RBE 1001: Introduction to Robotics
C-Term 2019-20
HW 2.2:Solutions

Solution

1. A cyclist in the George Street Bike Challenge for Major Taylor is shooting to complete the short, but steep, climb in 30 seconds. Figure 1 shows the elevation profile for the climb and the cyclist has done some experiments to determine that he can output 775 W for 30 seconds. The bike and rider weigh 85 kg, combined. The diameter of the rear tire is 65 cm.

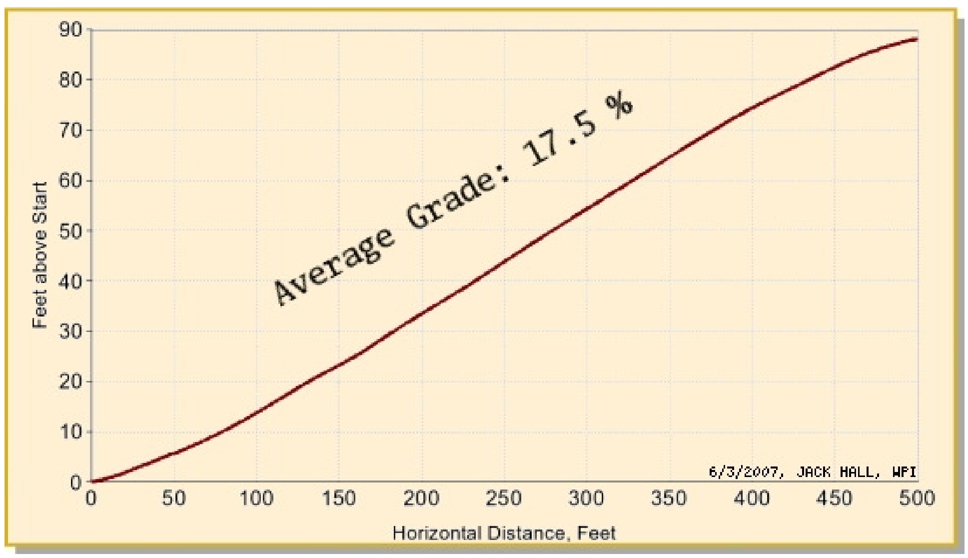

Figure 1: Elevation profile of George Street.

Figure 2 shows the geometry of the system.

(a) Given the power and neglecting air resistance (a reasonable assumption for steep climbs), how long will it take our hero to go up the hill?
Solution. There are two ways to approach this problem:

i. Starting with

$$P = F \cdot u$$

we can solve for the cyclist's speed,

$$u = \frac{P}{F}$$

The force is the force of the wheel on the road, F_A in Figure 2, which is found from balancing the forces in the x-direction (remember, even though he is moving, if we ignore accelerations, the forces balance!):

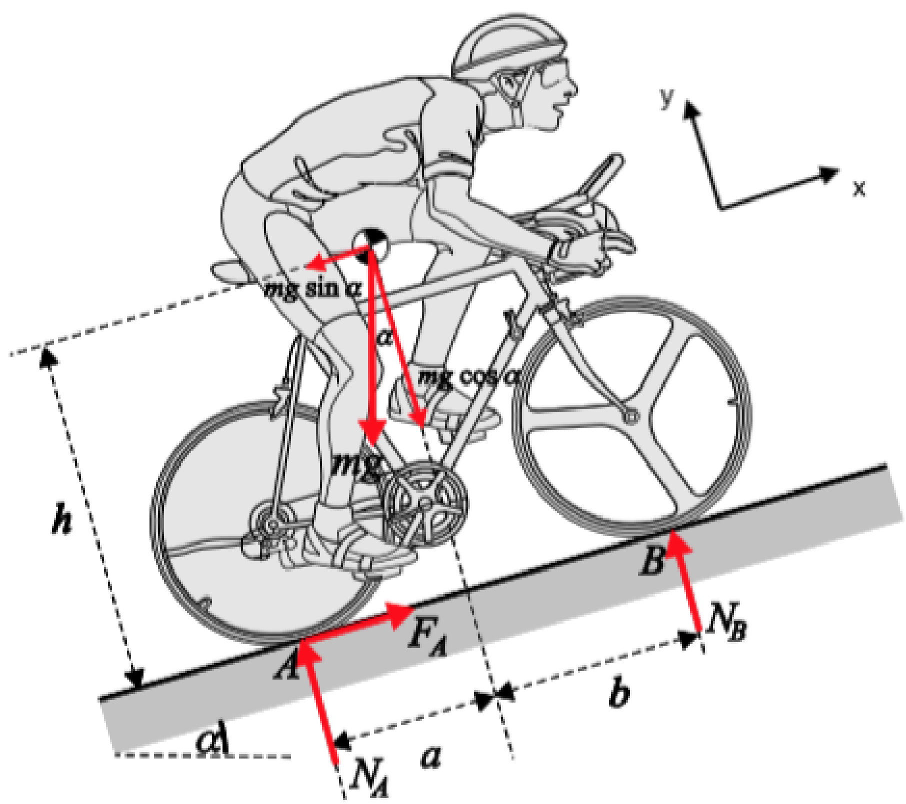

Figure 2: Cyclist climbing a hill.

