What you need to know

What are we doing in Module #3 (Analysis and Design)?
We are **analyzing** the response of 1st and 2nd **order systems** in terms of **transient response** ($y(t)$) and **steady-state response** (y_{ss}) to different inputs where $y_{ss} = \lim_{t\to\infty} y(t) = \lim_{s\to 0} s.Y(s)$.
Note: Deriving $y(t)$ itself is not our focus in this module because we already learned how to do it from previous modules ([CLTF=Y(s)/U(s)] → Y(s) → PFE + Inv. L → y(t))

Why are we doing transient analysis?
Transient Response gives us the **performance** of the system i.e. how it reacts to different inputs over time. For example, how fast it reaches and settles at the steady-state response.

Why do we care about Closed-Loop Poles' Location?
There are multiple reasons (Poles are shown on the s-plane). Having Poles' location (ζ, ω_n), we can do the following without solving for y(t).

1) conclude about **stability**/convergence without solving for y(t)
    - Any Pole on the RHP means that the system is unstable
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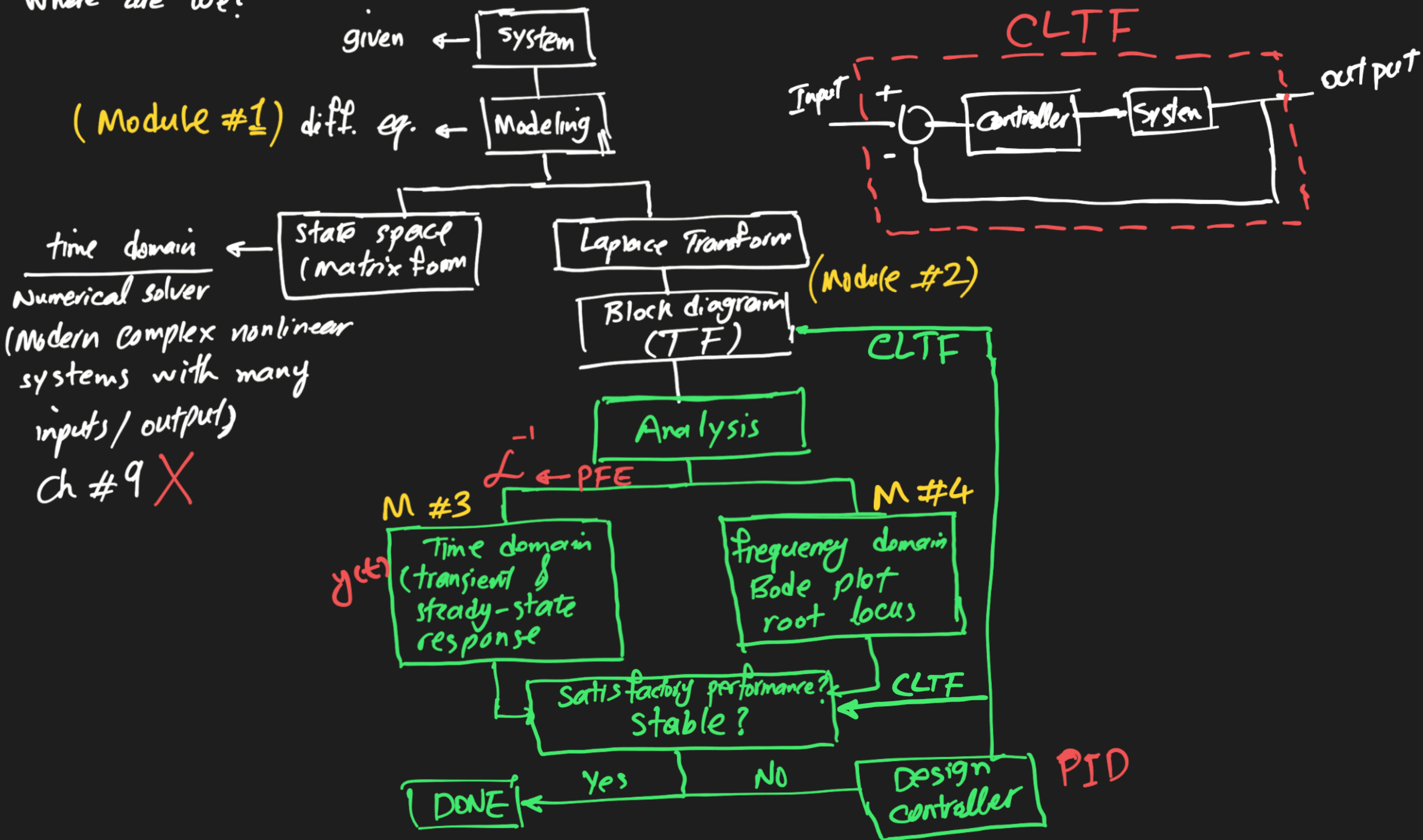
Why do we add a Controller to a system? (<u>to change the Poles' Location!</u>)

