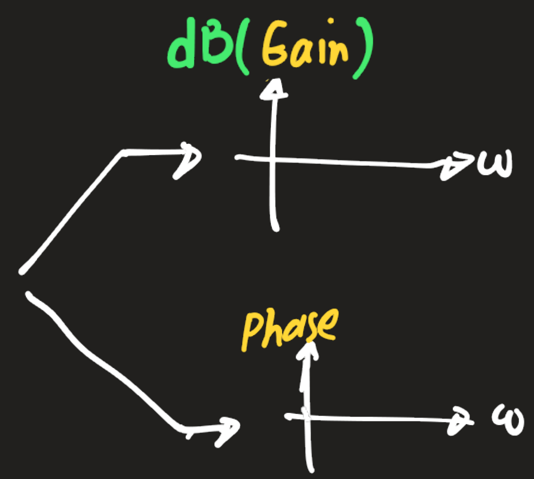
Recap

$$u(t) = A \sin(\omega t) \longrightarrow \boxed{\begin{array}{c} G(s) \\ \text{(LTI system)} \end{array}} \longrightarrow y_{ss} = A \underbrace{|G(j\omega)|}_{\text{Gain}} \sin(\omega t + \underbrace{\angle G(j\omega)}_{\text{phase}})$$

$(0 < \omega < \infty)$ differnet "ω" $\longrightarrow$ different Gain & Phase

Bode Plots



$$dB(\text{Gain}) = dB(|G(j\omega)|) = 20 \cdot \log |G(j\omega)|$$

$$G(s) = \frac{\overset{A}{10}\,\overset{C}{(s+3)}}{\underset{B}{s}\,\underset{D}{(s^2+4s+5)}}$$

$$dB = 20 \log |G(j\omega)| = dB(A) + dB(C) - dB(B) - dB(D)$$

$$\phi = \angle(G(j\omega)) = \phi(A) + \phi(C) - \phi(B) - \phi(D)$$

Reminder : (log & dB)

$\log(0)$: $\underbrace{\text{undefined}}$

$$\log_{10}(a) = b \Rightarrow \boxed{10^b = a} \Rightarrow \begin{cases} a = 0 \Rightarrow b = -\infty \leftarrow 10^{-\infty} = \frac{1}{10^{\infty}} = 0 \\ 0 < a < 1 \Rightarrow b < 0 \\ a = 1 \Rightarrow b = 0 \\ a > 1 \Rightarrow b > 0 \end{cases}$$

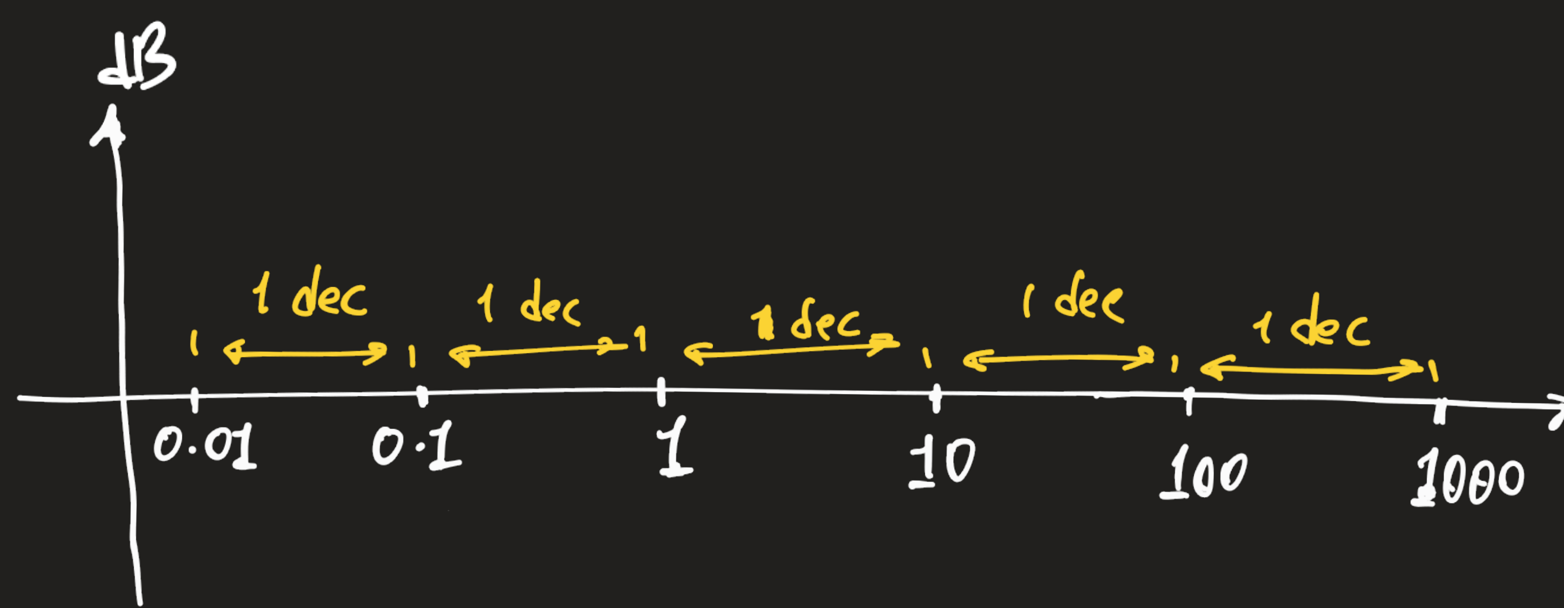
$$\left.\begin{aligned} \log(0.1) &= \log(10^{-1}) = -1 \\ \log(1) &= \log(10^{0}) = 0 \\ \log(10) &= \log(10^{1}) = 1 \\ \log(100) &= \log(10^{2}) = 2 \\ \log(1000) &= \log(10^{3}) = 3 \end{aligned}\right\} \Rightarrow \log(10^n) = n \log(10) = \underline{\underline{n}}$$

