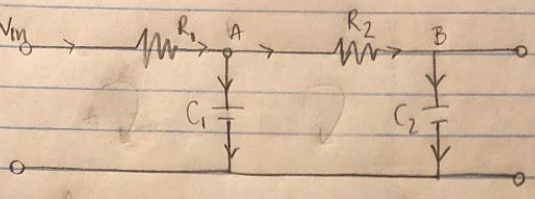
Homework 2

1. 

KCL @ node A

$$\frac{V_{in}-V_A}{R_1} = C_1\frac{dV_A}{dt} + \frac{V_A-V_{out}}{R_2}$$

KCL @ node B

$$\left(\frac{V_A-V_{out}}{R_2} = C_2\frac{d(V_{out})}{dt}\right)R_2 \Rightarrow V_A - V_{out} = C_2(\dot{V_{out}})R_2$$

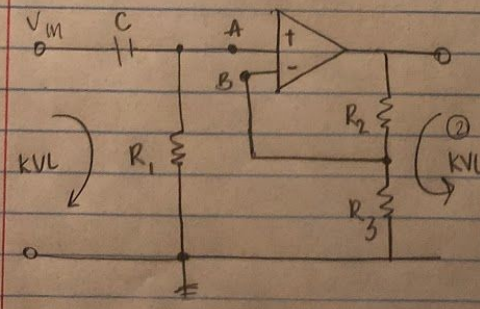
$$V_A = V_{out} + C_2R_2\dot{V_{out}}$$

$u = V_{in} \quad y = V_{out}$

$$\frac{V_{in}-V_{out}+C_2R_2\dot{V_{out}}}{R_1} = C_1\frac{d(V_{out}+C_2R_2\dot{V_{out}})}{dt} + \frac{V_{out}+C_2R_2\dot{V_{out}}-V_{out}}{R_2}$$

$$u - y - C_2R_2\dot{y} = \left(C_1(R_2C_2\ddot{y}+\dot{y}) + \frac{(R_2C_2\dot{y}+\dot{y})-\dot{y}}{R_2}\right)R_1$$

$$\boxed{u = C_1R_2C_2\ddot{y}R_1 + R_1\dot{y} + R_1C_2\dot{y} + C_2R_2\dot{y} + y}$$

2. 

$i = V_A/R_1$

$$V_A = e_i - \frac{1}{C}\int i_1\,dt$$

$$V_B = e_o\cdot\frac{R_3}{R_2+R_3}$$

$$e_i = V_C + V_{R_1}$$

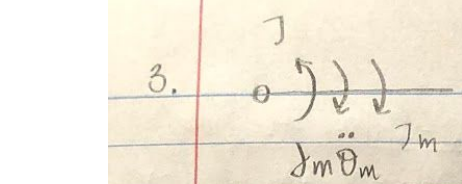
$$e_i = \frac{1}{C}\int i_1\,dt + R_1i_1$$

$$e_i = \frac{1}{CR_1}\int V_A\,dt + V_A$$

$$e_i = \frac{1}{CR_1}\cdot\frac{R_3}{R_2+R_3}\int e_o\,dt + e_o\frac{R_3}{R_2+R_3}$$

$$\boxed{\dot{e_i} = \frac{1}{CR_1}\cdot\frac{R_3}{R_2+R_3}e_o + \dot{e_o}\cdot\frac{R_3}{R_2+R_3}}$$

3.



$J\theta \Rightarrow$ moment of inertia

$$J_m\ddot{\theta}_m = T_m - T$$
$$T_L + J_L\ddot{\theta} = 0$$

gear ratio $= n = \frac{T}{T_L} = \frac{\theta}{\theta_m} \Rightarrow \frac{T}{T_L} = \frac{\theta}{\theta_m} \Rightarrow T = \frac{\theta}{\theta_m} T_L$

$$T_m = J\ddot{\theta}_m + T$$
$$T_m = k_T i$$
$$T = J\ddot{\theta}_m + k_T i$$
$$J_m\ddot{\theta}_m = T_m - k_T i$$
$$k_T i = J_m\ddot{\theta} + \frac{\theta}{\theta_m} T_L$$

