RBE 1001: Introduction to Robotics
C-Term 2019-20
HW 2.2: Fundamentals of Mechanics – Work and Power

Last modification: January 26, 2020

1 Work and power

In your physics classes, you've undoubtedly discussed the concepts of work and power. Here, we'll extend those concepts to robotics applications, for example motor performance and robot stability.

1.1 Work

Work, which we'll denote as W, is formally defined as the application of a force through a distance. In the one-dimensional case, consider the incremental work done by a force, F, acting over an infinitesimal distance, δx,

$$\delta W = F\delta x \tag{1}$$

By integrating Equation 1, we get the total amount of work done over a given distance, in this case, from $x_1 \rightarrow x_2$,

$$W = \int_{x_1}^{x_2} F dx \tag{2}$$

You showed in your physics classes that work can be related to the change in energy of a system, for example through lifting a weight, compressing a spring, or increasing the speed of a mass. In case you forgot, we've given you a couple of homework problems to refresh your memory.

1.2 Power

In robotics, the concepts of mechanical work and energy are less useful than power. That's not to say they aren't important – robots are still governed by physics! – but in robotics, you'll mostly concern yourself with forces, torques, and *power*.

Power is most often defined as the *rate at which work is done*. For example, if the the work done in Equation 2 is done over the time period, Δt, then the *average* power, P, needed to do the work is,

$$P = \frac{W}{\Delta t} = \frac{\Delta E}{\Delta t} \tag{3}$$

where we've used the fact that work leads to a change in energy.

When the focus is on robot motion, whether driving up a hill or lifting an object, it is more useful to relate power to speed. To do so, reconsider the amount of work done over a small distance, δx:

$$\delta W = F \delta x \tag{4}$$

If that work is done over an infinitesimal amount of time, δt, then we can write the power as,

$$P = \frac{\delta W}{\delta t} = \frac{F \delta x}{\delta t} \tag{5}$$

Note, however, that as we take $\delta x \to 0$ and $\delta t \to 0$, then the expression becomes simply,

$$P = F \cdot u \tag{6}$$

where u is the velocity in the x direction.[1]

It is this form of (mechanical) power that is the most useful in robotics. For example, consider the problem of a mobile robot driving up a ramp, as shown (with a car) in Figure 6.41 at the end of this document. For a constant speed, u, the wheels must provide enough force to cancel that of gravity (assuming frictional losses, f_D, are zero), and so the amount of power needed is immediately calculated as,

$$P = F \cdot u = mgu \sin\theta \tag{7}$$

You will find that when selecting motors and calculating performance, power will generally be the first consideration. You will see that by using gears, force (and torque) are rarely constraints on motor selection (you can always gear a motor way down), but if you don't have enough power to move your robot at a reasonable speed, gears can't help you.

1.3 Rotational power

In the previous example, we calculated the power needed to climb a grade, but we glossed over the fact that motors create rotational motion while the robot in the example moved linearly. To relate the two, imagine that the propulsion force is being provided by a single motor, spinning with angular speed, ω, attached directly to a wheel with radius, r, as shown in Figure 1.

From the geometry, the linear speed of the robot, u, is related to the motor speed by,

$$u = \omega r \tag{8}$$

where ω has units of radians per unit time (typically rad/s). Further, the force generated at the contact point of the wheel and the ground is,

$$F = \frac{\tau}{r} \tag{9}$$

[1] Since we'll be working a lot with 2-D problems, we'll adopt the convention that u is the velocity in the x direction and typically reserve v for the velocity in the y direction.

