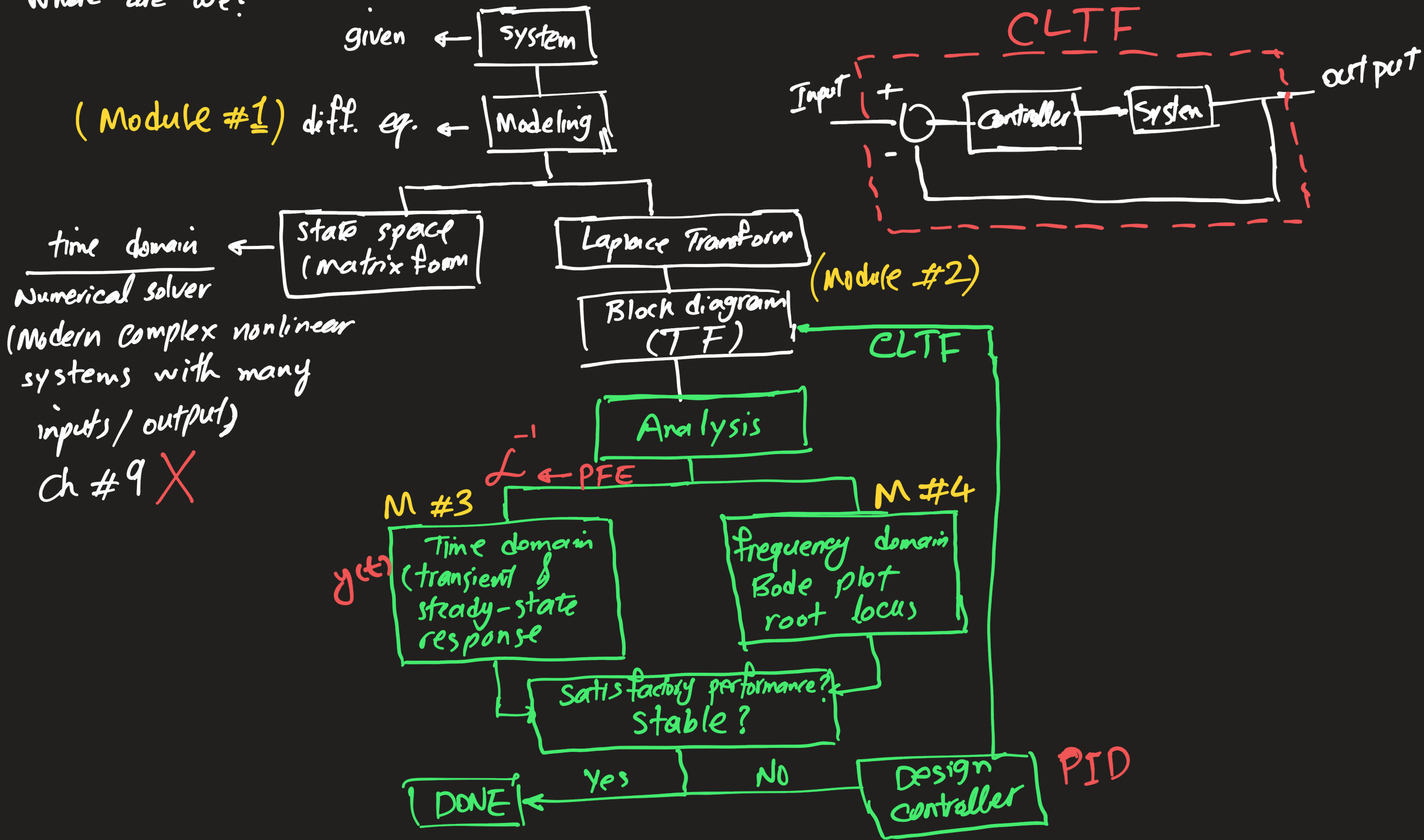
Where are we?



given
System
(Module #1)
diff. eq.
Modeling
time domain
Numerical solver
(Modern complex nonlinear systems with many inputs/outputs)
Ch #9
State space (matrix form)
Laplace Transform
(Module #2)
Block diagram (T F)
CLTF
Analysis
$\mathcal{L}^{-1}$
PFE
M #3
M #4
y(t)
Time domain (transient & steady-state response
frequency domain
Bode plot
root locus
Satisfactory performance?
stable?
Yes
No
DONE
Design controller
PID
CLTF
Input
+
-
controller
System
output

Chapter 5: Transient and Steady-State Response

(Module #3)

Steady state Response: The response of a system when $t \to \infty$ $\Rightarrow$ and all derivatives $\to 0$

$\to$ If it converges to a value/function $\Rightarrow$ equilibrium stability

$\to$ stability

- $\to$ tendency of the system to return to the equilibrium from any disturbance or initial value (Absolute stability)
- $\to$ can be a relative concept: How far away from being unstable?