$$\Sigma F_x = 0 = F_A - mg\sin\alpha$$

or,

$$F_A = mg\sin\alpha$$

The angle α is found from the profile of the hill:

$$\alpha = \arctan\left(\frac{88ft}{500ft}\right) = 9.98^\circ$$

Then,

$$F_A = 85kg \cdot 9.81m/s^2 \cdot \sin 9.98^\circ = 144.5\text{N}$$

Thus,

$$u = \frac{775\text{W}}{144.5\text{N}} = 5.36\text{m/s}$$

We can find the distance travelled from our good friend Pythagoras to get,

$$t = \frac{d}{u} = \frac{154.7\text{m}}{5.36\text{m/s}} = 28.9\text{s}$$

ii. Alternatively, use energy considerations,

$$P = \frac{\Delta E}{\Delta t}$$

which can be rearranged to find the amount of time

$$\Delta t = \frac{\Delta E}{P}$$

From the problem description,

$$\Delta E = mg \cdot 88\text{ft} \times \frac{1\text{m}}{3.28\text{ft}} = 22370\text{J}$$

So that,

$$P = \frac{22370\text{J}}{775\text{W}} = 28.9\text{s}$$

(Your answer may vary slightly, depending on how you interpreted the elevation profile.)

(b) Using a proper free body diagram of the rear wheel, calculate the angular speed of the rear wheel and the torque needed at the wheel axle to produce the expected time. Use the average grade in your calculations and assume the speed is constant (that is, ignore the time needed to get up to full speed).

Solution. We can use the figure from the handout, reproduced in Figure 3. From Figure 2, we can find the propulsive force needed:

$$\Sigma F_x = 0 = F_A - mg\sin\alpha$$

so that,

$$F_A = mg \sin \alpha \tag{1}$$

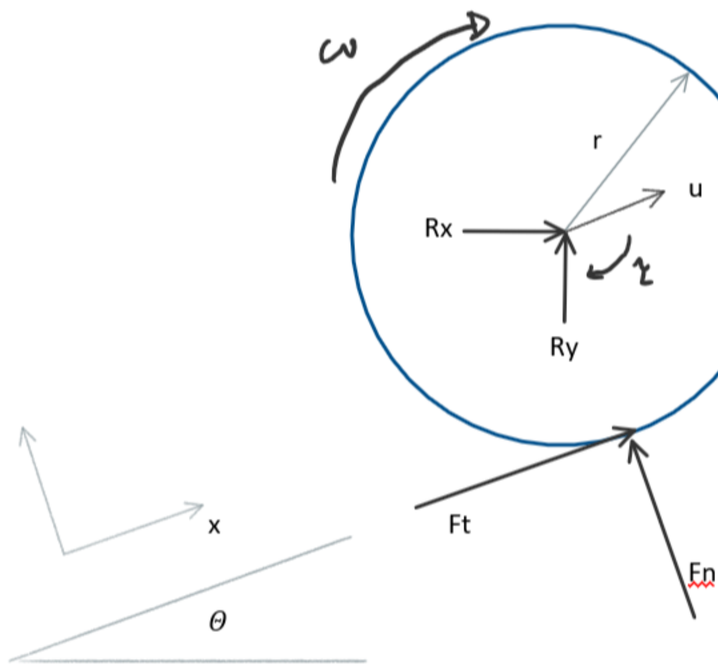

Figure 3: Free body diagram for a bike wheel.

Summing the moments about the axle,

$$\Sigma M_{axle} = 0 = -\tau + F_t \cdot r$$

where we note that $F_A = F_t$ across the two figures.
Or,

$$\tau = F_t \cdot r \tag{2}$$

Substituting Eq. 1 into Eq. 2, and using the geometry of the problem, we can find the torque. Note that,

$$\alpha = \arctan(0.175) = 9.93°$$

$$\tau = F_t \cdot r = mgr \sin \alpha = 85\text{kg} \cdot 9.81\text{m/s}^2 \cdot \frac{0.65\text{m}}{2} \cdot \sin(9.93°)$$

$$\boxed{\tau = 46.75\text{N} \cdot \text{m}}$$

The speed of the wheel can be found from the geometry of the problem or the power considerations. We'll do the latter:

$$\omega = \frac{P}{\tau} = \frac{775\text{W}}{93.5\text{N}} = \boxed{8.28\text{rad/s}}, or \boxed{79.1\text{rpm}}$$

(c) The center of gravity of the cyclist/bike combination is 60 cm in front of the rear axle (dimension 'a' in Figure 2) and 80 cm above the ground, measured perpendicularly (dimension 'h'). What is the steepest grade he could climb without tipping over backwards? At what speed would he climb such a grade?

Solution. If we sum the moments about point A in Figure 2, we can eliminate a lot of unknowns:

$$\Sigma M_A = 0 = h \cdot mg \sin\alpha - a \cdot mg \cos\alpha + N_B(a + b)$$

The cyclist will just begin to tip over when $N_B = 0$, which leads to

$$h \cdot mg \sin\alpha = a \cdot mg \cos\alpha$$

Or,

$$\tan\alpha = \frac{a}{h} = \frac{60\text{cm}}{80\text{cm}}$$

Or,

$$\boxed{\alpha = 36.9°}$$

At that slope the speed if found from

$$P = F \cdot u$$

Or,

$$u = \frac{P}{F} = \frac{P}{mg \sin\alpha} = \frac{775\text{W}}{85\text{kg} \cdot 9.81\text{m/s}^2 \sin(36.9°)}$$

$$\boxed{1.55\text{m/s}}$$