Where are we?


given
System
(Module #1) diff. eq.
Modeling
time domain
Numerical solver
(Modern complex nonlinear systems with many inputs / outputs)
Ch #9
State space (matrix form)
Laplace Transform
(Module #2)
Block diagram (TF)
CLTF
Analysis
$\mathcal{L}^{-1}$
PFE
M #3
M #4
y(t)
Time domain (transient & steady-state response
frequency domain
Bode plot
root locus
Satisfactory performance?
stable?
CLTF
Yes
NO
DONE
Design controller
PID
CLTF
Input
+
-
controller
System
output

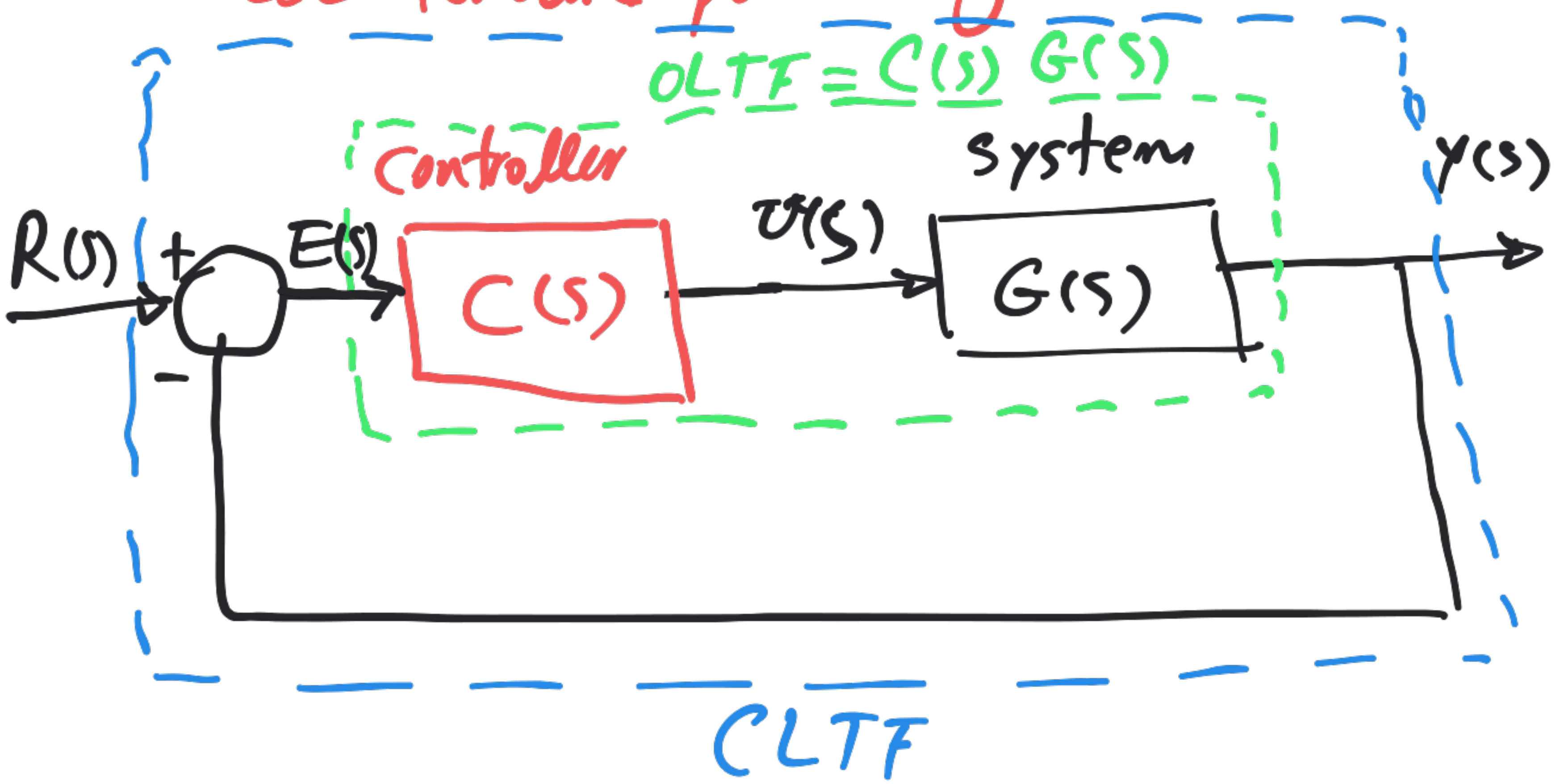
Controller Design

system is unstable?
TR not satisfactory?
SS not satisfactory?