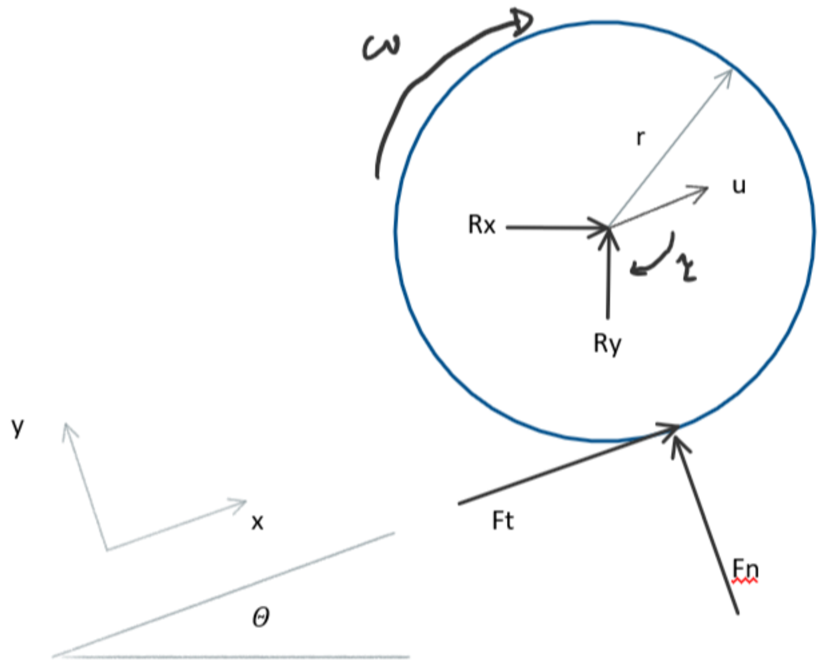

Figure 1: Free body diagram for a robot wheel.

where τ is the torque produced by the motor. We can then relate the power of the motor to more useful terms:

$$P = F \cdot u = \frac{\tau}{r} \cdot \omega r = \tau\omega \tag{10}$$

That is, the power produced by a motor is the rotational speed times the torque. Word of caution: When applying Equation 10, it is essential that your units are consistent! For example, if the rotational speed is given in *rpm* and the torque in $in \cdot oz$, you'll have to manage a fair bit of unit conversions to get to, say, $Watts$.

1.4 Practice Problems

In Section 6.8 of *Undergraduate Lecture Notes in Physics*, A. Radi and J. O. Rasmussen (appended to this document), do:

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- 41(a). Note that two forces are acting on the elevator: gravity and friction.
- 43

2 Assigned Problems

1. A cyclist in the George Street Bike Challenge for Major Taylor is shooting to complete the short, but steep, climb in 30 seconds. Figure 2 shows the elevation profile for the climb and the cyclist has done some experiments to determine that he can output 775 W for 30 seconds. The bike and rider weigh 85 kg, combined. The diameter of the rear tire is 65 cm.

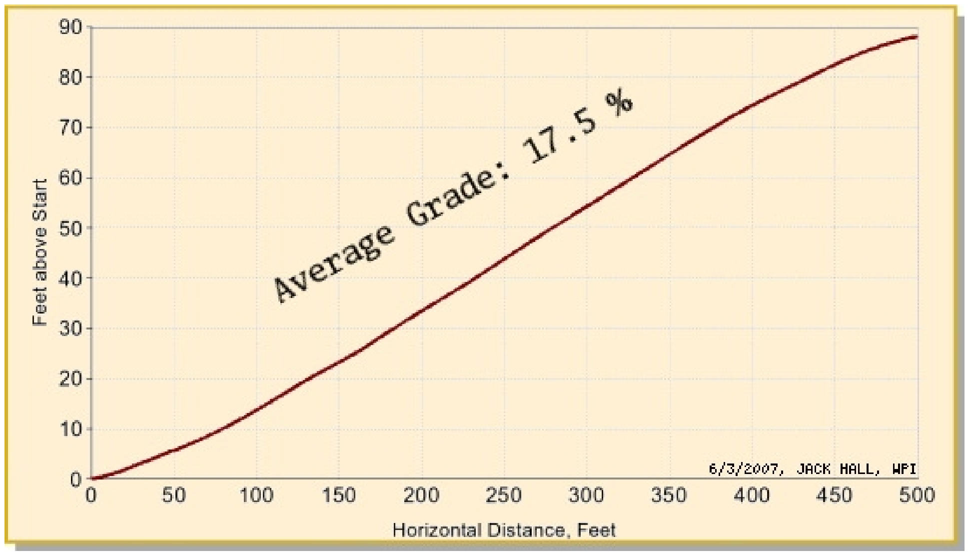

Figure 2: Elevation profile of George Street.

Figure 3 shows the geometry of the system.

(a) Given the power and neglecting air resistance (a reasonable assumption for steep climbs), how long will it take our hero to go up the hill?

(b) Using a proper free body diagram of the rear wheel, calculate the angular speed of the rear wheel and the torque needed at the wheel axle to produce the expected time. Use the average grade in your calculations and assume the speed is constant (that is, ignore the time needed to get up to full speed).

(c) The center of gravity of the cyclist/bike combination is 60 cm in front of the rear axle (dimension 'a' in Figure 3) and 80 cm above the ground, measured perpendicularly (dimension 'h'). What is the steepest grade he could climb without tipping over backwards? At what speed would he climb such a grade?