$$\boxed{dB(a) = 20 \log(a)}$$

→ decade: logarithmic scale/unit w/ ratio of 10:1

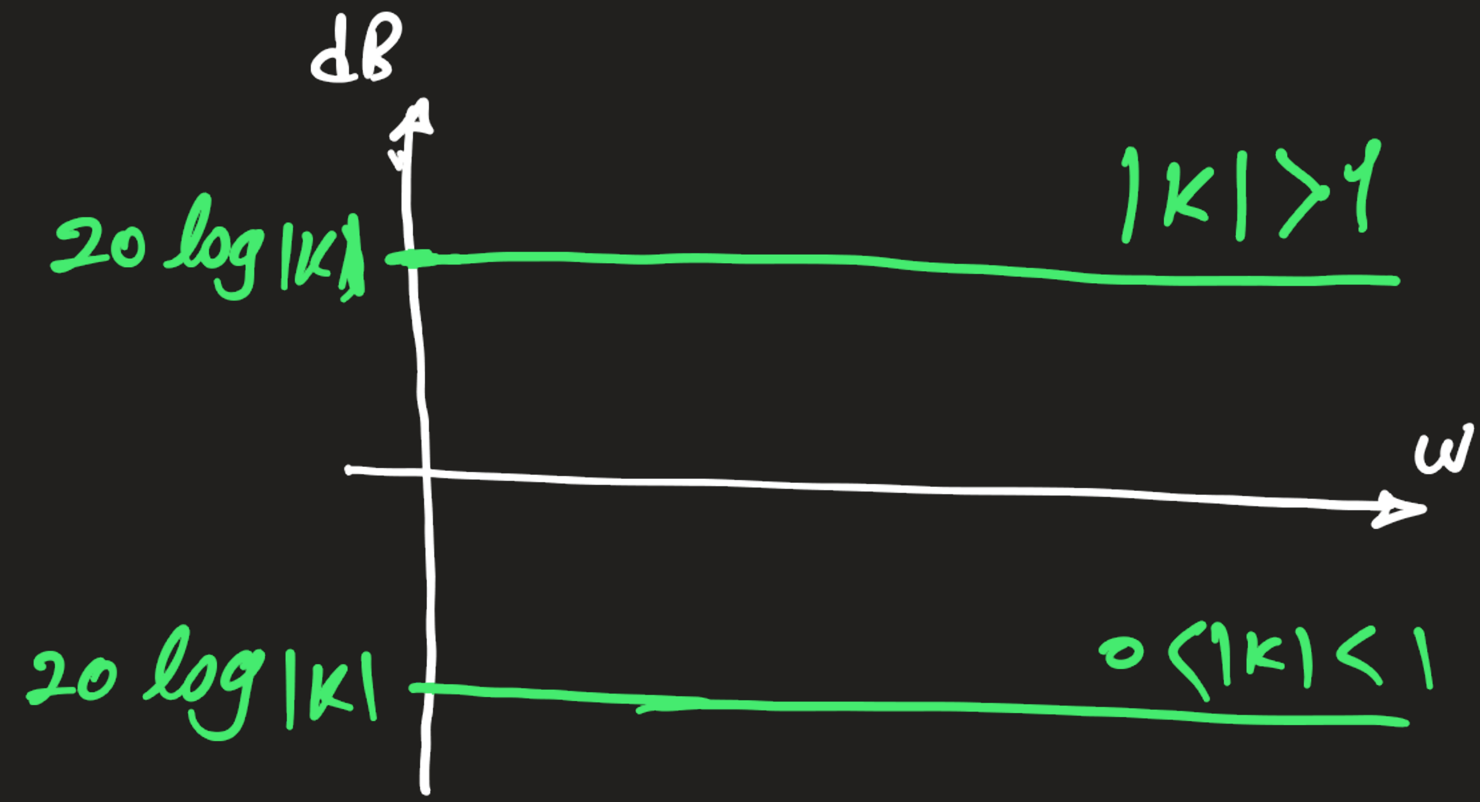
→ Slope : N dB/dec :

