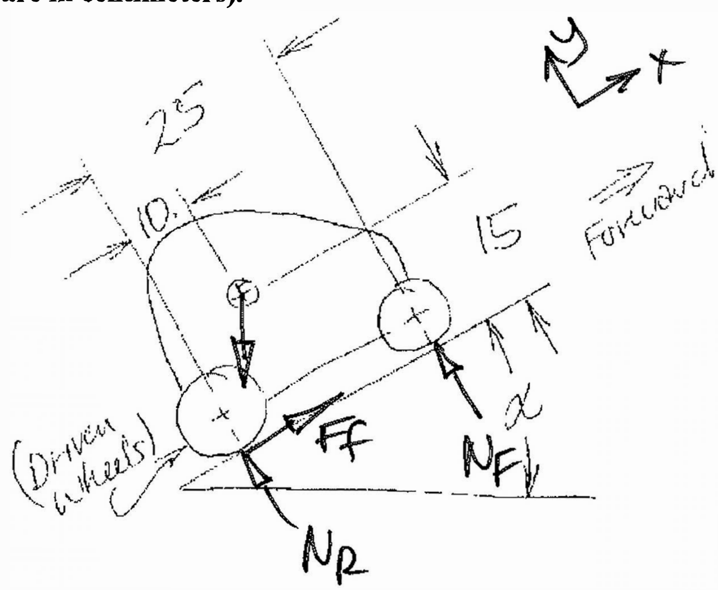
RBE 1001 Homework #2 **Name:** Solutions

Problem statement:

Consider a 25N, 4-wheeled robot with the two rear wheels driven and the front wheels undriven. You would like to determine the maximum grade (in degrees from horizontal) that it can climb when traveling both forward and backwards. You are confident that it will not be motor torque limited but are unsure if it will lose traction (ie spin out) or lose stability (ie tip over) at the maximum climb angles. **Complete the following sketches to make FBDs, then develop the appropriate EoE (Equations of Equilibrium) to answer the following questions (all dimensions are in centimeters).**



1) When traveling forward, compute the stability-limited maximum climb angle (hint: when the normal force on the front wheel goes to zero). (~ ans: α = 35 degrees.)

$$\Sigma M_{Z_R} = 0 = \cancel{N_F}^{\,0}(25) + 25\sin\alpha(15) - 25\cos\alpha(10)$$

$$\therefore \frac{\sin\alpha(15)}{\cos\alpha(10)} = 1$$

$$\tan\alpha = \frac{1}{1.5}$$

$$\alpha = \boxed{33.7^\circ}$$

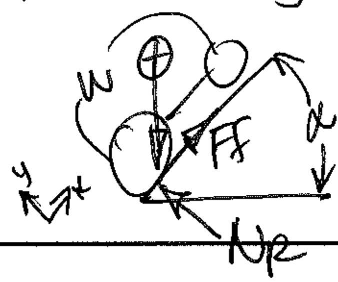
2) Given a coefficient of static friction (μ) of 1.0, compute the friction-limited climb angle. (~ ans: $\alpha = 48$ degrees)

Would occur when 100% of the weight is supported solely by driven wheels – for this to actually happen (in this case), the CG would have to be altered to prevent tipping.

$$\Sigma F_x = 0 = F_f - w\sin\alpha \quad \therefore F_f = w\sin\alpha$$

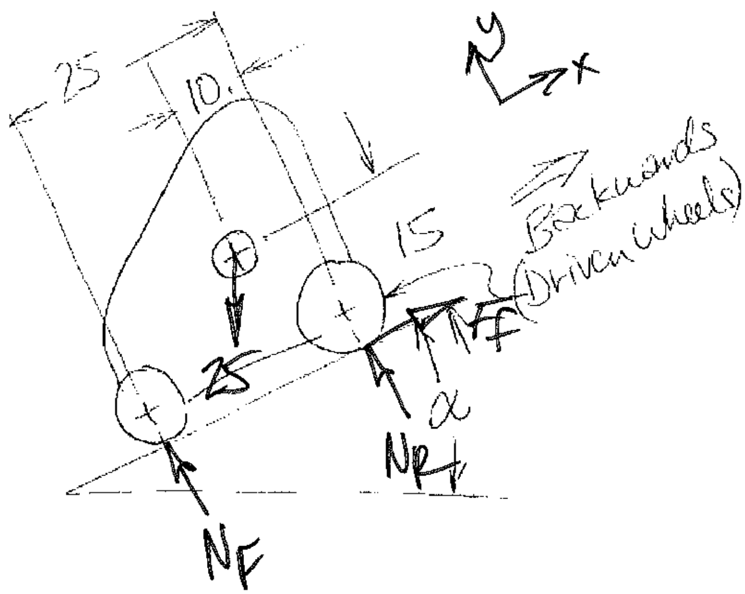
$$\Sigma F_y = 0 = N_R - w\cos\alpha \quad \therefore N_R = w\cos\alpha$$

SO...



$$F_f = \mu N \Rightarrow w\sin\alpha = (1.0)\, w\cos\alpha$$

$$\therefore \boxed{\alpha = 45^\circ}$$



3) When traveling backwards, compute the stability-limited maximum climb angle (hint: when the normal force on the rear wheels goes to zero). (~ ans: α = undefined.)

At stability limit, $N_R \Rightarrow 0 \quad \therefore F_f \Rightarrow 0$

$$\Sigma F_x = 0 = -25\sin\alpha + \cancel{F_f}^{\,0}$$

$$\therefore \alpha = 0^\circ$$

$$\Sigma M_{z_\oplus} = 0 = -N_F(25-10) + \cancel{N_R}^{\,0}(10) + \cancel{F_f}^{\,0}(15)$$

$$\therefore N_F = 0$$

$$\Sigma F_y = 0 = N_F + \cancel{N_R}^{\,0} - 25\cos\alpha$$

$$\therefore 25\cos\alpha = N_F = 0$$

$$\therefore \alpha = 90^\circ$$

Can't happen!

4) Given a coefficient of static friction (μ) of 1.0, compute the friction-limited climb angle. (~ ans: α = 20 degrees)

From #3:

ΣF_x : $F_f = 25 \sin\alpha$

ΣF_y : $N_F + N_R = 25\cos\alpha$

if $\mu = 1.0$ then, at friction limit, $F_f = N_R$

$\therefore N_F + 25\sin\alpha = 25\cos\alpha$ ⓐ

$N_F = 25\cos\alpha - 25\sin\alpha$

$\Sigma M_{Z_R} = 0 = 25\sin\alpha(15) + 25\cos\alpha(10) - N_F(25)$

$\therefore N_F = \sin\alpha(15) + \cos\alpha(10)$

combined with ⓑ...

$N_F = \sin\alpha(15) + \cos\alpha(10) = 25\cos\alpha - 25\sin\alpha$

divide by $\cos\alpha$:

$\tan\alpha(15) + 10 = 25 - 25\tan\alpha$

$40\tan\alpha = 15$

$\alpha = \tan^{-1}\left(\frac{15}{40}\right) = 20.6°$ ←