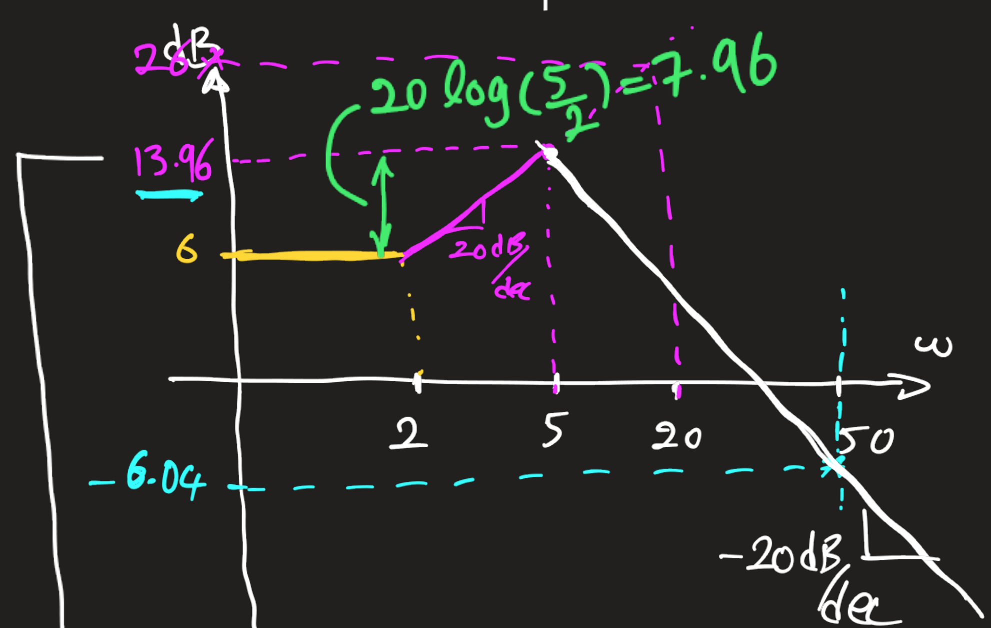
Name: ____________________________________ ID: ____________________________

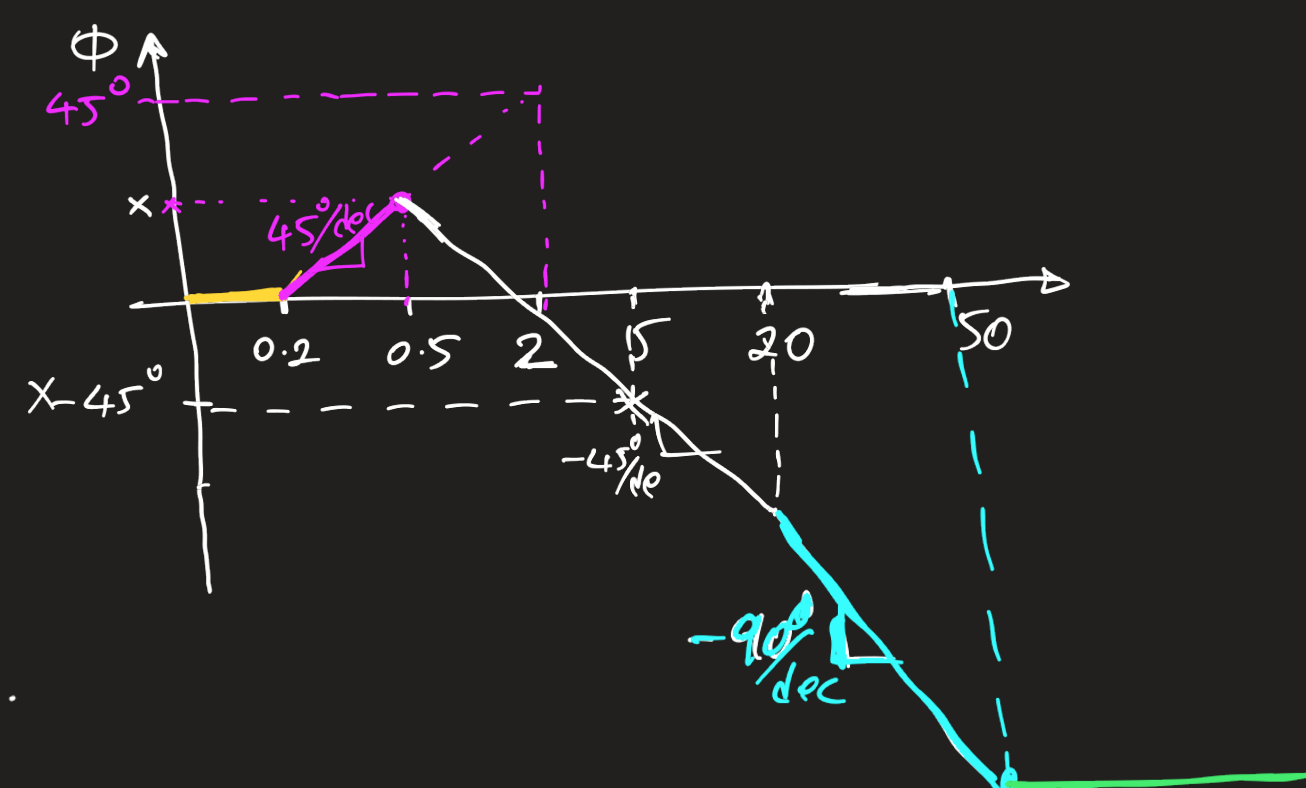
1.) Please draw the Bode diagram for the system described by the open-loop transfer function:

$$G(s) = \frac{100(s+2)}{4s^2 + 10s + 100}$$

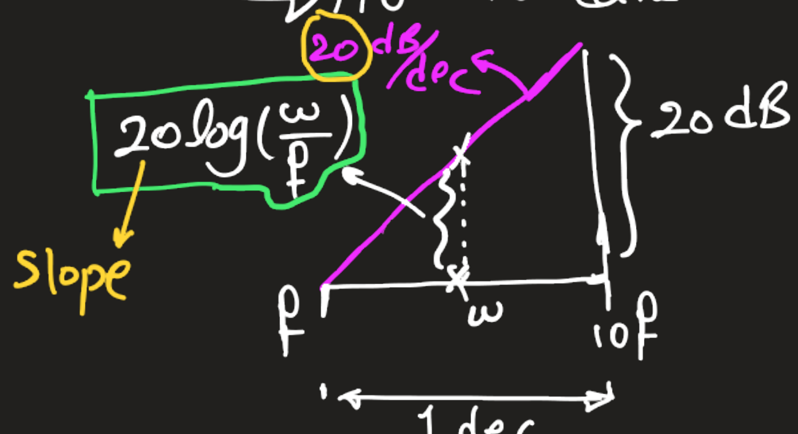
1) $G(s) = \dfrac{100(s+2)}{4s^2+10s+100} = \dfrac{(\cancel{100})(2)\,\underline{\frac{(s+2)}{2}}}{(\cancel{100})\,\underline{\frac{(4s^2+10s+100)}{100}}} = \dfrac{\overset{G}{2}\,\overset{1N\ (\omega_n=2)}{\left(\frac{s}{2}+1\right)}}{\underset{2D\ (\omega_n=5)}{\left(\frac{s^2}{25}+\frac{s}{10}+1\right)}}$

Factor	Gain	Phase
G: Gain $\underline{\underline{2}}$	$20\log 2 \approx 6$ dB	0°
1N: $\omega_n = 2$	$2 < \omega < \infty \Rightarrow 20$ dB/dec	$0.2 < \omega < 20 \Rightarrow 45^\circ/\text{dec}$
2D: $\omega_n = 5$	$5 < \omega < \infty \Rightarrow -40$ dB/dec	$0.5 < \omega < 50 \Rightarrow -90^\circ/\text{dec}$





How to calculate?



