RBE 1001: Introduction to Robotics
C-Term 2019-20
HW 4.1: Control

Last modification: February 11, 2020

1. You're designing an altitude control system for precision landing of a quadrotor. The system uses an ultrasonic sensor that points downward so that it measures the height above the landing surface. Through some experiments, you determine that the lift force produced by the motors is correlated to the voltage supplied to the motors, squared:

$$F_{lift} = \beta V_{motor}^2$$

where $\beta = 15\ \frac{\mathrm{N}}{\mathrm{V}^2}$. The quadrotor weighs 800g.

To read the ultrasonic sensor, you will use a 10-bit ADC with a reference voltage of 5V.

   (a) The ultrasonic sensor outputs 1 V for every 4 meters away an object is. What is the ADC reading when $z$, the distance of the quadrotor above the landing surface, is 8 meters? Be exact.

$$ADC = \text{floor}\left[\frac{2V}{5V} \cdot 1024\right] = \text{floor}\,(409.6) = \boxed{409}$$

   (b) Let the error, $e$, be defined as the difference between the target altitude, $z_{target}$, and the current altitude, i.e.,

$$e = z_{target} - z$$

   Using a proportional control algorithm with,

$$V_{motor} = K_p \cdot e$$

   write an expression for steady-state height of the quadrotor above the ground as a function of target height and proportional gain, $K_p$. Explain your reasoning.

At steady-state, the thrust will just balance the weight

$$mg = F_{lift} = \beta V_m^2 = \beta (K_p e)^2 = \beta K_p^2 (z_t - z)^2$$

Rearranging,

$$(z_t - z)^2 = \frac{mg}{\beta K_p^2} \Rightarrow \boxed{z = z_t - \sqrt{\frac{mg}{\beta K_p^2}}}$$

(c) If the target height is 10 m and $K_p = 2.0$, what will the steady-state height of the quadrotor be?

$$z = 10\,\text{m} - \sqrt{\frac{0.8\,\cancel{\text{kg}} \cdot 9.81\,\frac{\text{m}}{\text{s}^2}}{15\,\frac{\cancel{\text{N}}}{\cancel{\text{V}^2}} \cdot \left(2.0\,\frac{\cancel{\text{V}}}{\text{m}}\right)^2}} = 10\,\text{m} - 0.342\,\text{m} \approx \boxed{9.64\,\text{m}}$$

(d) How can you reduce the steady-state error to zero?

Add integral control!