N dB increase per decade

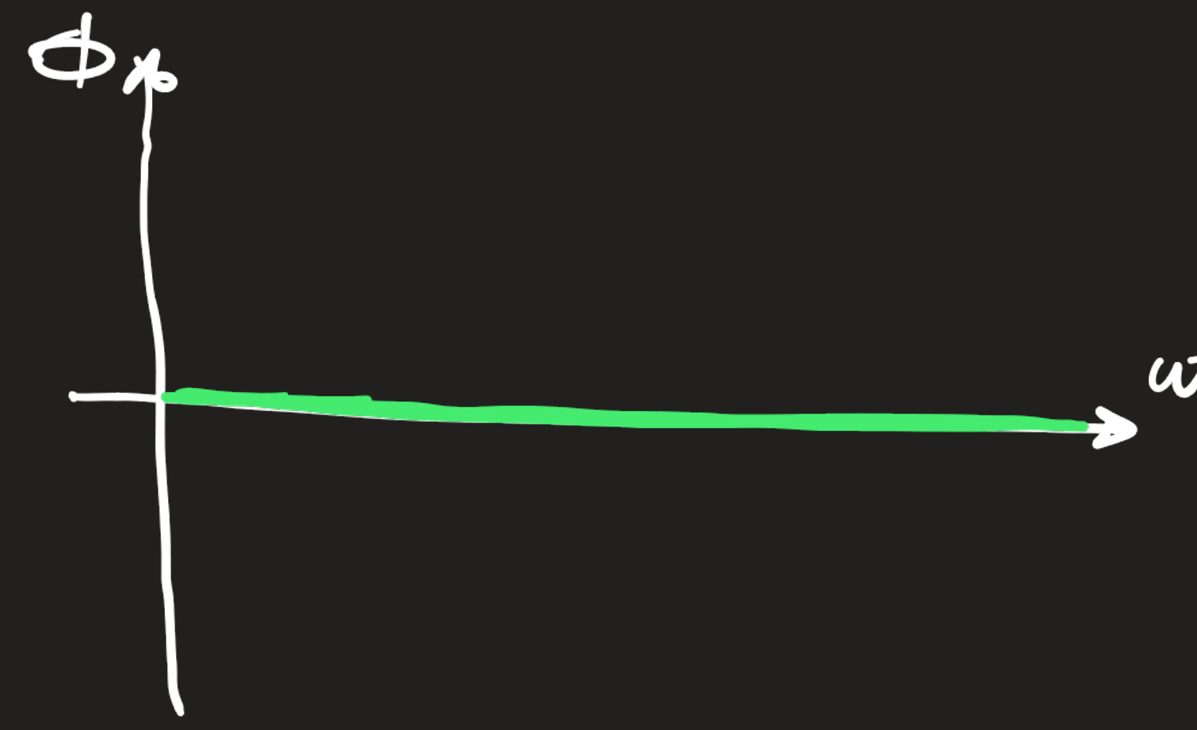


A) Constant Gain (K)

$$dB = 20 \log_{10} |K|$$



$$\Phi = \angle (K + 0j) = \tan^{-1} 0 = 0$$



Not function of ω

⇒ straight horizontal line

B) $\underline{\underline{S=0}}$ (Zero or pole at origin)

$\hookrightarrow \boxed{G(s) = s \quad (\text{zero})}$ $\quad G(j\omega) = j\omega$

$dB = 20 \log |j\omega| = 20 \log_{10} |\omega| \leftarrow$

$\Rightarrow \underbrace{\text{cross } \omega\text{-axis}}_{dB=0}$ at $\omega = 1$