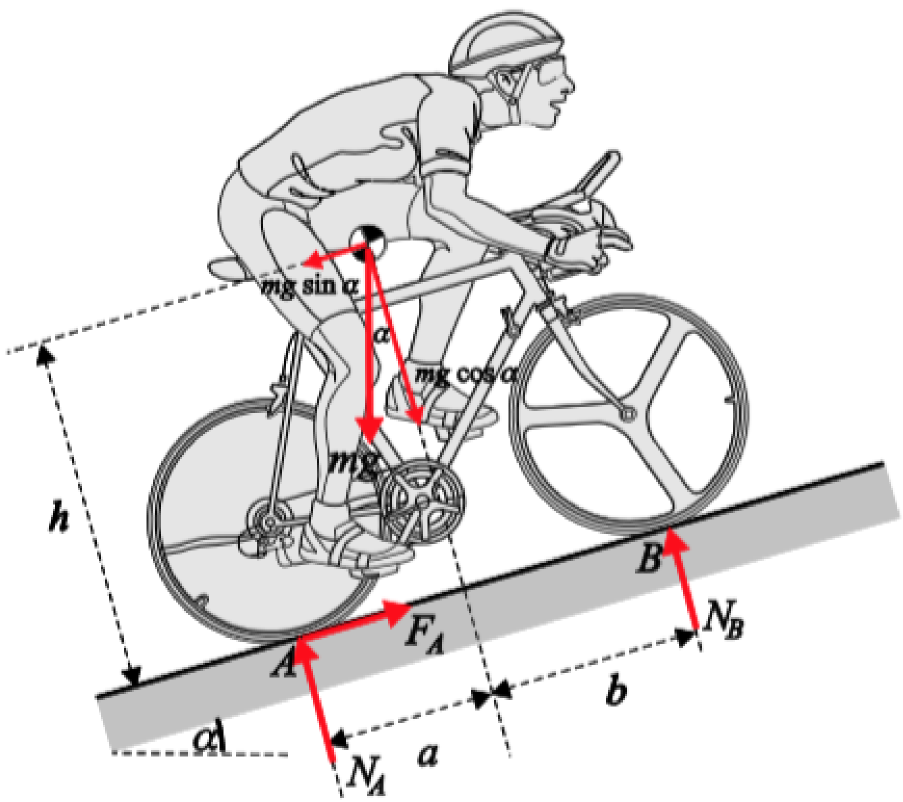

Figure 3: Cyclist climbing a hill.

Practice problems referenced in Section 1.4 above. You do not need to submit these, but solutions to select problems have been placed on canvas.

Section 6.8 Power

(39) How much average power in kilowatts and horsepower is required to lift a block of 100 kg to a height of 10 m in 30 s?

(40) At 30 piasters (Egyptian pound $= 100$ Piaster) per kilowatt-hour of electricity, what is the cost of operating a 5-hp motor for 2 h?

(41) An elevator fully loaded with passengers has a mass $M = 2{,}000$ kg. As the elevator descends, an almost constant frictional force $f = 4{,}000$ N acts against its motion. What power must be delivered by the motor to descend the elevator at: (a) a constant speed v of 4 m/s, and (b) a constant acceleration a of 1.5 m/s^2 that produces a speed $v = at$?

(42) A constant horizontal force $F = 20$ N acts on a block of mass $m = 4$ kg resting on a horizontal plane. The block starts from rest at $t = 0$. Show that the instantaneous power delivered by the force at any time t is given by $P = F^2 t/m$, and find its value at $t = 5$ s.

(43) A car generates 20 hp when traveling at a constant speed of 100 km/h. What is the total resistive force that acts on the car?

(44) A car of mass $m = 1{,}500$ kg accelerates from rest to 100 km/h in 8 s. What is the average power delivered by its engine?

(45) A car of mass m accelerates with acceleration a up an inclined plane of angle θ as in Fig. 6.41. The drag force f_D consists of rolling friction α (N) and air drag βv^2(N), i.e. $f_D = \alpha + \beta v^2$, where α and β are constants and v is the speed of the car. (a) Find the force F that propels the car. (b) Show that $P = mva + mvg \sin\theta + \alpha\, v + \beta\, v^3$ is the power delivered to the wheels by the engine, where mva is the power delivered to accelerate the car, $mvg \sin\theta$ is the power to overcome gravity, $\alpha\, v$ is the power to overcome rolling friction, and $\beta\, v^3$ is the power to overcome air drag. (c) Calculate the various components of P and hence the total P if we take $m = 1{,}000$ kg, $a = 2$ m/s^2, $v = 20$ m/s, $\alpha = 200$ N, $\beta = 0.5$ kg/m, and $\theta = 15°$.

Fig. 6.41 See Exercise (45)