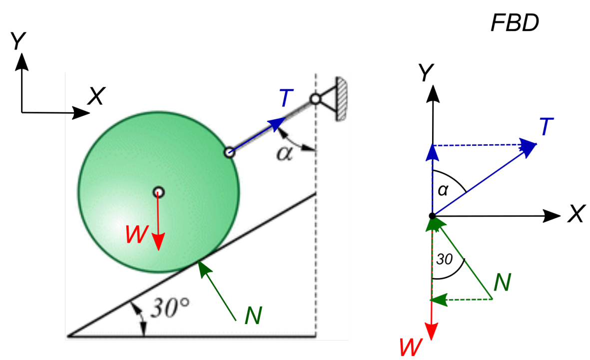
Last modification: January 27, 2020

RBE 1001: Introduction to Robotics
C-Term 2019-20
HW 2.1: Solutions

Solutions

From Statics, Learning from Engineering:

1. (4.34) A 10 N ball is supported by an incline and a cable making an angle α with the vertical direction. Knowing the force in the cable equal to 5 N, determine the angle α and the force exerted by the ball on the incline. Consider the ball as a particle.



W = 10N
T = 5N

$$\Sigma F_x = 0$$

$$T \cdot \sin(\alpha) - N \cdot \sin(30) = 0$$

$$N \cdot \sin(30) = T \cdot \sin(\alpha)$$

$$N = 2T \cdot \sin(\alpha) \tag{1}$$

$$\Sigma F_y = 0$$

$$0 = T \cdot \cos(\alpha) + N \cdot \cos(30) - W$$

$$W = T \cdot \cos(\alpha) + N \cdot \cos(30) \tag{2}$$

Substituting 1 into 2 gives,

$$W = T \cdot \cos(\alpha) + 2T \cdot \sin(\alpha)\cos(30)$$

Knowing that

$$\sin(30) = \frac{1}{2}$$

we get,

$$W = 2T \cdot \sin(30)\cos(\alpha) + 2T \cdot \cos(30) \cdot \sin(\alpha)$$

which allows us to use the trigonometric identity,

$$\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$$

to get,

$$W = 2T \cdot \sin(30 + \alpha)$$

Noting that $W/(2T) = 1$, we can solve for α:

$$(30 + \alpha) = \arcsin(1)$$

$$(30 + \alpha) = 90$$

$$\boxed{\alpha = 60^\circ}$$

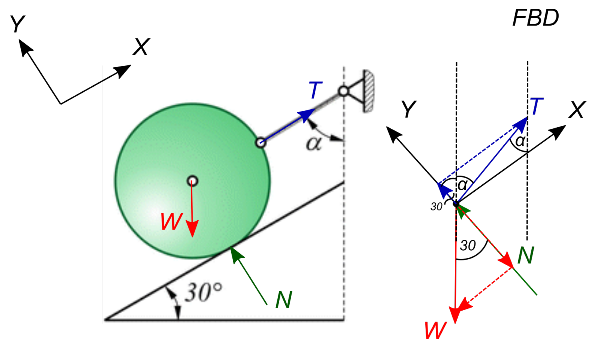
Note that, although the diagram implies $\alpha = 60$, it's not given that the cable is parallel to the ramp.

From Eq. 1,

$$N = 2T \cdot \frac{\sqrt{3}}{2}$$

$$\boxed{N = 8.66\text{N}}$$

Alternatively, align the coordinate system with the inclined plane. It's not given that the cable is parallel to the ramp, then,



$$\Sigma F_x = 0 = -W \sin 30 + T \sin(\alpha + 30)$$

which can be rearranged to,

$$\sin(\alpha + 30) = \frac{W}{2T} = 1$$

which implies

$$\boxed{\alpha = 60^\circ}$$

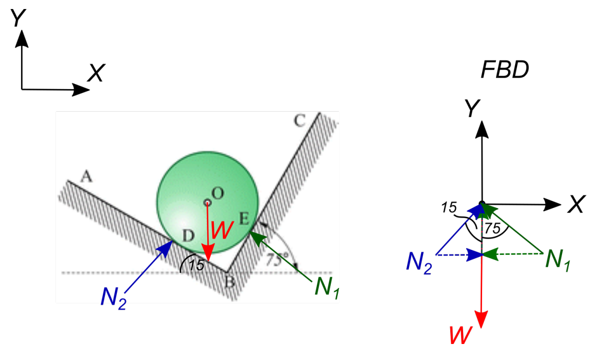
Then in the y-direction,

$$\Sigma F_y = 0 = N - W \cos 30 + 0 \cdot T$$

Which leads to,

$$\boxed{N = 8.66\mathrm{N}}$$

2. (4.35) Disc O (weighing 10 N) is supported by two frictionless planes which are perpendicular one to another. Determine the force exerted by the disc on each plane. Solve the problem analytically.



W = 10N

$$\Sigma F_x = 0$$

$$-N_1 \cdot \sin(75) + N_2 \cdot \sin(15) = 0$$

$$N_1 = N_2 \cdot \frac{\sin(15)}{\sin(75)} \tag{3}$$

$$\Sigma F_y = 0$$

$$-W + N_1 \cdot \cos(75) + N_2 \cdot \cos(15) = 0$$

$$N_1 \cdot \cos(75) + N_2 \cdot \cos(15) = W \tag{4}$$

Substituting 3 into 4

$$N_2 \cdot \frac{\sin(15)}{\sin(75)} \cdot \cos(75) + N_2 \cdot \cos(15) = W$$

$$N_2 \cdot (\frac{\sin(15)}{\sin(75)} \cdot \cos(75) + \cdot \cos(15)) = 10$$

$$\boxed{N_2 = 9.66N}$$

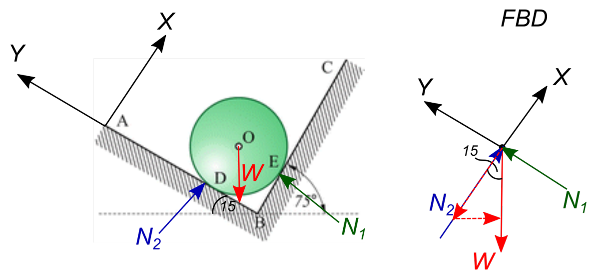
Substituting N_2 into eq.3 We get:

$$\boxed{N_1 = 2.59N}$$

Alternatively, align the coordinate system with the planes:

$$\Sigma F_x = 0 = N_2 - W \cos 15$$

Then the answer is immediately,



$$N_2 = 10\text{N} \cdot \cos 15 = 9.66\text{N}$$

Similarly,

$$\Sigma F_y = 0 = N_1 - W \cos 75$$

$$N_1 = 10\text{N} \cdot \cos 75 = 2.59\text{N}$$