$\Rightarrow$ slope : $\underline{\underline{20}}$ dB/dec $\Leftarrow$

$\phi = \angle(0 + j\omega) = \tan^{-1}\left(\frac{\omega}{0}\right) = \pi/2$

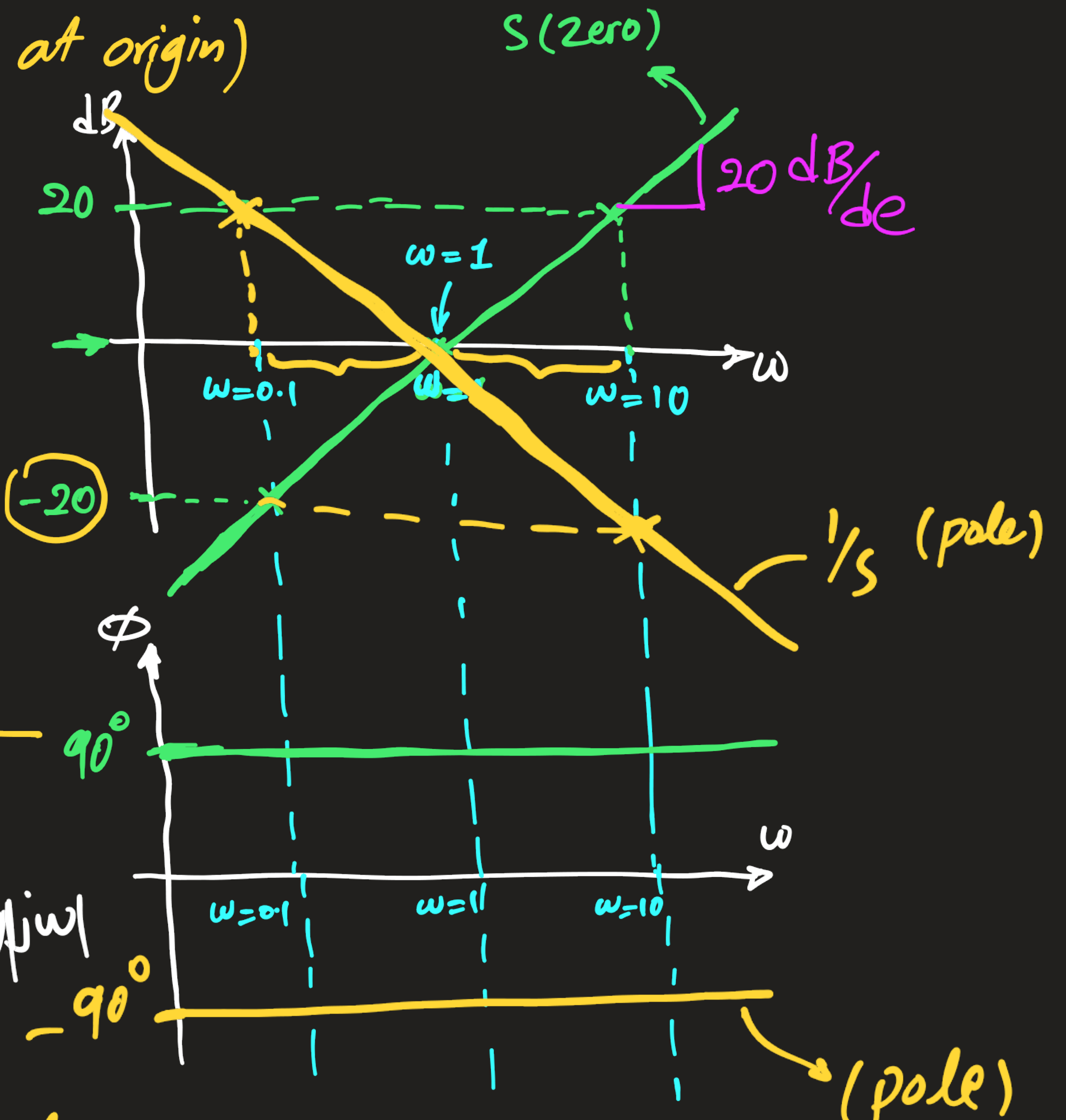
$\hookrightarrow \boxed{G(s) = \frac{1}{s} \quad (\text{pole})}$

$dB = 20 \log \left|\frac{1}{j\omega}\right| = \cancel{20 \log 1}^{\,0} - 20 \log |j\omega|$

$dB = - 20 \log |\omega|$

$\overset{dB}{\Rightarrow}$ cross ω-axis at $\underset{dB=0}{\underline{\omega = 1}}$ and slope -20 dB/de

$\phi = \angle(1) - \angle(j\omega) = 0 - 90^\circ = -90^\circ$



c) Firs-order :

w_n = corner frequency

$\boxed{\text{first-order Zero}}$: $G(s) = \tau s + 1 \overset{w_n = \frac{1}{\tau}}{=\!=} \left(\frac{s}{\omega_n} + 1\right) \Rightarrow \boxed{G(jw) = j\,\frac{\omega}{w_n} + 1}$

if given as $H(s) = s + \omega_n \Rightarrow$ factor as $H(s) = \omega_n \left(\frac{s}{\omega_n} + 1\right)$

deal with this separately as a gain

standard

$dB = 20\log|G(j\omega)| = 20\log\left(\sqrt{1 + \left(\frac{\omega}{\omega_n}\right)^2}\right)$

when $\omega << \omega_n \Rightarrow dB \cong 20\log(1) = 0$

when $\omega >> \omega_n \Rightarrow \underbrace{dB \cong 20\log\left(\frac{\omega}{\omega_n}\right)}$ → this is a line with slope of 20 dB/dec

