[Controls]

28.5

Homework #1

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1. control input: desired level of brightness
Output: actual level of brightness
disturbance: Light from outside the window

In this control system the objective / goal is to maintain a constant level of light / brightness in the room by adjusting the dimmer.
This is a closed / feedback loop (always measuring light and adjust to disturbance).
~~The dimmer is the plant. It recieves a signal from the contoller and adjusts the brightness of the light~~

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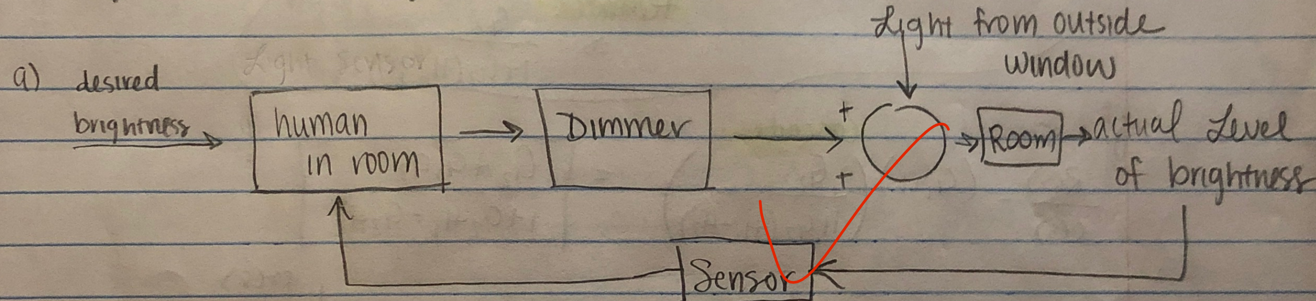
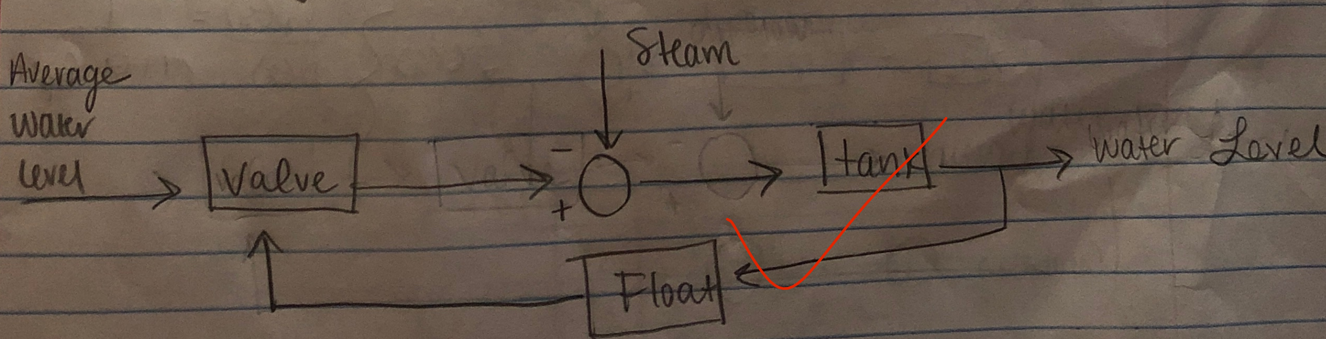
2. a)



b) ~~control input: desired water level~~
~~output: actual water level~~
~~disturbance: outflow~~
~~valve: plant~~



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6. $\dddot{y} + 3\ddot{y} + 2\dot{y} = u$

$x_1 = y \qquad \dot{x}_1 = x_2$

$x_2 = \dot{y} \qquad \dot{x}_2 = x_3$

$x_3 = \ddot{y} \qquad \dot{x}_3 = -3x_3 - 2x_2 + u$

$y = x_1$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \cdot u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$\dot{x} = Ax + Bu$

$y = Cx + Du$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \quad D = 0$$

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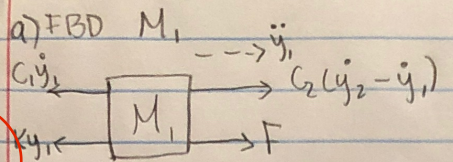
5. a) FBD M_1

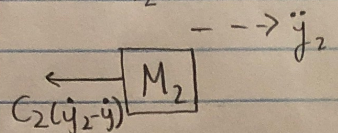


$\sum F = m\ddot{y}$

$$-c_1\dot{y}_1 - ky_1 + c_2(\dot{y}_2-\dot{y}_1) + F = m_1\ddot{y}_1$$

$$\boxed{m_1\ddot{y}_1 + (c_1+c_2)\dot{y}_1 + ky_1 - c_2\dot{y}_2 = F} \quad (1)$$