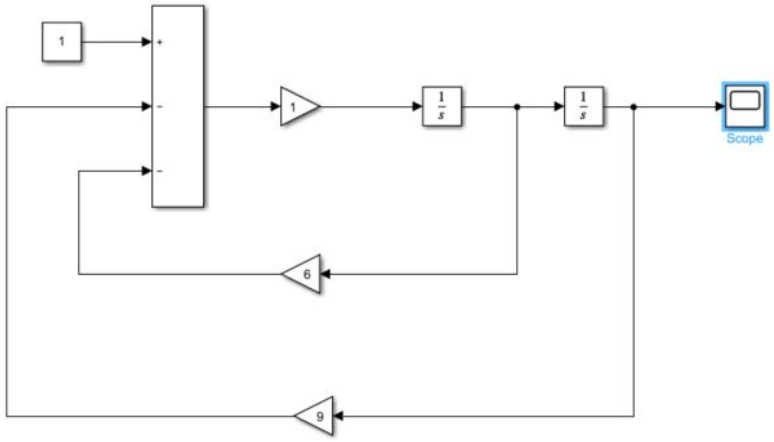
$$\frac{T}{T_L} = \frac{\theta}{\theta_m} \Rightarrow T = \frac{\theta}{\theta_m}(T_L)$$
$$\theta_m = \frac{n}{\theta} \Rightarrow \frac{\theta}{\theta_m} = n$$

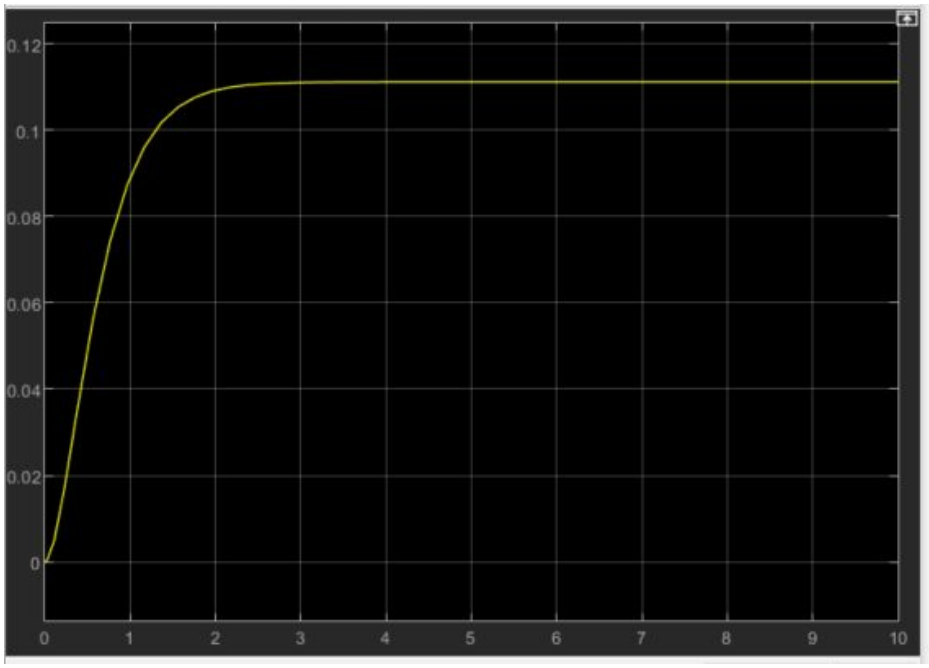
$$T_m = T + J_n\ddot{\theta}$$
$$k_T i = T + J_n\ddot{\theta}$$
$$k_T i = \frac{\theta}{\theta_m} T_L + J_m\ddot{\theta}$$
$$k_T i = \frac{\theta}{\frac{\theta}{n}} J_c\ddot{\theta} + J_m\frac{\theta}{n}$$
$$k_T i = n \cdot J_c\ddot{\theta} + J_m\frac{\dot{\theta}}{n}$$

$$\frac{\theta}{\theta_m} = n \Rightarrow \theta_m = \frac{\theta}{n}$$
$$T_L = J_L\ddot{\theta}$$

4.