$\Rightarrow$ in our example: $f = 2, \omega = 5$

$\Rightarrow 20\log\left(\frac{5}{2}\right) = 7.96$

$\Rightarrow 6 + 7.96 = 13.96$

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2.) **a)** If the phase margin of a system is found to be 45.6 degrees at a gain crossover frequency of $\omega_G = 10$ rad/s, please estimate the maximum overshoot it will exhibit under unity feedback.

b) Please design a lead compensator for this system for a desired phase margin PM = 60^o. Hint: Please remember that, by definition, the system will have a gain of 1 (or 0 dB) at the gain crossover frequency.

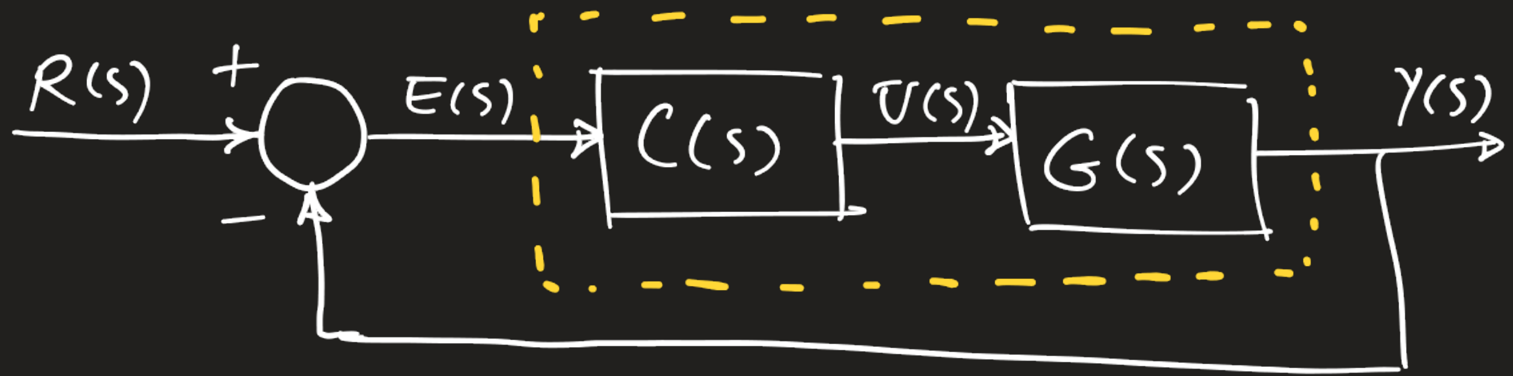
2 a) $PM = 45.6^\circ$ $\boxed{\omega_G = 10}$

$$\xi \simeq \frac{PM}{100} = 0.456 \Rightarrow M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \approx 0.2 = 20\%$$

2 b) desired $PM = 60^\circ$

Controller Analysis: (PD and Lead compensator)

* PD controller: $C(s) = k_p + k_D \cdot s$ $\Rightarrow C(s) = K_p\left(1 + \frac{K_D}{K_p} \cdot s\right)$



$T_D = \frac{1}{\omega_D}$

$\Rightarrow \boxed{C(s) = K_p\left(\frac{s}{\omega_D}+1\right)}$

OLTF: $C(s)G(s) = K_p\left(\frac{s}{\omega_D}+1\right)G(s)$

$\Rightarrow$ PD controller adds a gain & 1-st order term to the numerator of OLTF