→ $\omega = \omega_n \Rightarrow dB = 0$

→ $\omega > \omega_n \Rightarrow$ 20 dB/dec

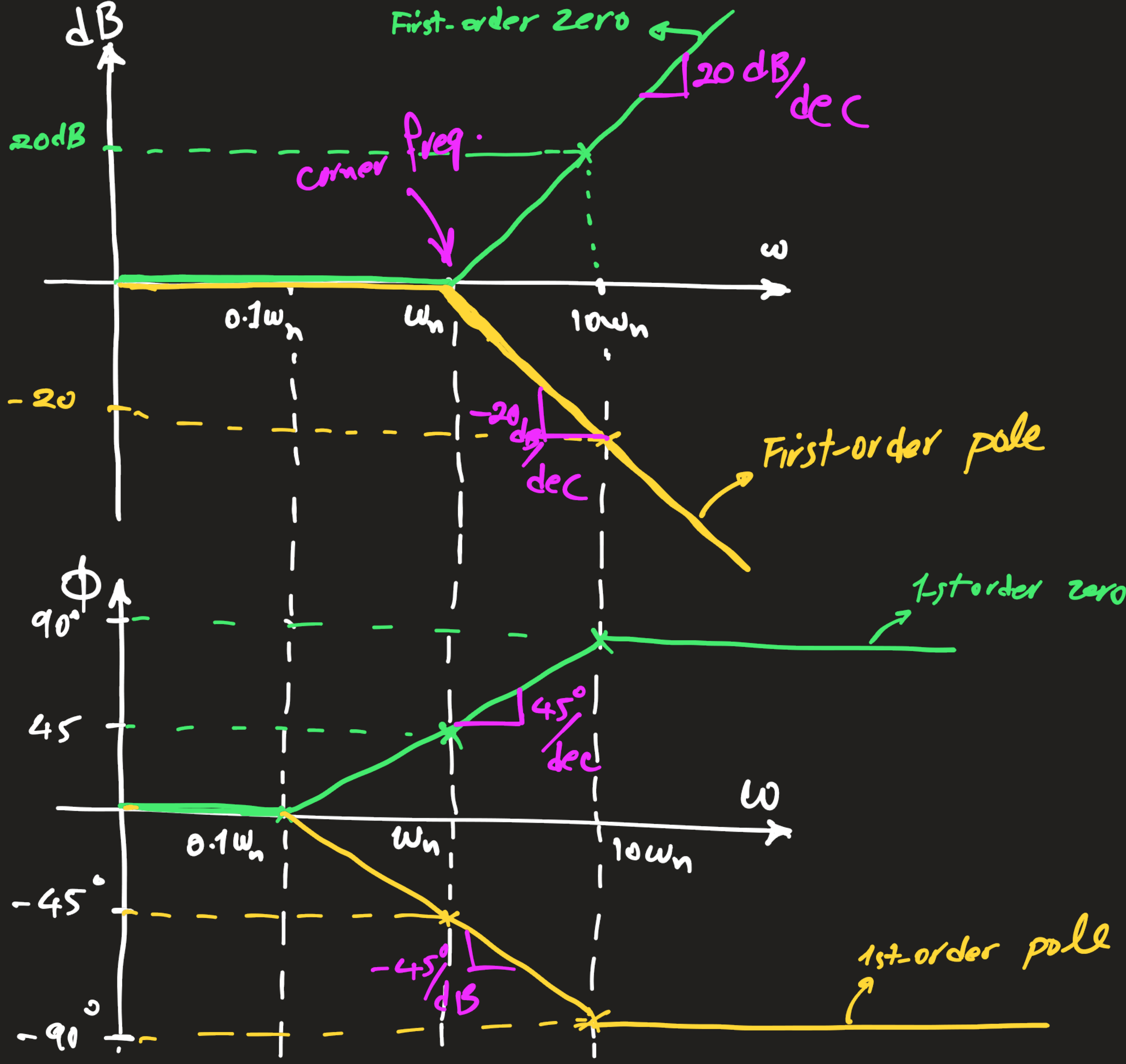
$\phi = \angle\left(1 + j\,\frac{\omega}{\omega_n}\right)$

when $\omega << \omega_n \Rightarrow \phi = \angle(1) = 0$

$\omega = \omega_n \Rightarrow \phi = \tan^{-1} 1 = 45°$

$\omega >> \omega_n \Rightarrow \phi = \tan^{-1}\infty = 90°$

$\boxed{\text{first-order pole}}$: $G(s) = \dfrac{1}{\frac{s}{\omega_n} + 1} = \left(\frac{s}{\omega_n} + 1\right)^{-1}$



dB
First-order zero
20 dB/dec
20dB
corner freq.
ω
0.1ωn
ωn
10ωn
-20
-20 dB/dec
First-order pole
Φ
90°
1st order zero
45
45°/dec
ω
0.1ωn
ωn
10ωn
-45°
-45°/dB
-90°
1st-order pole

D) Second-order:

Second-order zero:

$$s^2+2\zeta\omega_n s+\omega_n^2 = \omega_n^2\left(\frac{s^2}{\omega_n^2}+\frac{2\zeta s}{\omega_n}+1\right)$$

standard

deal separately

unity DC gain

$$\Rightarrow G(s) = \frac{s^2}{\omega_n^2}+\frac{2\zeta s}{\omega_n}+1$$

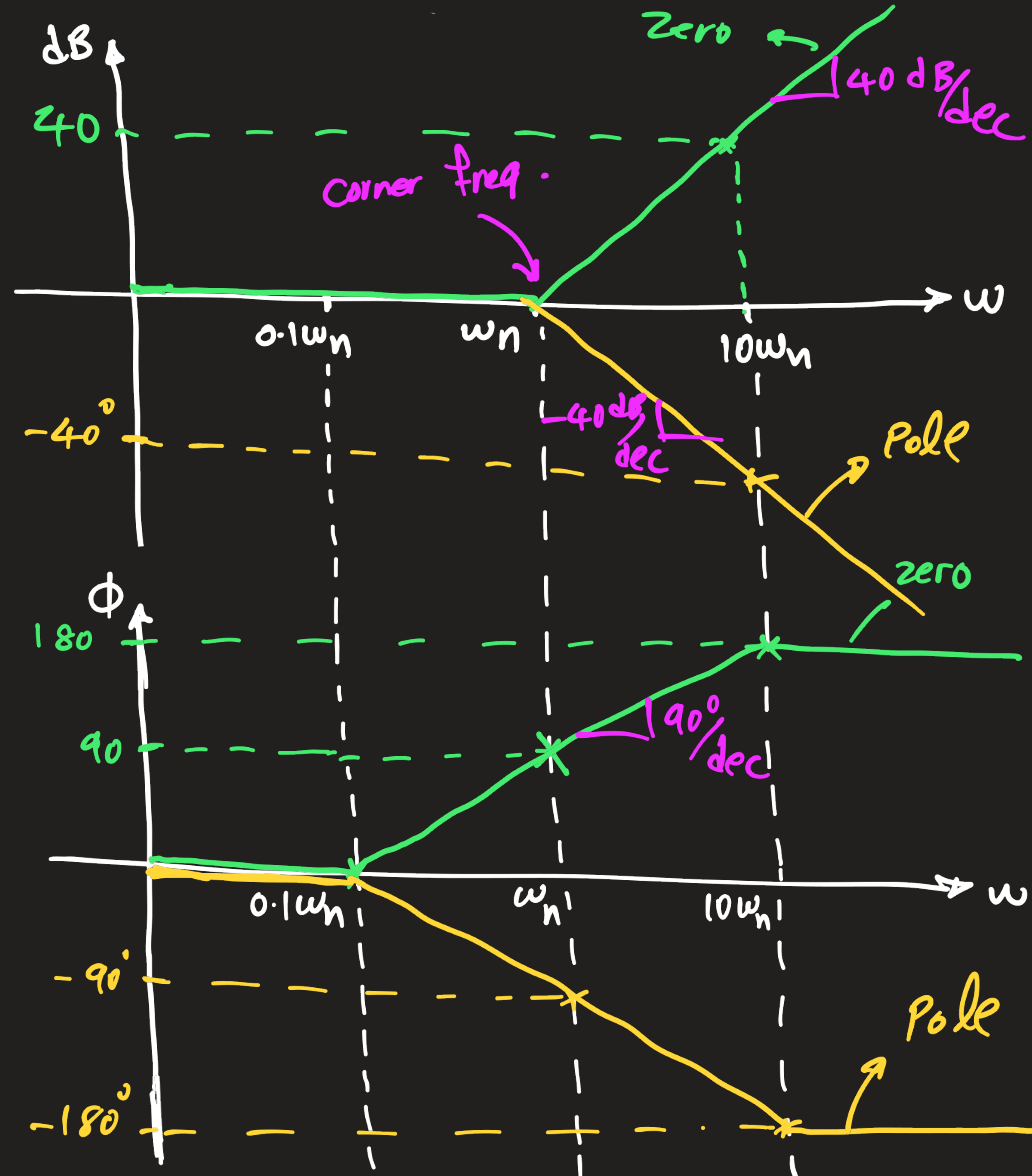
$$G(j\omega) = \frac{\overbrace{(j\omega)^2}^{-\omega^2}}{\omega_n^2}+\frac{2\zeta(j\omega)}{\omega_n}+1 = \left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)+j\left(\frac{2\zeta\omega}{\omega_n}\right)$$

$$dB = 20\log|G(j\omega)| = \begin{cases} 0 & ,\ \omega \ll \omega_n \\ 20\log\left(\frac{\omega}{\omega_n}\right)^2 & ,\ \omega \gg \omega_n \\ = 40\log\left(\frac{\omega}{\omega_n}\right) \rightarrow \text{line w/ slope of } 40\ \text{dB/dec} \end{cases}$$

$$\boxed{\log a^c = c\log a}$$

$$\phi = \angle G(j\omega) = \begin{cases} 0^\circ & ,\ \omega \ll \omega_n \\ 90^\circ & ,\ \omega = \omega_n \\ 180^\circ & ,\ \omega \gg \omega_n \end{cases}$$

second-order pole: $G(s) = \left(s^2/\omega_n^2 + 2\zeta s/\omega_n + 1\right)^{-1}$



dB
Zero
40 dB/dec
240
Corner freq.
ω
0.1ωn
ωn
10ωn
-40°
-40dB/dec
Pole
zero
Φ
180
90
90°/dec
0.1ωn
ωn
10ωn
ω
-90°
Pole
-180°