Two additional properties of Laplace Transform:

1) IVT (Initial Value Theorem):

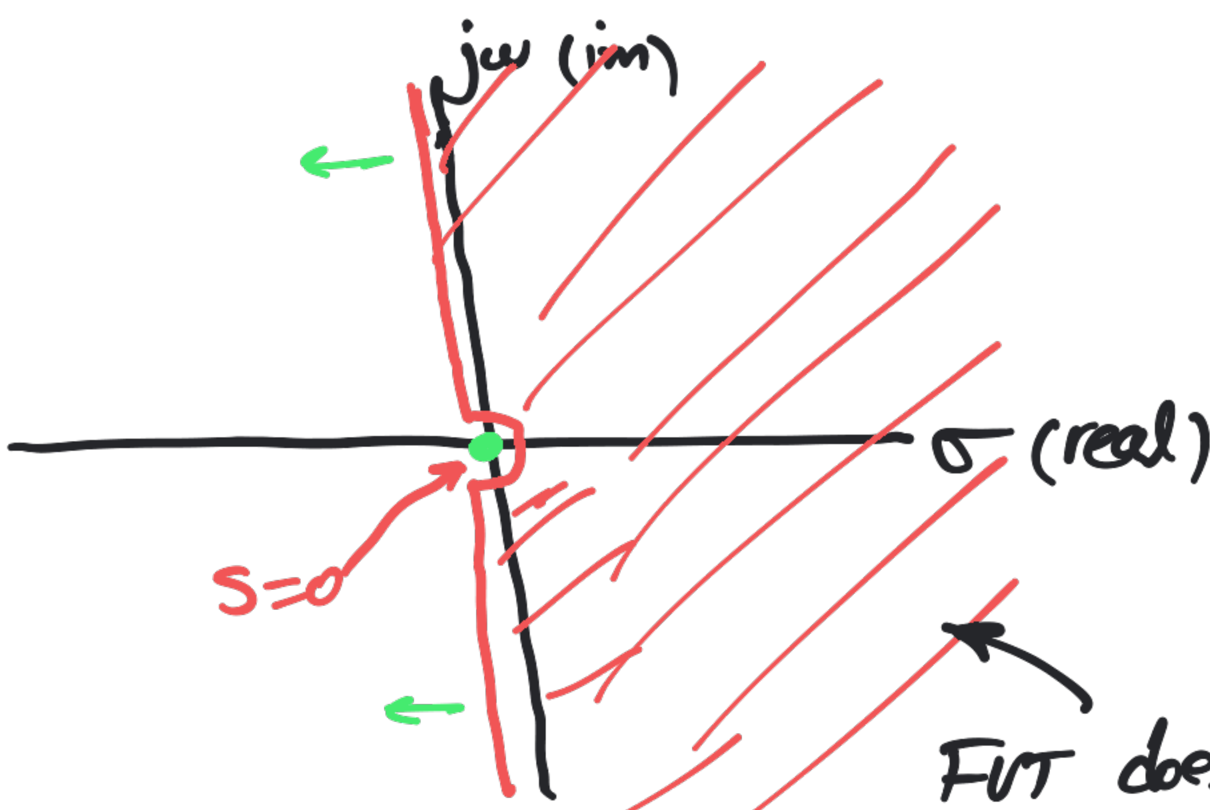
$$f(0) = \lim_{t \to 0} f(t) = \lim_{s \to \infty} s F(s)$$

2) FVT (Final Value Theorem)

$$f(\infty) = \lim_{t \to \infty} f(t) = \lim_{s \to 0} s F(s)$$ } steady-state response

$s = \sigma + j\omega$

Poles:



FVT does not give a correct result here.

The final value theorem works for systems with <u>all</u> poles either on the LHP or at the origin or Combination

(FV=0) $F(s) = \frac{1}{s+2}$ $\Rightarrow$ S-S : $\lim_{s \to 0} s \frac{1}{s+2} = 0$ (No poles at the origin)

$s=-2$

(FV= cte) $F(s) = \frac{1}{s}$ $\Rightarrow$ S-S : $\lim_{s \to 0} \cancel{s} \frac{1}{\cancel{s}} = 1$ (One pole at the origin)

$s=0$

(FV=∞) $F(s) = \frac{1}{s^2}$ $\Rightarrow$ S-S : $\lim_{s \to 0} \cancel{s} \frac{1}{s^{\cancel{2}}} = \infty$ (Two or more poles at the origin)

$s=0$

Pole : roots of denominator of TF

denominator = 0 ← characteristic eq.