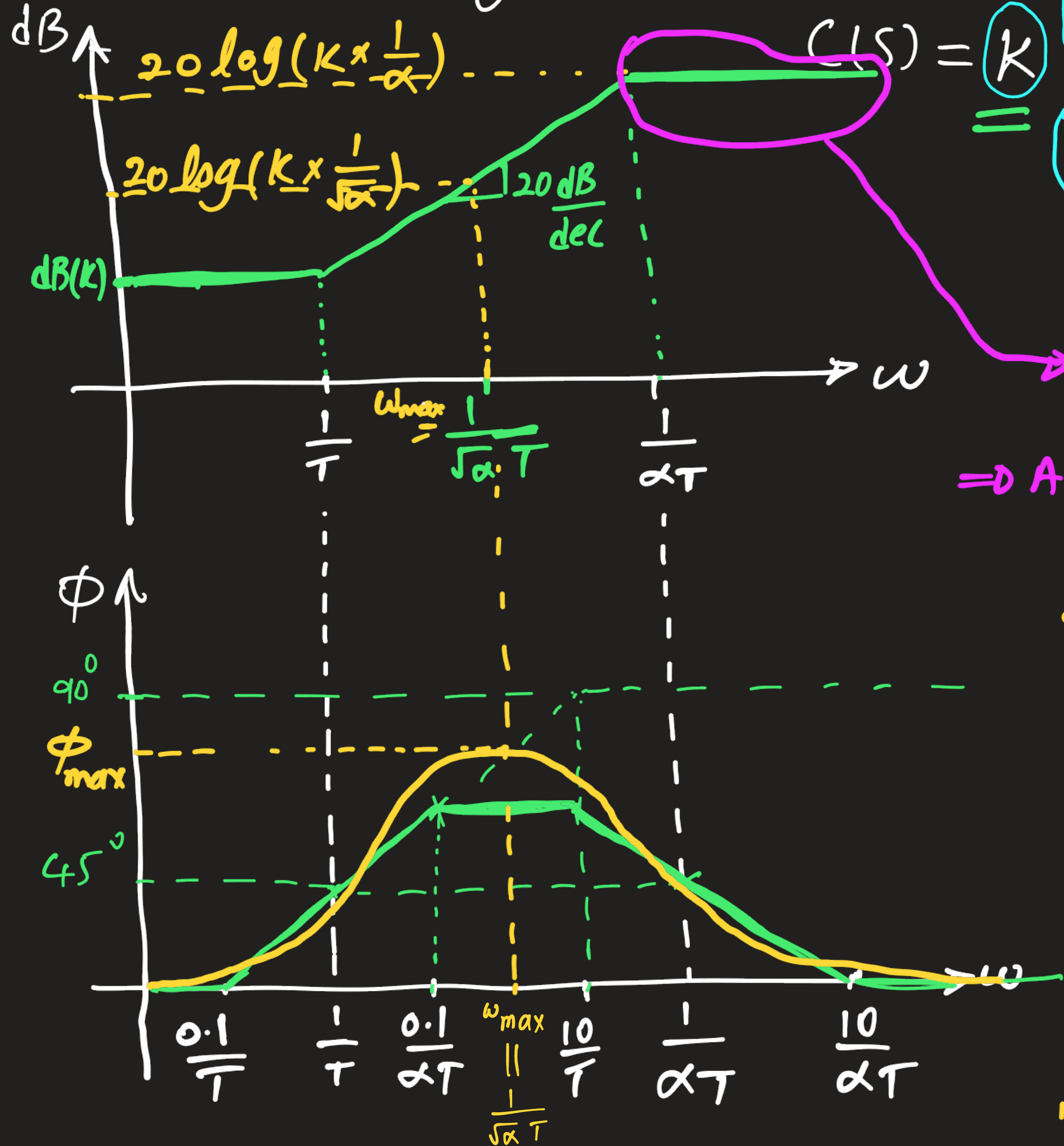
$$\underbrace{a}_{C(s)} + \underbrace{b}_{G(s)} = D$$

✓

desired

* LEAD compensator : $C(s) = K\left[\frac{Ts+1}{\alpha Ts+1}\right], \quad 0<\alpha<1$

A controller that maintains the good parts of PD control, without amplifying noise.