$\Longrightarrow$ Add a controller to your system

How?

We add a block with TF of C(s) in the feed forward path right befor the system's block:



$$C(s) = \frac{U(s)}{E(s)}$$

$$OLTF = C(s)\,G(s)$$

OLTF: $C(s)\,G(s)$

$$CLTF = \frac{Y(s)}{R(s)} = \frac{C(s)G(s)}{1 + C(s)G(s)}$$

$\leftarrow$ numerator

$\leftarrow$ denominator (char. polyn.)

diff $C(s)$ $\rightarrow$ diff char. poly $\rightarrow$ diff. pole's location

stability ↙ ↓ TR ↘ SS

In control systems, we are usually interested in

Steady-state Error (e_{ss}) instead of S-S response (y_{ss})

$$E = R - Y \Rightarrow \frac{E}{R} = 1 - \underbrace{\boxed{\frac{Y}{R}}}_{CLTF} \Rightarrow \frac{E}{R} = 1 - \frac{CG}{1+CG} = \frac{1}{1+CG}$$

$$\Rightarrow \frac{E(s)}{R(s)} = \frac{1}{1 + C(s)G(s)} \Rightarrow \boxed{E(s) = \frac{R(s)}{1 + C(s)G(s)}}$$

$e_{ss} = ?$

$$E(s) = \frac{R(s)}{1+C(s)G(s)} \Rightarrow e_{ss} = \lim_{s\to 0} s\cdot E(s) = \lim_{t\to\infty} e(t)$$

$$\Rightarrow \boxed{e_{ss} = \lim_{s\to 0} \frac{sR(s)}{1+C(s)G(s)}}$$

steady-state error

$$E(s) = \frac{R(s)}{1+C(s)G(s)}$$

- No poles at $s=0 \rightarrow e_{ss} = 0$
- $\underline{1}$ pole at $s=0 \rightarrow e_{ss} = cte$
- $\underline{2}+$ poles at $s=0 \rightarrow \underbrace{e_{ss} = \infty}_{\text{unstable}}$

PID :

P_controller: $u(t) = K_p e(t) \xrightarrow{\mathcal{L}} U(s) = K_p E(s) \Rightarrow \frac{U(s)}{E(s)} = \boxed{K_p}$ C(s)

I- controller: $u(t) = K_i \int e(t)\,dt \xrightarrow{\mathcal{L}} U(s) = K_i \cdot \frac{1}{s} E(s) \Rightarrow \frac{U(s)}{E(s)} = \boxed{\frac{K_i}{s}}$

D-controller: $u(t) = K_d \frac{de(t)}{dt} \xrightarrow{\mathcal{L}} U(s) = K_d s E(s) \Rightarrow \frac{U(s)}{E(s)} = \boxed{K_d \cdot s}$

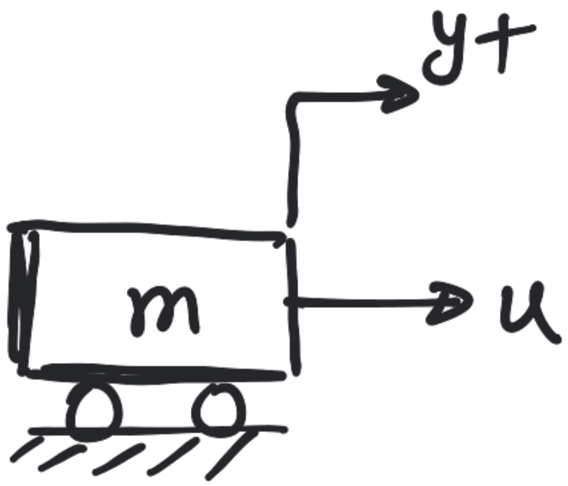
PID : $P + I + D \Rightarrow \boxed{C(s) = K_p + \frac{K_i}{s} + K_d \cdot s}$

$K_p \rightarrow$ proportional gain

$K_d \rightarrow$ derivative gain

$K_i \rightarrow$ integral gains

Case study (second-order system)



Control the following system!

