RBE 1001: Introduction to Robotics
C-Term 2019-20
Final Exam

Last modification: February 28, 2020

You work for a robotics company doing lots of mechanical design and some electrical analysis, often on several projects at once. You are getting ready to head out on your annual ski trip to the Alps and your manager has asked that you finalize some calculations before you go. She has asked for answers to the problems below – well documented, of course – and you'll meet with her on Tuesday at noon to go over them. Your manager has a reputation for asking lots of "what if" questions – changing parameters and such – so you'll need to be able to solve variations on them in your meeting.

That is, the final exam will be drawn from the following questions. You may discuss the problems with anyone you want up to the start of the exam and we will discuss solution strategies in class and the review session (no, I won't work them for you, but I'll be happy to send you down the correct path). I will change the parameters and I will also ask additional questions based on the prompts herein. I may also add one "stretch" question.

Problems

1. Despite the fact that you've already tested your robot's ability to get into the construction zone, you've been asked to go back and do the theoretical analysis to see how the friction coefficient affects the ability to get over the speed bump.

   Figure 1 shows the geometry of the system as the BaseBot is starting the maneuver. The worst case scenario occurs when the front wheels *just* start to lift off the ground – if there is enough torque/friction/stability at this point, then it should be able to at least get its front wheels up on the ledge (see the next problem for the rear wheels).

   Use $L = 30cm$, $m = 4kg$, and $R = 5cm$.

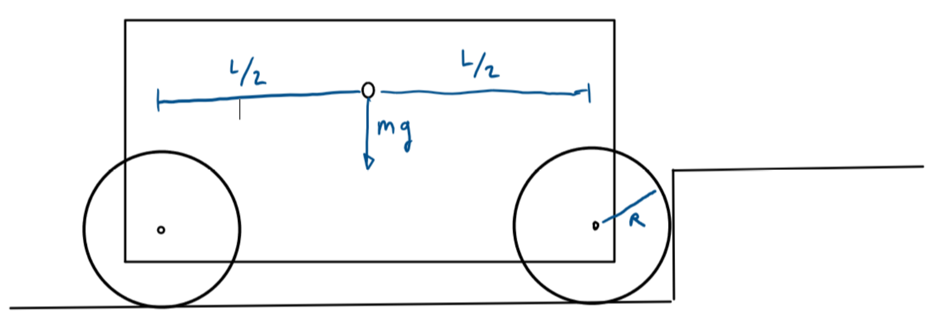

Figure 1: BaseBot beginning to climb the speed bump.

   (a) Draw out a proper free body diagram of the system. Pay particular attention to the reaction forces on the front wheel.

(b) Write out the Equations of Equilibrium for the system. Where will you sum the moments (and why)?

(c) Write an expression for the traction force at the each wheel. Assume $\mu = 1.0$ is the same for each wheel.

(d) With the CoG as shown, will the robot get up the bump?

Interesting questions:

- How does the friction coefficient affect the ability to climb the speed bump? What is the minimum value of μ that is needed?
- How does the weight affect the ability to climb? What about the location of the CoG?
- What happens if the vehicle is front wheel drive? What about rear wheel drive?

2. Following on from the last problem, you now want to determine if the robot can get the rear wheels up the speed bump. Figure 2 shows the geometry of the system. To simplify the problem, the height of the speed bump for this problem is exactly the radius of the wheels and the location of the CoG is in horizontal coordinates (to save you from having to take too many tangents).

   The questions in the previous problem hold (with minor edits for the rear wheel), with the worst case scenario being when the rear wheel just starts to lift off the ground.

   Use $a = 4in$, $b = 6in$, $h = R = 2in$, and $m = 4kg$. Does it get up the bump if $\mu = 1.0$ for each wheel? If I give you the friction coefficient for the front wheel, can you find the minimum friction for the rear wheel?

