Drawing Bode's Diagrams:

procedure: Given a TF $G(s) = \dfrac{B(s)}{A(s)}$

1. Find its factors: Gain, s, first-order, second-order
2. Rearrange to make sure each factor has DC gain of 1.
3. Find Bode information for each factor $(\underset{\text{Gain}}{dB}, \underset{\text{Phase}}{\phi})$ and plot
4. Add them up:

order

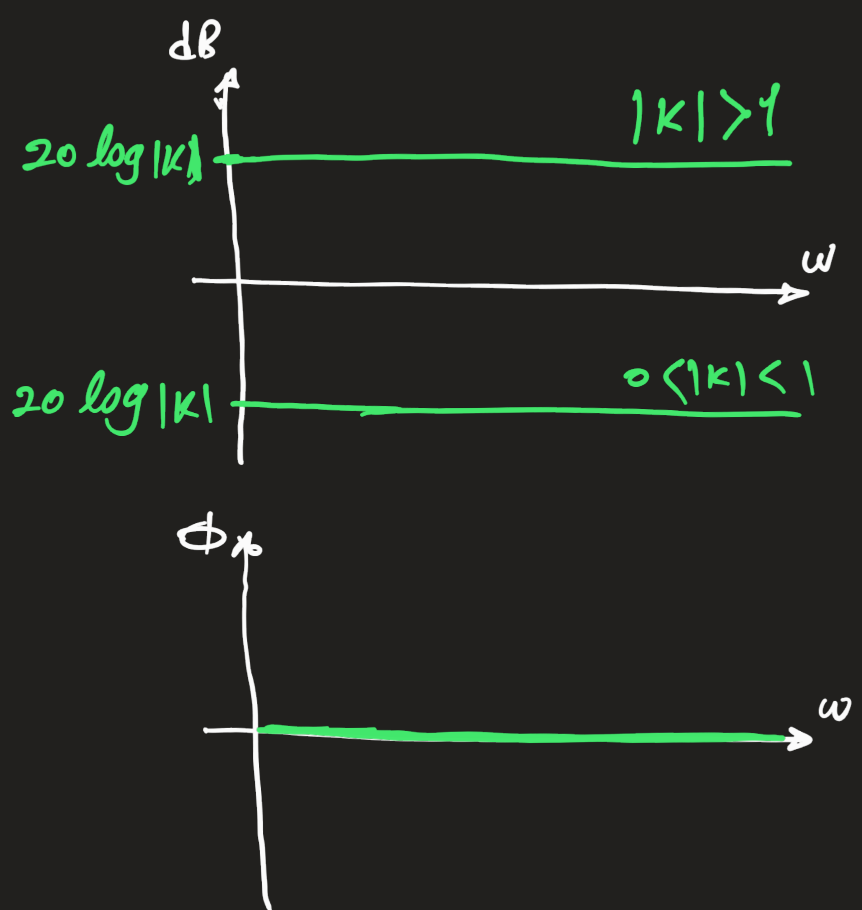
→ Draw $s=0$ zero/pole factor

→ Add gain

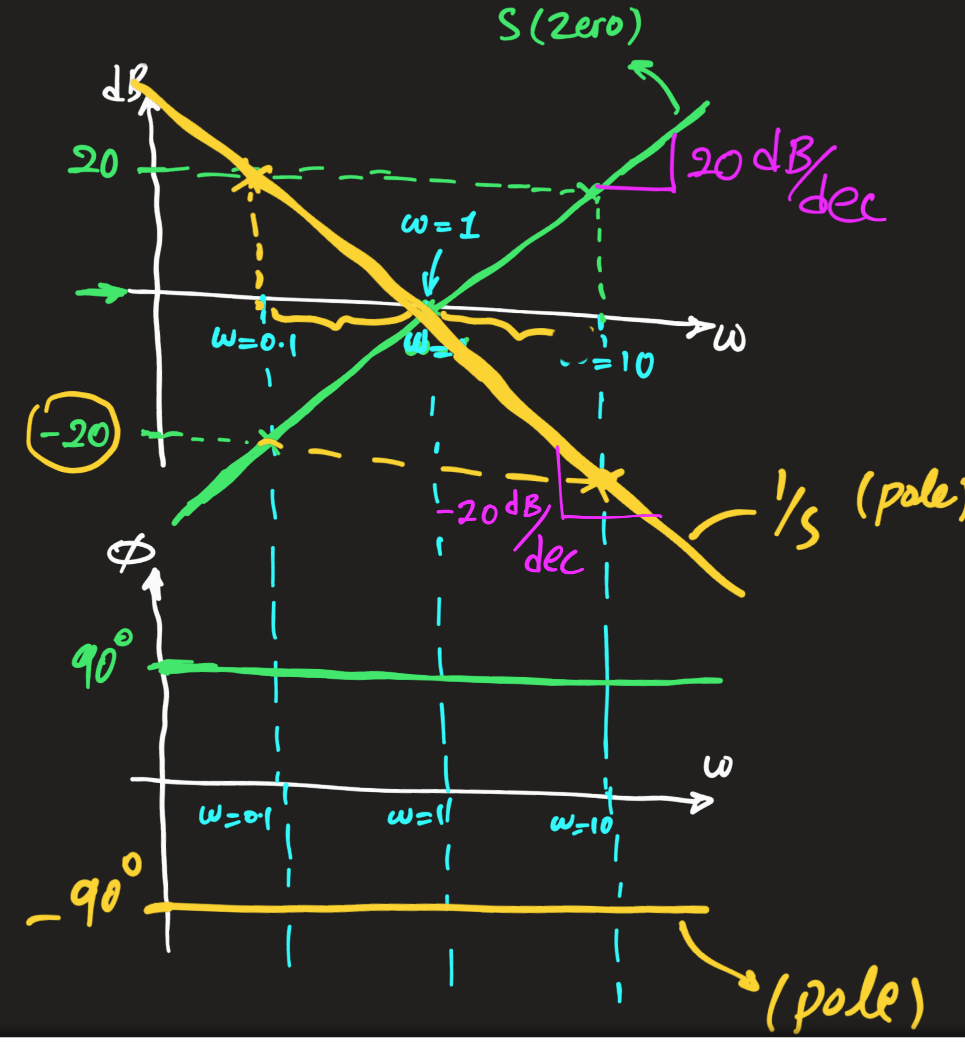
→ Add first- and second-order terms from <u>smallest</u> corner frequency (ω_n) to the <u>largest</u>

