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RBE 1001: Introduction to Robotics
C-Term 2019-20
HW 2.0: Fundamentals of Mechanics – Free body diagrams

## 1 Introduction

Mechanics is an important discipline in the development of robots. Throughout your robotics career, you will apply concepts in statics (the study of forces and deformation), kinematics (the study of motion), dynamics (how forces cause those motions), and control (how to produce desired motions). Here, we introduce some of the terminology and fundamental principles that are important in robotics. We expect that you have had a course in physics – at least in high school – and are comfortable with the concepts of forces, moments, work, and energy. We will quickly review those concepts, develop others, and then apply them to common problems in robotics.

## 2 Forces

As you have seen before, a force is a vector quantity that is primarily defined by the effect it has on objects: forces alter the motion of the body, as famously described by Newton's Laws of Motion. Forces can be classified as body forces – gravity being the most important to us – and contact forces, which act on the surface of a body.

An important thing to remember is that even though a force may formally act at a certain point, for rigid bodies, where we ignore internal deformations, a force can act anywhere along a line coincident with the force vector – such freedom may seem counter intuitive, but taking advantage of this property can greatly simplify a problem in mechanics.

## 3 Moments and torque

A torque, or moment, is a rotational action caused by a force. When driving a screw, you apply forces to the outside of a screwdriver handle, but in doing so you create a twisting effect – a torque – that turns the screw. There is no real difference between "torque" and "moment" in this context; however, "torque" tends to be used when there is an axle or pivot that is the axis of motion (particularly with motors), while "moment" tends to be used in a more general sense, such as when analyzing the motion of a rigid body.

Formally, a moment, $\mathbf{M}$, is calculated as a vector quantity,

$$\mathbf{M} = \mathbf{r} \times \mathbf{F} \tag{1}$$

where $\mathbf{r}$ is a vector from the axis about which the moment is being calculated to a point on the line of action of the force, $\mathbf{F}$. For the two-dimensional problems that we'll encounter in this class, a moment can be calculated as the product of magnitude of the force and the moment arm, a line through the axis of rotation that is perpendicular to the line of action of the force. For convenience, we'll typically drop the vector notation for such moments, but it must be remembered that a moment is a vector quantity. In particular, the sign of the moment is found using the *right hand rule*.

Another important note is that when applied to a rigid body, moments can be thought of as free vectors – they can be applied at any point on the body. Again, this property may seem counterintuitive, but remember: the *angular velocity* of a rigid body is the same at any point on or in the body; therefore, from dynamic principles, it makes sense that the applied moments are the same.

## 4 Free body diagrams

An essential tool to analyzing mechanical systems is the *free body diagram*. The free body diagram is used to isolate the system or a part of the system so that the equations of motion can be applied and solved. A proper free body diagram includes the following:

- A rough depiction of the object,
- A proper (right-handed) coordinate system,
- All applied forces and moments, including,
  - Restraining forces due to contact with fixed surfaces, typically called *reaction forces* and denoted with an $R$,
  - Other forces and moments that are applied to the object, for example the moment (torque) from a motor or the force due to snow on top of a car, and
  - Body forces, including gravity.
- Important geometric dimensions or quantities, such as lengths or angles.

Applied forces and moments are typically drawn in the direction that they are expected to act (it's OK if your expectation is wrong; you'll simply get a negative magnitude) at the point where they act. Remember that while gravity acts over the entire body, it can be drawn as a simple force acting at the *center of mass*, which must be included when gravity is being considered.

Further resources on how to create free body diagrams can be found in:

- *Statics For Dummies*,[1] Chapters 13 - 15, available online through the Gordon Library.
- *Statics, Learning from Engineering Examples*, Chapter 3, available online through the Gordon Library.

In this course, we will concern ourselves with two-dimensional analysis.

You will use free body diagrams extensively to perform *static analysis*, analysis of non-accelerating bodies that will determine, for example, if your robot can climb a given hill or lift a given object.

[1] I really, really don't like the title and its implications...but it's a well written guide to statics!

### 4.1 Practice Problems

Do the following from *Statics, Learning from Engineering Examples*. Note that the problem statements include drawing both a physical diagram and a free body diagram – we're only interested in the FBD for these exercises.

- Prob. 3.1
- Prob. 3.4
- Prob. 3.8, but include a side force due to wind.
- Prob. 3.10
- Prob. 3.18
- Prob. 3.21, assume the brake is engaged.

Solutions to practice problems are provided in a separate document.

## 5 To submit

In *Statics, Learning from Engineering Examples*:

Create free body diagrams for the following (you don't need the physical model that is asked for in the problem statements):

- Prob. 3.21, assume the brake is engaged.
- Prob. 3.23