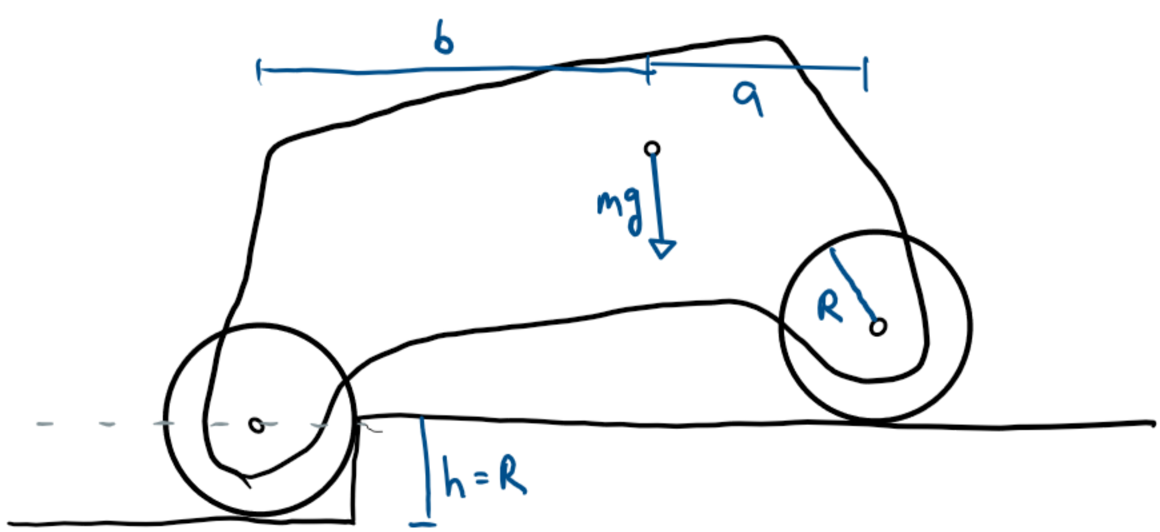

Figure 2: BaseBot beginning to lift the rear wheel.

3. Your challenge is to create a conceptual design circuit for a "convoy robot" that uses only analog parts. That is, similar to how the Braitenberg vehicle performed line following without a microcontroller, this circuit would enable a vehicle to maintain an offset from a vehicle that is driving in front of it.

   Your design will incorporate an ultrasonic sensor that produces a voltage that is proportional to the distance to an object by the formula,

$$V_{out} = d$$

   where V_{out} is in volts and d is in meters. The motors are wired in a standard fashion such that increasing the voltage increases the speed.

   When powered at 6V, the motor has a no-load speed of $\omega_{NL} = 300\,\mathrm{rpm}$ and a stall torque of $T_{st} = 0.1\,\mathrm{N \cdot cm}$. The direct drive wheel has a diameter of $d = 12.5/\pi$ cm and your design conditions are for the target distance to be 40 cm when driving 0.5 m/s.

   You have a battery that produces 5V and your op-amp is limited by the rails, so you can't produce negative voltages.

   (a) What motor voltage corresponds to the design speed?
   (b) What flavor of op-amp circuit will you use? You do not need to size components (yet), but keep in mind you want your robot to slow down when it gets closer to the lead vehicle. Draw your circuit with generic resistors.
   (c) Size the resistors such that the target speed is met at the target distance.
   (d) Make graph of the output voltage of the amplifier stage as a function of the sensor voltage.
   (e) What is the speed of the wheel when the distance is 1m?
   (f) What are the limits on the speed of the motor?

   Interesting questions:

   - What happens if the ultrasonic outputs a voltage scaled by,

$$V_{sensor} = \alpha \cdot d$$

   - What op-amp circuit would you use if the voltage were inversely related to distance,

$$V_{sensor} = \beta / d$$

   - How does the stall torque affect the problem?

4. Your professor has built a motorized bicycle trailer for carrying loads of groceries up to his house. His worst case scenario (i.e., the design condition) is a 10 kg trailer (including batteries and motor) carrying 20 kg of energy drink up Sagamore Road, which has a maximum pitch of 18%.

   Cleverly, he has incorporated a force sensor into his trailer hitch, and he uses the sensor output, which is a voltage, to control the amount of *power* that the motor produces. His first attempt uses proportional control,

$$P_{motor} = K_p \cdot V_{sensor}$$

   (a) According to the datasheet, the force sensor outputs 1 V for every 20 N force on the hitch. To test it, he hooks the sensor up to a 10-bit ADC on a microcontroller with an ADC reference voltage of 5V.

   In the design scenario, *without any power to the motor*, what does the ADC read? Be exact.

   (b) Your professor implements his control algorithm and hooks up the motor. In his test, he rides a mere 1.5 m/s up the hill in the design scenario. For a proportional constant of $K_p = 30$, how much force is there on the trailer hitch? Assume steady-state conditions so that you can do a proper force balance.

   (c) How much *torque* is the motor producing at this point? The wheels have a diameter of 50 cm.

   (d) What can your professor do to eliminate the force at the hitch (besides just park the bike and drive like a civilized person...)?

   Interesting questions:

   - How would changes in K_p affect the performance? What about the sensitivity of the sensor?
   - What happens on a flat road?

5. Your professor's daughter is designing a robotic snow plow to make her winter chores easier (maybe for next year...). The driveway at her house is quite steep, up to a 12% grade. Her current design is shown in Figure 3. It is *all-wheel drive*, meaning that each wheel will generate as much torque as it can without slipping, with the following dimensions (also shown in the figure):

   - The robot has a mass of 5 kg,
   - The wheelbase is 40 cm,
   - The center of gravity is half-way between the axles and 20 cm above the ground, and
   - The wheels have a diameter of 20 cm.