Example: $G(s) = \frac{Y(s)}{U(s)} = \frac{10}{s+2}$

Determine s-s response (y_{ss}) for unit-step input:

$U(s) = \frac{1}{s} \Rightarrow y_{ss} = ?$

$$Y(s) = \frac{10\,U(s)}{s+2} = \frac{10}{s(s+2)}$$

$$y_{ss} = \lim_{s \to 0} \cancel{s} \frac{10}{\cancel{s}(s+2)} = \frac{10}{2} = 5$$

Time-domain:

$$(s+2)\,Y(s) = 10\,U(s)$$

$$\Rightarrow s\,Y(s) + 2\,Y(s) = 10\,U(s)$$

$$\dot{y} + 2y = 10\,u(t) \quad 1$$

$$\underset{0}{\cancel{\dot{y}}} + 2y = 10 \Rightarrow \boxed{y = 5}$$

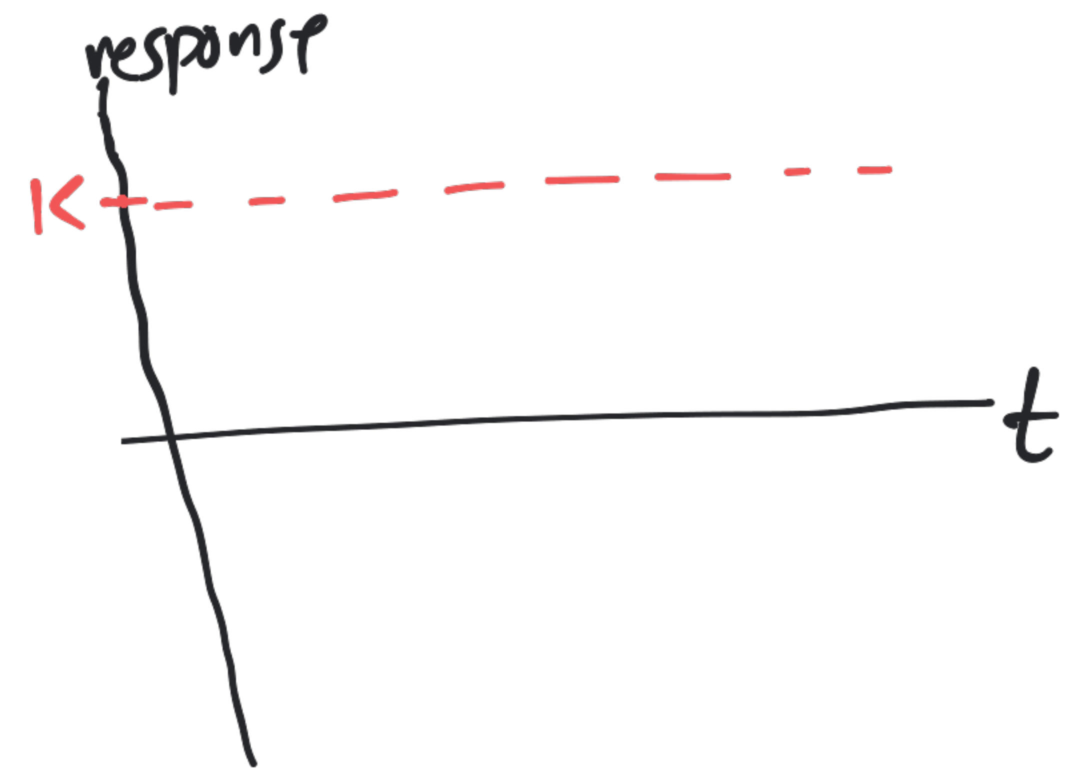
$G(s) = \frac{10}{s+2} \Rightarrow$ the s-s response is not clear

A better way : $G(s) = \frac{5}{0.5s+1}$

$$\boxed{G(s) = \frac{K}{\tau s + 1}}$$ standard first-order form

K: steady-state gain (DC gain)

τ : time constant



First_order system :

$$\frac{Y(s)}{U(s)} = \frac{K}{\tau s+1} \Rightarrow \underbrace{Y(s) = \frac{K}{\tau s+1} U(s)}$$

$$\boxed{y(t)}$$?

