(c) Given a gear stage efficiency of 0.95, how much torque must the motor provide to steadily raise the load in the scenario shown in the sketch?