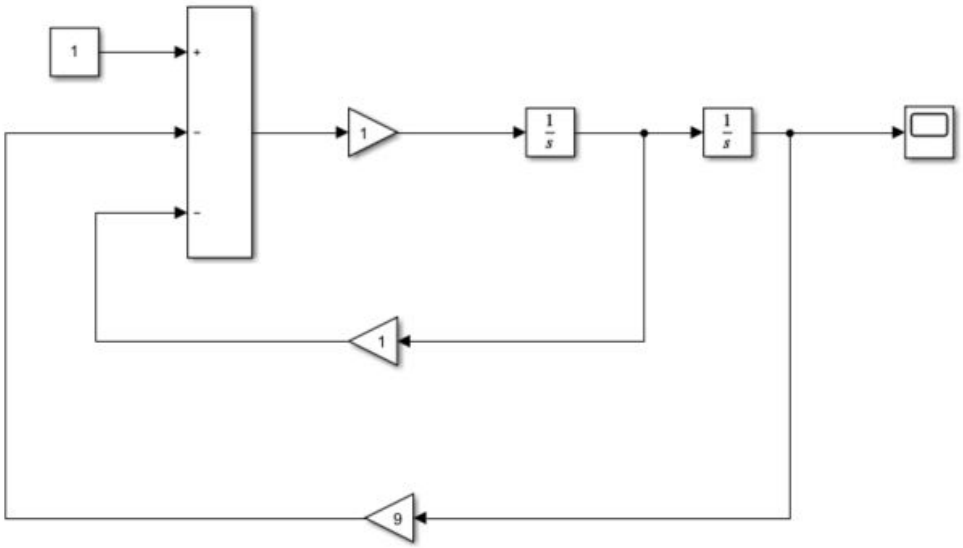
a)

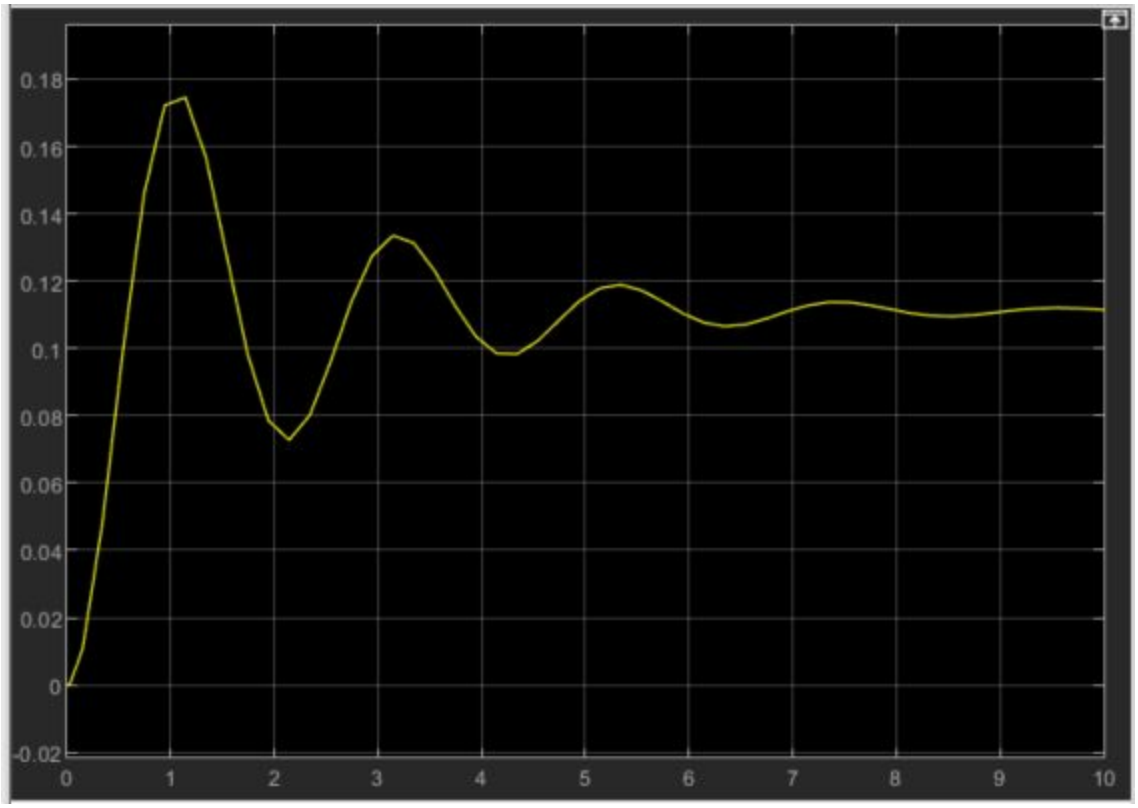




4.