Unit_step Response :

Unit-step $U(s) = \frac{1}{s} \Rightarrow Y(s) = \frac{1}{s}\frac{K}{\tau s+1}$

$$\Rightarrow Y(s) = \frac{C_1}{s} + \frac{C_2}{\tau s+1} = \underbrace{\frac{K}{s} - \frac{\tau K}{\tau s+1}} = \underbrace{K\left(\frac{1}{s} - \frac{\tau}{\tau s+1}\right)}$$

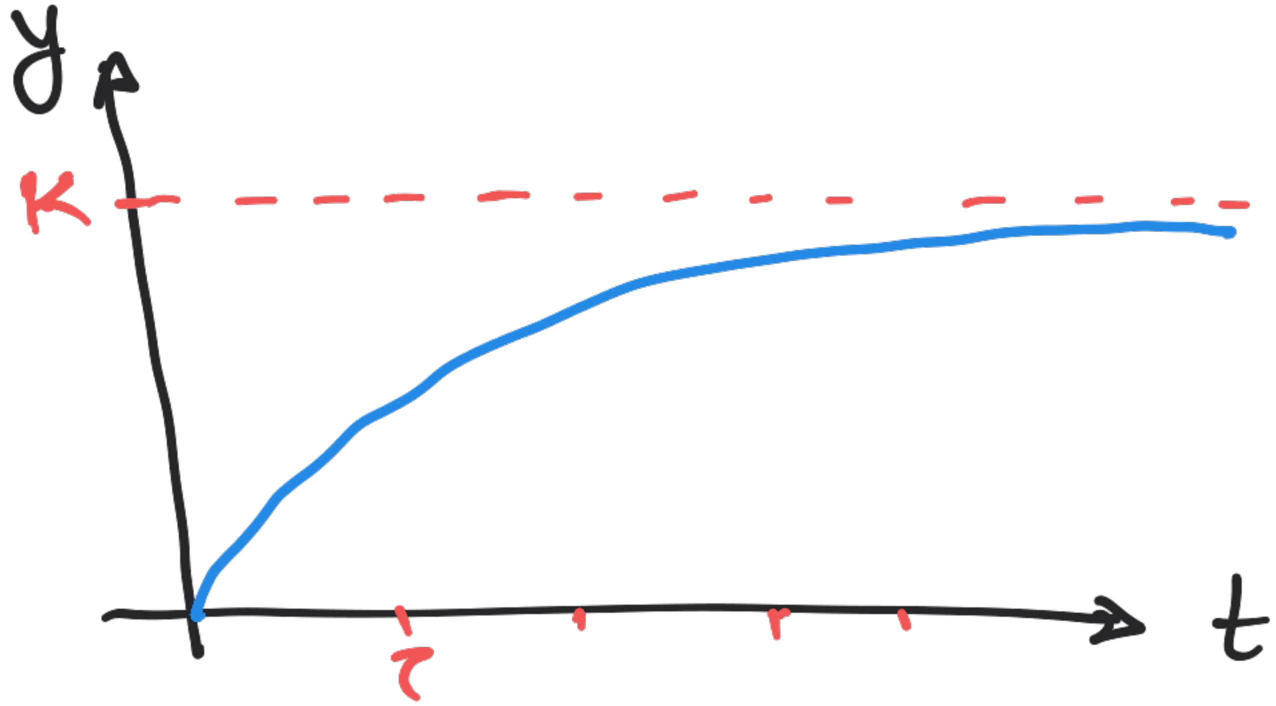
$$C_1 = s\frac{1}{s}\frac{K}{\tau s+1}\Big|_{s=0} = K$$

$$\boxed{Y(s) = K\left(\frac{1}{s} - \frac{1}{s+1/\tau}\right)}$$

$$C_2 = (\tau s+1)\frac{1}{s}\frac{K}{\tau s+1}\Big|_{s=-1/\tau} = -\tau K$$

$$\mathcal{L}^{-1} \Rightarrow \boxed{y(t) = K\left(1 - e^{-t/\tau}\right)}, \; t > 0$$

$$y(t) = K\left(1 - e^{-t/\tau}\right), \quad t > 0$$



t	% final value	
0	0	
τ	63.2%	
2τ	86.5%	
3τ	95%	$\rightarrow$ 5%
4τ	98.2%	$\rightarrow$ 2%
5τ	99.3%	$\rightarrow$ 1%
⋮		

settling time

time required for unit step response to remain within 2% or 5% of final valu

Unit-Ramp Response: $(K=1)$

$$\frac{Y(s)}{R(s)} = \frac{1}{\tau s+1} = G(s) \qquad U(s) = \frac{1}{s^2} \leftarrow u(t) = t$$

$$\Rightarrow Y(s) = \frac{1}{s^2(\tau s+1)} = \frac{c_1}{s^2} + \frac{c_2}{s} + \frac{c_3}{\tau s+1}$$