Drawing Bode's Diagrams:

procedure: Given a TF $G(s) = \frac{B(s)}{A(s)}$

1. Find its factors: Gain, s, first-order, second-order
2. Rearrange to make sure each factor has DC gain of 1.
3. Find Bode information for each factor $(\underset{\text{Gain}}{\underset{\downarrow}{dB}}, \underset{\text{Phase}}{\underset{\downarrow}{\phi}})$ and plot
4. Add them up:
   - → Draw $s=0$ zero/pole factor
   - → Add gain
   - → Add first- and second-order terms from smallest corner frequency (ω_n) to the largest
     - ↳ Add slopes.

Example: $G(s) = \dfrac{1}{(s+2)(s+100)}$

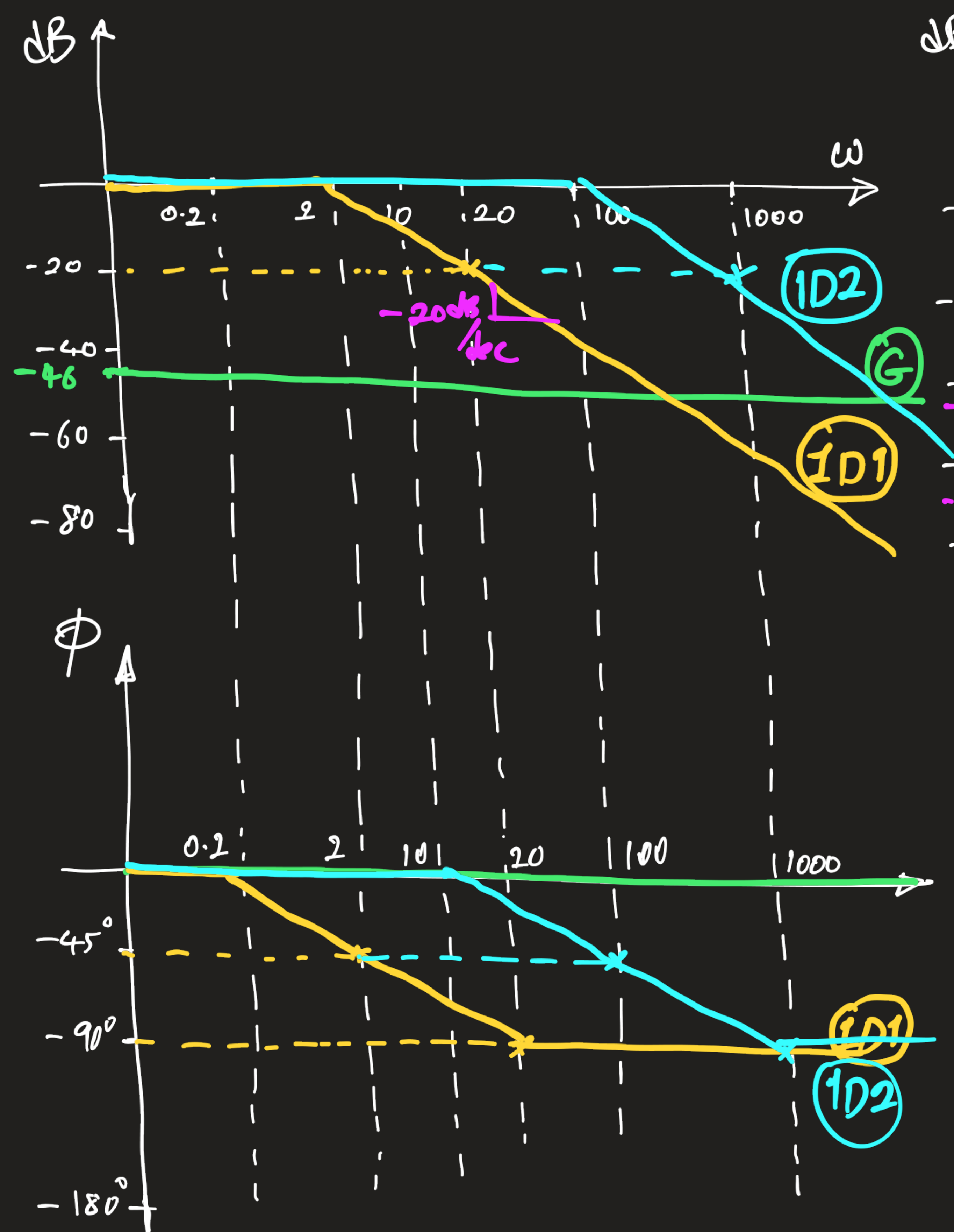
1- Already factored.