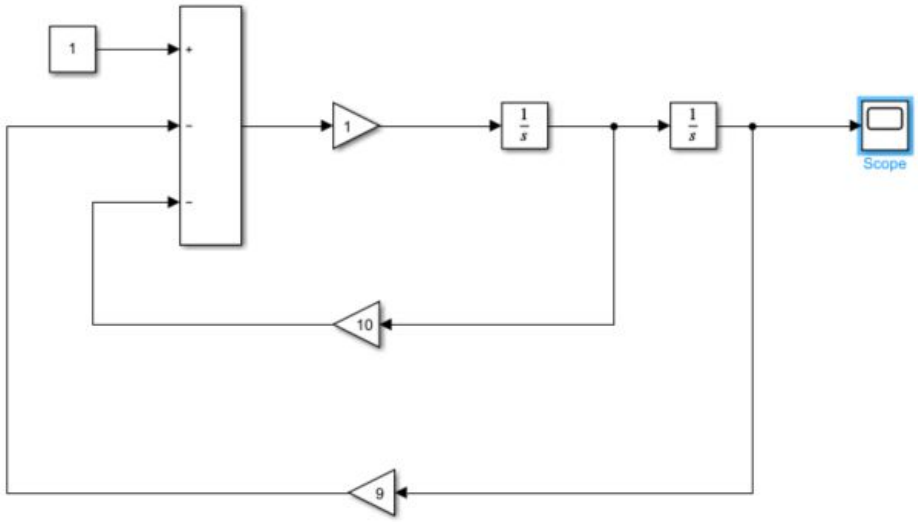
b) (i)

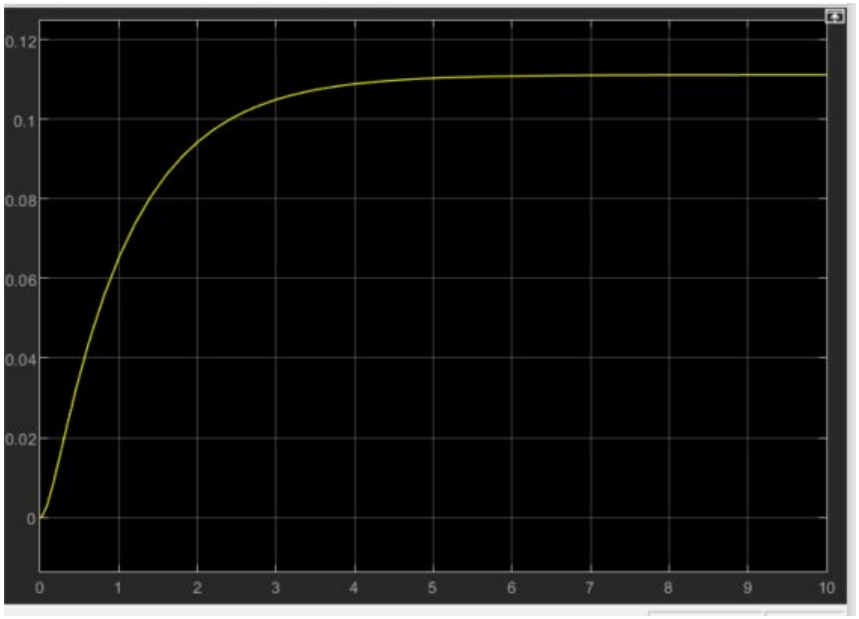

1
+
-
-
1
1/s
1/s
1
9


0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
-0.02
0
1
2
3
4
5
6
7
8
9
10

4.

b) (ii)





As the c-value increases, the graph steadies/levels out. The slope increases exponentially and then steadies out. There is less deviation and the y-values stay about the same as the time increases.