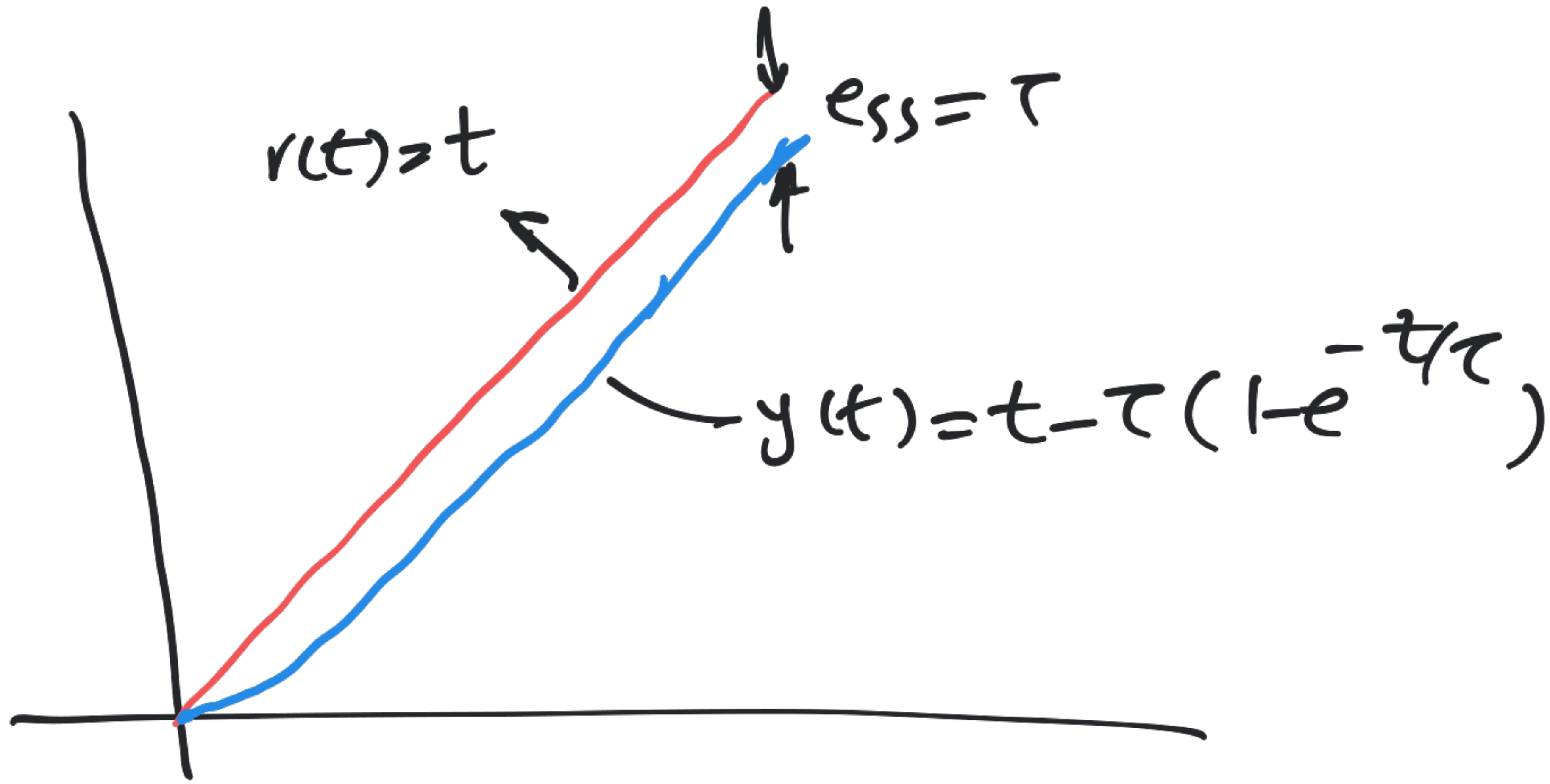
$$= \frac{1}{s^2} - \frac{\tau}{s} + \frac{\tau^2}{\tau s+1}$$

$$\xRightarrow{\mathcal{L}^{-1}} \boxed{y(t) = t - \tau + \tau e^{-t/\tau}}$$

Error: $e(t) = r(t) - y(t)$

$$e(t) = \cancel{t} - \cancel{t} + \tau(1 - e^{-t/\tau})$$

$$\Rightarrow \boxed{e_{ss} = \tau}$$



$r(t) = t$
$e_{ss} = \tau$
$y(t) = t - \tau(1 - e^{-t/\tau})$