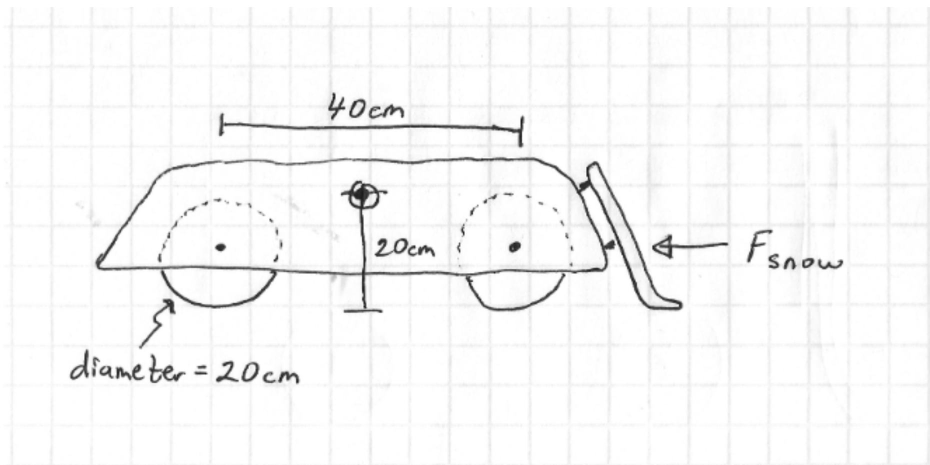

Figure 3: Snow plow design.

The motors are big enough that the robot is not expected to be torque-limited. To simplify the analysis, she approximates the force of the snow as follows:

- The force acts at the height of the wheel axles and is horizontal, and
- The magnitude is equal to 5 N for every cm of snow depth.

Through some tests, she estimates that the coefficient of friction will be $\mu = 0.25$.

(a) Assuming the robot is traction-limited, what is the maximum depth of snow the plow can push *uphill* on the steepest grade?

(b) Eventually, she finds some studded wheels that give the robot much better traction: she estimates the coefficient of friction will be a whopping $\mu = 3.0$! How much snow can the robot push *uphill* without slipping?

Interesting questions:

- How can you solve for the amount of snow that can be pushed downhill? (Hint: You don't need *any* new equations for this part!)
- With the high-grip tires, will it still be traction limited? (Hint: You can either solve for the angle at which it tips or you can test if it tips at the maximum traction angle.)

6. Shown in Figure 4, your 4-wheeled robot with 14 in wheels needs to pick up a 20 lb. weight from ground level and raise it to 6 ft. You are provided a motor that is rated for continuous torque output of 228 in-lbs and stalls at 310 in-lbs. The center of gravity of the robot chassis is as shown; it weighs 100 lbs. The arm weighs an additional 10 lbs. with a center of gravity midway along its length. You will rotate the arm around the shoulder joint using a single strand of #25 roller chain. Your job is to spec the sprockets.

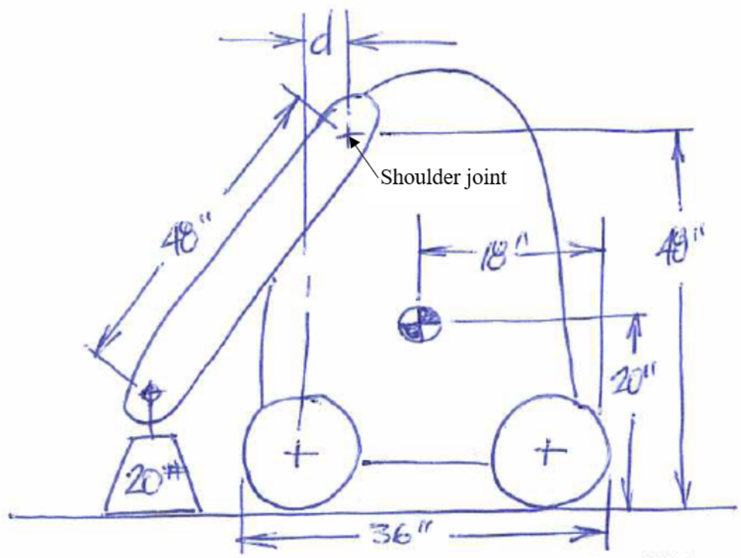

Figure 4: Robot with arm.

(a) Draw a free body diagram of the loaded arm and use the Equations of Equilibrium to determine the maximum torque required at the shoulder joint. (Hint: First think of what position the arm will be in to maximize the torque.)

(b) What is the overall speed reduction required? Assume perfect efficiency.

(c) Using a single chain and a 12-tooth sprocket on the motor, what size sprocket would you need at the shoulder joint? You may specify sprockets of any integer number of teeth.

(d) What is the maximum load (tension) on the chain at the design condition? If you use a safety factor of 2.0 (meaning your chain has to be rated for twice the design load), will a #25 chain be sufficient?

Interesting questions:

- Whether or not the #25 chain is sufficient, how would using a #35 chain affect the design? What safety factor would you get?
- How does changing the arm length affect the analysis? Or the weights involved?
- How does inefficiency in the system affect the design?
- If you used a 'quick link,' would that affect your choice of chain?
- How far in front of the shoulder joint does the wheel need to be (dimension, d) to prevent your robot from tipping?