$$C(s) = \underset{G}{K}\,\frac{\overset{1N}{\left(\frac{s}{1/T}+1\right)}}{\underset{1D}{\left(\frac{s}{1/\alpha T}+1\right)}}$$

$\omega_n = \frac{1}{T}$

$\omega_n = \frac{1}{\alpha T}$

Better noise response

$\Rightarrow$ Approximates PD control until $\omega = \frac{1}{\alpha T}$

ϕ_{max} : maximum phase shift occurs at $\boxed{\omega_{max} = \frac{1}{\sqrt{\alpha}\,T}}$

Geometric mean of $\frac{1}{T}$ & $\frac{1}{\alpha T}$

$$\phi_{max} = \angle C(j\omega_{max}) = \sin^{-1}\left(\frac{1-\alpha}{1+\alpha}\right)$$

invert $\Rightarrow \alpha = \dfrac{1-\sin(\phi_{max})}{1+\sin(\phi_{max})}$

How to design a Lead Compensator:

$C(s) = K\dfrac{Ts+1}{\alpha Ts+1}$ $\Rightarrow$ Need to determine 3 parameters (K, T, α)

Design Specs:
→ Desired ω_{G_d} (usually selected at system's ω_G)

→ Desired PM_d (found from ζ or M_p)

Calculate the gain and phase of plant $G(s)$ at ω_{G_d}.

$\xrightarrow{G(s)} K_G = |G(j\omega_{G_d})|$ → this is not the dB value $dB = 20\log K_G$

we need this value

$\xrightarrow{G(s)} \phi_G = \angle G(j\omega_{G_d})$

Phase lead required: $\left(PM_d = \underbrace{\phi_G + 180}_{PM_s} + \phi_{max}\right) \Rightarrow$ ① $\boxed{\phi_{max} = PM_d - 180^\circ - \phi_G}$ $\Rightarrow \phi_{max} = PM_d - PM_s$

$\Rightarrow$ ② $\boxed{\alpha = \dfrac{1-\sin(\phi_{max})}{1+\sin(\phi_{max})}}$

$\omega_{max} = \dfrac{1}{\sqrt{\alpha}\,T} = \omega_{G_d} \Rightarrow$ ③ $\boxed{T = \dfrac{1}{\sqrt{\alpha}\;\omega_{G_d}}}$ $\omega_{max} = \omega_{G_d}$

Gain at ω_{max} should cancel out system gain $K_G \Rightarrow$ because dB @ $\omega_{G_d} = 0$

$\Rightarrow 20\log\left(\dfrac{K}{\sqrt{\alpha}}\right) + 20\log(K_G) = 0 \Rightarrow 20\log\left(\dfrac{K}{\sqrt{\alpha}}\right) = -20\log K_G^{-1}$

$\Rightarrow 20\log\left(\dfrac{K}{\sqrt{\alpha}}\right) = 20\log\left(\dfrac{1}{K_G}\right) \Rightarrow$ ④ $\boxed{K = \dfrac{\sqrt{\alpha}}{K_G}}$

2b) Approach $\quad C(s) = K \dfrac{Ts+1}{\alpha Ts+1} \quad 0<\alpha<1$

$\omega_G = 10$

$K_G = 1 \rightarrow 0\,dB \quad K(\omega_G)$

$PM_s = 45.6°$

$PM_d = 60°$

1) calculate additional phase:

$$\phi_{max} = PM_d - PM_s = 60 - 45.6 = \boxed{14.4°}$$

2) Calculate α:

$$\alpha = \frac{1-\sin\phi_{max}}{1+\sin\phi_{max}} \boxed{= 0.6017}$$

3) $T = \dfrac{1}{\sqrt{\alpha}\,\omega_G} \Rightarrow \boxed{T = 0.129}$ $\quad$ want $\omega_{max} = \omega_G$

4) $K = \dfrac{\sqrt{\alpha}}{K_G} = \sqrt{\alpha} = \sqrt{0.6017} \boxed{= 0.776}$