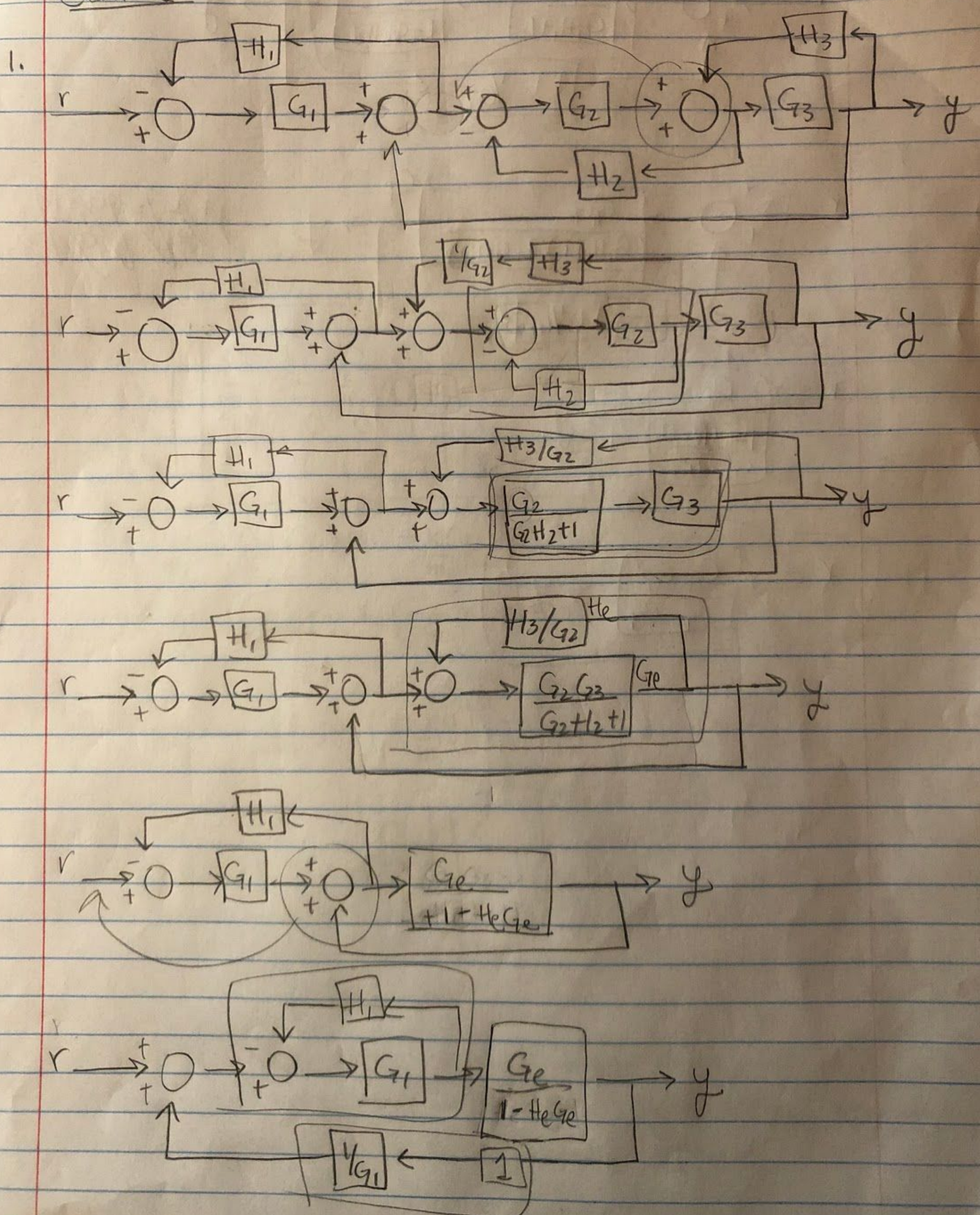
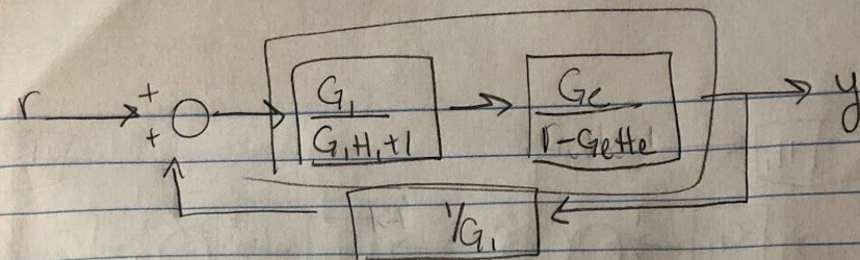
Exam 1

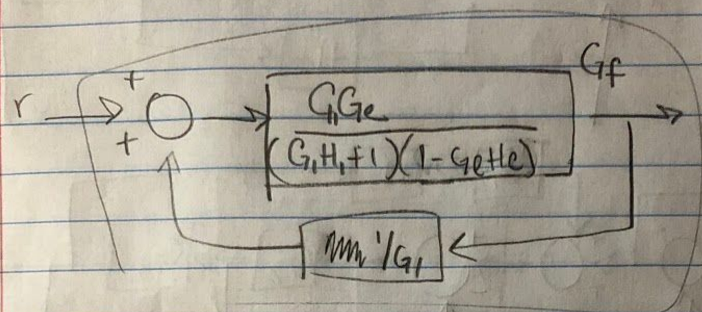
1.