$$m\ddot{y} = u \xrightarrow{\mathcal{L}} ms^2 Y(s) = U(s)$$

$$\Rightarrow \frac{Y(s)}{U(s)} = \frac{1}{ms^2} = G(s)$$

$$CLTF = \frac{Y(s)}{R(s)} = \frac{C(s)G(s)}{1 + C(s)G(s)} = \boxed{\frac{C(s)}{ms^2 + C(s)}}$$

P-Control $(C(s) = K_p)$

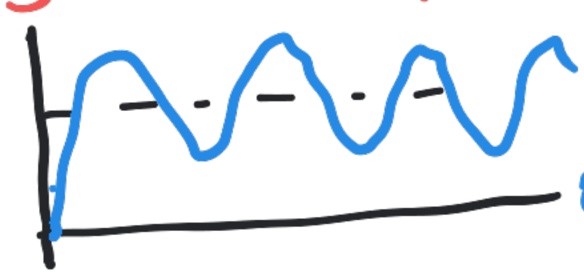
$$\frac{Y(s)}{R(s)} = \frac{K_p}{ms^2 + K_p} = \frac{K_p/m}{s^2 + 0.s + K_p/m} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n + \omega_n^2}$$

oscillatory $\leftarrow \xi = 0$ $\qquad \omega_n^2$

$P_{1,2} = \pm j\omega_n$

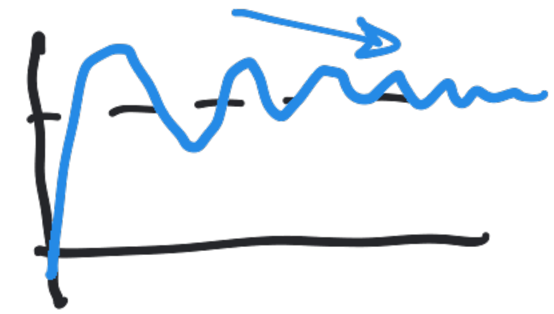
$\boxed{K_p \rightarrow \omega_n \rightarrow TR}$

side note



marginally stable

P-D- Controller: $(C(s) = K_p + K_d \cdot s)$



$$\frac{Y(s)}{R(s)} = \frac{C(s)G(s)}{1+C(s)G(s)} = \frac{K_p + K_d \cdot s}{ms^2 + K_d s + K_p}$$

mass damping stiffness

for positive K_p & K_d $\Longrightarrow$ system is stable!

$\boxed{K_d \to \xi \to TR}$
side note

$$E(s) = \frac{R(s)}{1+C(s)G(s)} \qquad R(s) = \frac{1}{s} : \text{unit-step}$$

$$E(s) = \frac{1}{s} \cdot \frac{1}{ms^2 + K_d s + K_p} \Longrightarrow 1 \text{ pole at } s=0 \Longrightarrow e_{ss} = cte$$

$$e_{ss} = \lim_{s\to 0} s \cdot E(s) = \lim_{s \to 0} \cancel{s} \cdot \frac{1}{\cancel{s}} \cdot \frac{1}{m\cancel{s^2} + K_d \cancel{s} + K_p} = \boxed{\frac{1}{K_p} = e_{ss}}$$

P-I- Controller: $C(S) = K_p + \frac{K_i}{S}$

$$E(S) = \frac{R(s)}{1 + C(S)G(S)} = \frac{1/s}{1 + \frac{K_p + K_i/s}{ms^2}} = \frac{m\,s}{ms^2 + K_p + K_i/s}$$

$$\Rightarrow \underline{\underline{E(S)}} = \frac{ms^2}{ms^3 + K_p \cdot S + K_i}$$ $\Rightarrow \underbrace{\text{No poles at } s=0}_{e_{ss}=0}$

$$e_{ss} = \lim_{S \to 0} S \cdot E(S) = \lim_{S \to 0} S \cdot \frac{ms^2}{ms^3 + K_p S + K_i} = 0 \quad \Big] \; e_{ss}$$

Outcomes:

proportional controller

↳ may result in oscillation

↳ may allow non-zero e_{ss}

↳ acts like a spring w/ coef. K_p

↳ correlates w/ ω_n

Derivative controller

↳ acts like a damper w/ coef. K_d

↳ correlates with ζ

Integral controller

↳ Removes the S-S error

To benefit from all, you may use PFD and tune your K_p, K_d, K_i gains for a desired output!

The effects of K_p, K_i and K_d on system performance is summarized in the table below

TR

Parameter	Rise time	Overshoot	Settling time	Steady-state error	Stability
K_p	Decrease	Increase	Small change	Decrease	Degrade
K_i	Decrease	Increase	Increase	Eliminate	Degrade
K_d	Minor change	Decrease	Decrease	No effect in theory	Improve if K_d small

https://sites.google.com/site/fpgaandco/pid