HW 2.3: Solutions

RBE 1001, A19

The following has the solutions from a previous edition of this problem, which has everything in this term's edition, plus a little more.

Solution

1. George Street Bike Challenge for Major Taylor

   (a) Where should the cyclist operate on the given curve to minimize his time to climb George Street? What is the pedalling torque and speed at that point? (Sanity check: A cyclist typically shoots for a *cadence* in the 90 - 120 rpm range. Don't forget the 2π to convert from 1/ s to rot/ s!)

   [Max power will lead to max speed, which leads to the fastest time. Torque is then read directly from the data (665 in-lb). Cadence, here given by ω, is a little trickier:

$$\omega = \frac{P}{T} = \frac{775 \text{ W}}{665 \text{ in} - \text{lb}} \times \frac{550 \text{ ft} - \text{lb/s}}{746 \text{ W}} \times \frac{12 \text{ in}}{\text{ft}} \times \frac{\text{rot}}{2\pi \text{ rad}} \times \frac{60 \text{ s}}{\text{min}} = 98.5 \text{ rpm}$$

   which is a perfectly reasonable cadence.]

   (b) Given the power at that point and neglecting air resistance (a reasonable assumption for steep climbs), how fast could our hero expect to go up the hill? Even though you haven't yet specified any gear sizes, don't forget to include the efficiency of the drive train in your calculation. Also, since we're neglecting air resistance, you can do this from energy principles alone – no FBD needed!

   [From energy principles:

$$\text{Time} = \frac{\text{change in potential}}{\text{output power}} = \frac{190 \text{ lbs} \cdot 0.175 \cdot 500 \text{ ft}}{775 \text{ W} \cdot 0.95} \times \frac{746 \text{ W}}{550 \text{ ft} - \text{lb/s}} = 30.6 \text{ s}$$

   A perfectly reasonable time (but your professor took 33.4 seconds).]

   (c) Using a proper free body diagram of the rear wheel, calculate the speed of the rear wheel and the torque needed at the wheel axle to produce the expected time. Use the average grade in your calculations and assume the speed is constant (that is, ignore the time needed to get up to full speed).

   [Torque of the wheel is found from the traction force (sometime referred to as the friction force.]

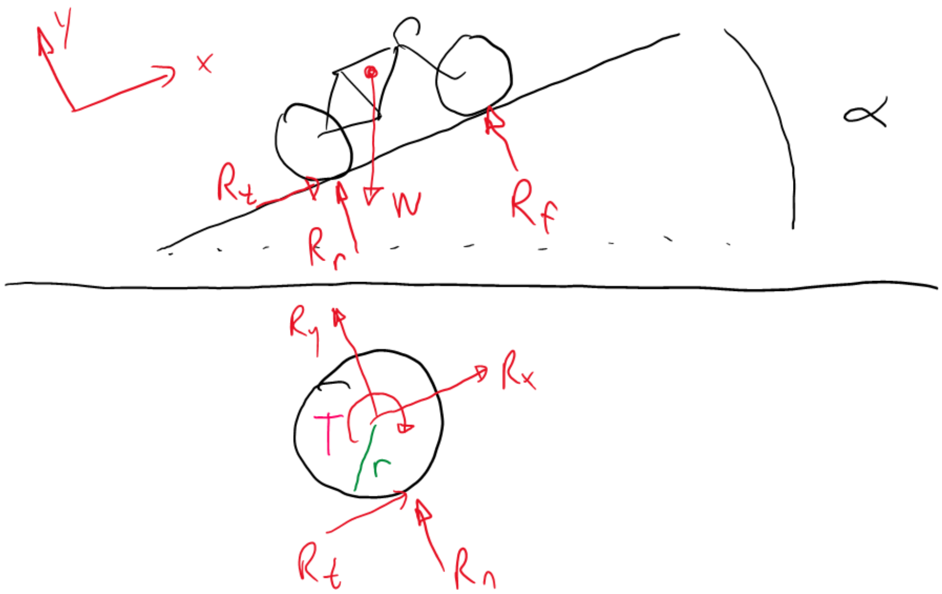

Figure 1: FBD of bike (top) and wheel (bottom)..

Consulting the FBD in Figure 1, we can quickly solve for the traction force as

$$R_t = W \sin \alpha$$

Then from the sum of the moments on the wheel,

$$T = rR_t = rW \sin \alpha$$

$$T = 12.5 \text{ in} \cdot 190 \text{ lbs} \cdot \sin[\tan^{-1}(0.175)] = 409.4 \text{ in} - \text{lbs}$$

Speed is found one of two ways, either from the distance, wheel circumference, and time, or from power principles. The former is easier:

$$\omega = \frac{\text{distance}}{\text{circumference} * \text{time}} = \frac{500 \text{ ft}/\cos[\tan^{-1}(0.175)]}{30.6 \text{ s}} \times \frac{1 \text{ rot}}{\pi \cdot 25 \text{ in}} \times \frac{12 \text{ in}}{\text{ft}} \times \frac{60 \text{ s}}{\text{min}} = 152.1 \text{ rpm}$$

]

(d) Still using the average grade, what size sprocket (gear) should be used on the rear wheel? (Sanity check: The hill climb can be done with a typical set of sprockets, which can range anywhere from, say, 11 - 32 teeth.)

[

Tooth count, n, is found from the speed ratio:

$$n = 34 \text{ teeth} \times \frac{98.5 \text{ rpm}}{152.1 \text{ rpm}} = 22 \text{ teeth}$$

Rounded down *slightly* to an integer number of teeth.

]

(e) The cyclist realizes that the sprocket set on his bike doesn't include the specified sprocket size, so he plans to climb using a rear gear with two *fewer* teeth. Will he go faster or slower than expected? Will his cadence be higher or lower than it would with the ideal sprocket?

[Any deviation from peak power will slow him down. With a smaller rear gear (20 teeth), his cadence will slow down, since it will take more force on the chain (and torque at the "motor") to produce the needed torque. Thus, he'll move to the right (higher torque side) of peak power.]

(f) Let's say the cyclist could find the *exact* gear ratio needed to be at maximum power on the *average* grade. If he didn't change gears during the climb, how would the fact that actual grade deviates both above and below the average grade affect his time? Explain.

[Again, any deviation from ideal will lead to less power output. Sometimes he'll spin faster, sometimes slower than ideal, but he'll go slower in both cases; therefore, slower overall.]