FBD M_2



$$-c_2(\dot{y}_2-\dot{y}_1) = m_2\ddot{y}_2$$

$$\boxed{m_2\ddot{y}_2 + c_2\dot{y}_2 - c_2\dot{y}_1 = 0} \quad (2)$$

b) Let

$x_1 = y_1 \qquad x_3 = y_2$

$\dot{x}_1 = x_2 = \dot{y}_1 \qquad x_4 = \dot{y}_2 = \dot{x}_3$

$\dot{x}_2 = \ddot{y}_1 \qquad \dot{x}_4 = \ddot{y}_2$

$$m_1\dot{x}_2 + (c_1+c_2)x_2 + kx_1 - c_2x_4 = F$$

$$\dot{x}_2 = -\left[\frac{c_1+c_2}{m_1}\right]x_2 - \frac{k}{m_1}x_1 + \frac{c_2}{m_1}x_4 + \frac{F}{m_1} \quad (3)$$

$$m_2\dot{x}_4 + c_2x_4 - c_1x_2 = 0$$

$$\dot{x}_4 = -\frac{c_2}{m_2}x_4 + \frac{c_1}{m_2}x_2 \quad (4)$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ -\frac{k}{m_1} & -\left[\frac{c_1+c_2}{m_1}\right] & 0 & \frac{c_2}{m_1}\\ 0 & 0 & 0 & 1\\ 0 & \frac{c_1}{m_2} & 0 & -\frac{c_2}{m_2}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \begin{bmatrix}0\\ \frac{1}{m_1}\\ 0\\ 0\end{bmatrix}F(t)$$

$$\boxed{\dot{x} = Ax + BF(t)}$$

output $q = y_2 - y_1$

$$q = \begin{bmatrix}-1 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + DF$$

$$\boxed{q = Cx + DF(t)}$$

3.

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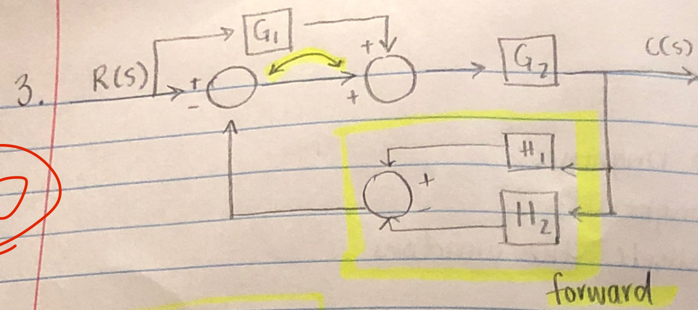
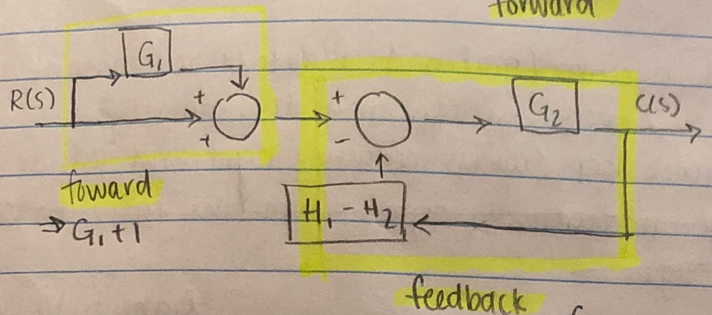
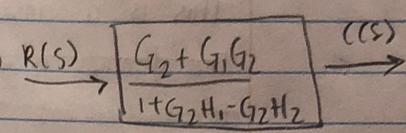




foward $\Rightarrow G_1 + 1$

feedback $\Rightarrow \dfrac{G_2}{1+G_2(H_1-H_2)}$



cascade

$$\frac{R(s)}{C(s)} = (G_1+1)\left(\frac{G_2}{1+G_2(H_1-H_2)}\right) = \boxed{\frac{G_2+G_1G_2}{1+G_2H_1-G_2H_2}}$$

4.

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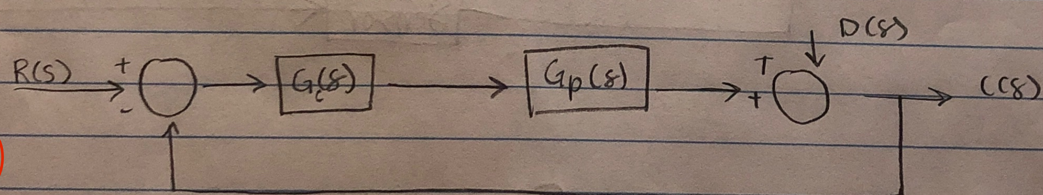
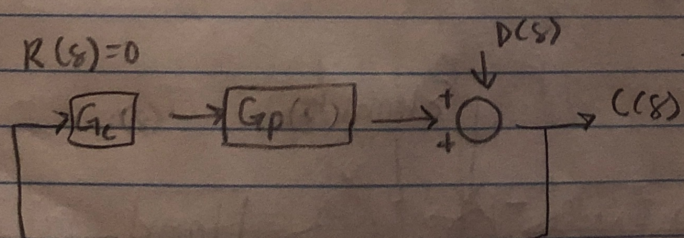


Keeping $D(s)=0$

$$\frac{C_R(s)}{R(s)} = \frac{G_c\,G_p}{1+G_c\,G_p} \quad (1)$$

When $R(s)=0$



$$\frac{C_D(s)}{D(s)} = \frac{1}{1+G_c\,G_p} \quad (2)$$

Combining (1) and (2):

$$C(s) = C_R(s) + C_D(s) = \frac{G_c\,G_p}{1+G_c\,G_p}R(s) + \frac{1}{1+G_cG_p}D(s) = \boxed{\frac{G_cG_pR(s)+D(s)}{1+G_c\,G_p}}$$