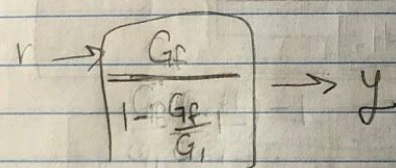


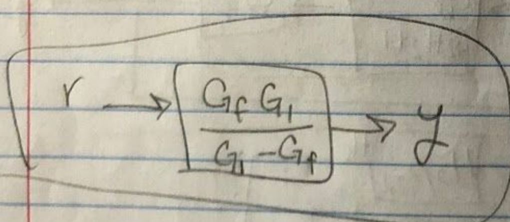
r
y
H1
G1
G2
G3
H2
H3
1/G2
G2 / (G2H2+1)
H3/G2
He
Ge
G2G3 / (G2H2+1)
Ge / (1+HeGe)
Ge / (1-HeGe)
1/G1
1

1 Cont.


r
+
+
$\frac{G_1}{G_1H_1+1}$
$\frac{G_e}{1-G_eH_e}$
y
$1/G_1$


r
+
+
$\frac{G_1G_e}{(G_1H_1+1)(1-G_eH_e)}$
G_f
y
$1/G_1$


r
$\frac{G_f}{1-\frac{G_f}{G_1}}$
y


r
$\frac{G_f G_1}{G_1-G_f}$
y

Exam 1 Cont.

2. a) $M_1\ddot{y}_1 = u_1 - C_1(\dot{y}_1 - 0) - k(y_1 - y_2)$

$M_2\ddot{y}_2 = -u_2 - C_2(\dot{y}_2 - 0) + k(y_1 - y_2)$

b) ★ $x_1 = y_1 \Rightarrow \dot{x}_1 = \dot{y}_1 = x_2$ ★ row 1

$x_2 = \dot{y}_1 \Rightarrow \dot{x}_2 = \ddot{y}_1$

★ $x_3 = y_2 \Rightarrow \dot{x}_3 = \dot{y}_2 = x_4$ ★ row 3

$x_4 = \dot{y}_2 \Rightarrow \dot{x}_4 = \ddot{y}_2$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k}{m_1} & -\frac{C_1}{m_1} & \frac{k}{m_1} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k}{m_2} & 0 & -\frac{k}{m_2} & -\frac{C_2}{m_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \frac{1}{m_1} & 0 \\ 0 & 0 \\ 0 & -\frac{1}{m_2} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\ddot{y}_1 = \frac{u_1}{m_1} - \frac{C_1}{m_1}(\overset{x_2}{\dot{y}_1} - \overset{x_4}{0}) - \frac{k}{m_1}(\overset{x_1}{y_1} - \overset{x_3}{y_2})$$

~~$\dot{x} = Ax + Bu$~~

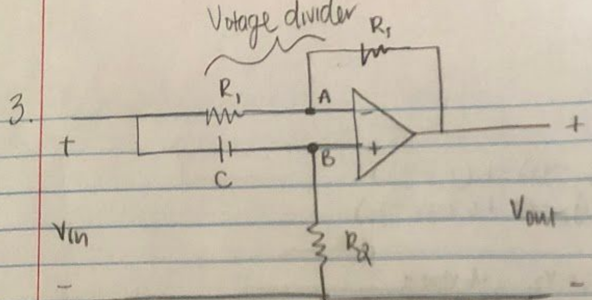
$$\ddot{y}_1 = \frac{1}{m_1}u_1 - \frac{C_1}{m_1}x_2 + \frac{C_1}{m_1}x_4 - \frac{k}{m_1}x_1 + \frac{k}{m_1}x_3$$

$$\ddot{y}_2 = \frac{-1}{m_2}u_2 - \frac{C_2}{m_2}\overset{x_4}{\boxed{\dot{y}_2}} + \frac{k}{m_2}\overset{x_1}{\boxed{y_1}} - k\overset{x_3}{\boxed{y_2}}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

c) If $F_1 > F_2$ then the system would slowly be pushed farther from the wall, to the right. The spring (k) would constrict and C_1 & C_2 would expand.

3.



$V_A = V_B$

$V_A = V_{out}\left(\frac{R_1}{R_1+R_1}\right) = V_{out}\left(\frac{\cancel{R_1}}{2\cancel{R_1}}\right)$

$V_A = \frac{V_{out}}{2}$

$V_{in} = V_C + R_2$

$V_{in} = \frac{1}{C}\int i_2 dt + R_2 i_2$

$\boxed{V_{in} = \frac{1}{CR_2}\int V_B\, dt + V_B}$

$V_{in} = \frac{1}{CR_2}\cdot\frac{1}{2}\int V_{out}\, dt + \frac{1}{2} V_{out}$

$$\boxed{\dot{V}_{in} = \frac{1}{2CR_2} V_{out} + \frac{1}{2}\dot{V}_{out}}$$

3 Bonus: The resistors (both of value R_1) are positioned such that they form a voltage divider. That means that the voltage going through node A (V_A) equals V_{out} times $\left(\frac{R_1}{R_1+R_2}\right)$ which equals $\frac{1}{2}$ as the R_1 values cancel out from the numerator & denominator. With that said, changing the values of both resistors won't change/affect the system because the values cancel out. So changing these values (picking a smaller/larger value) for these resistors doesn't make it more desirable nor less desirable. It makes no difference. Also the differential equation doesn't include the resistors with value R_1 (as they are canceled out)