2- $G(s) = \dfrac{1}{2\underbrace{(s+2)}_{2}\ \underbrace{(s+100)}_{100}100} = \dfrac{1/200}{(\frac{s}{2}+1)(\frac{s}{100}+1)}$

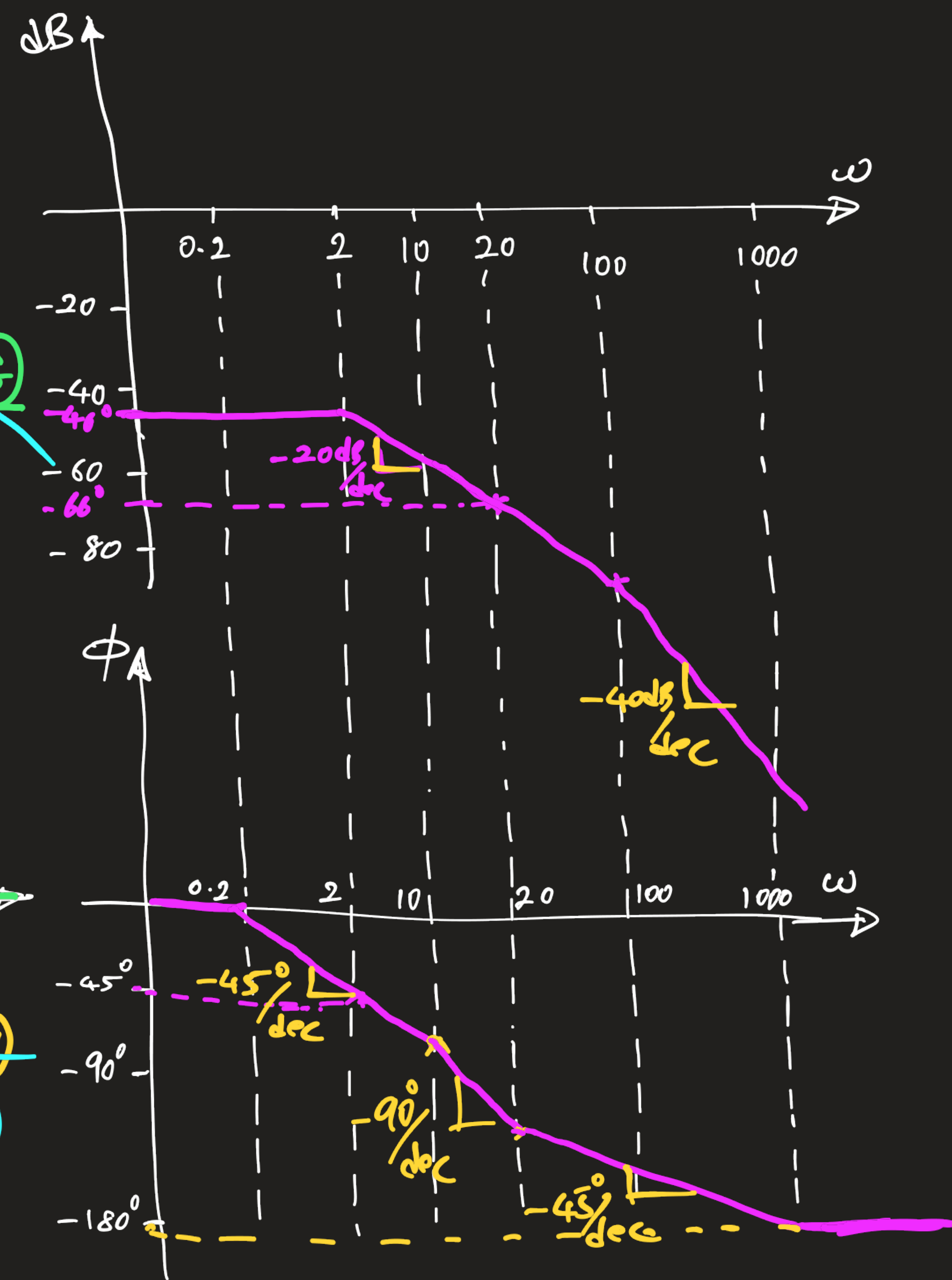
3-

	Factor	Gain	Phase
G	Gain: $1/200$	$\Rightarrow 20\log(\frac{1}{200}) \approx -46\,dB$	0
1D1	First order pole: $\omega_n = 2$	$\Rightarrow$ 0 dB $\omega < 2$ -20 dB/dec $\omega \geq 2$	$0.2 < \omega < 20$ $-45°$/dec
1D2	First-order pole: $\omega_n = 100$	$\Rightarrow$ 0 dB $\omega < 100$ -20 dB/dec $\omega \geq 100$	$10 < \omega < 1000$ $0.1\omega_n$ $-45°$/dec $10\,\omega_n$

3-Cont.) plot each factor separately



4) Add them up



Example: $G(s) = \dfrac{100}{s(s+10)}$

Factor	Gain	Phase