↳ Add slopes.

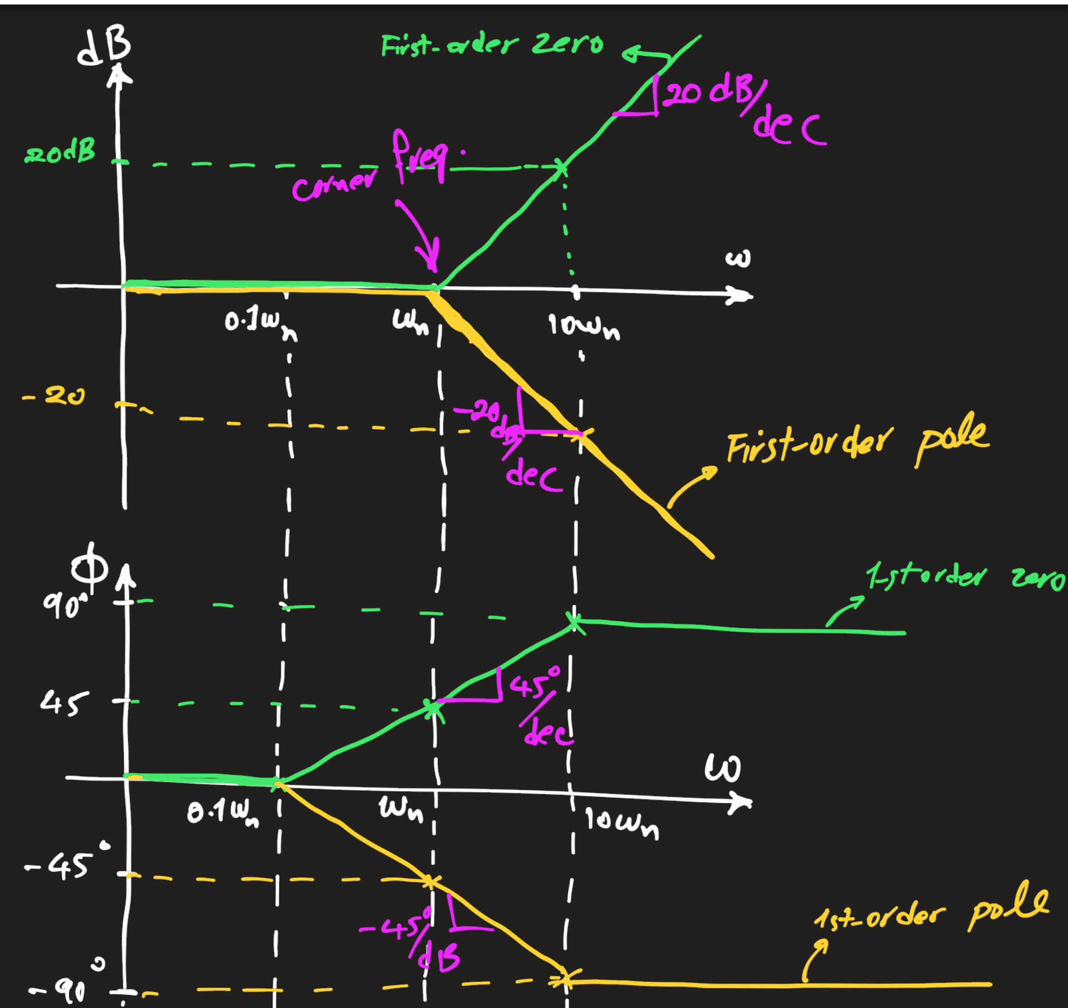
Constant gain



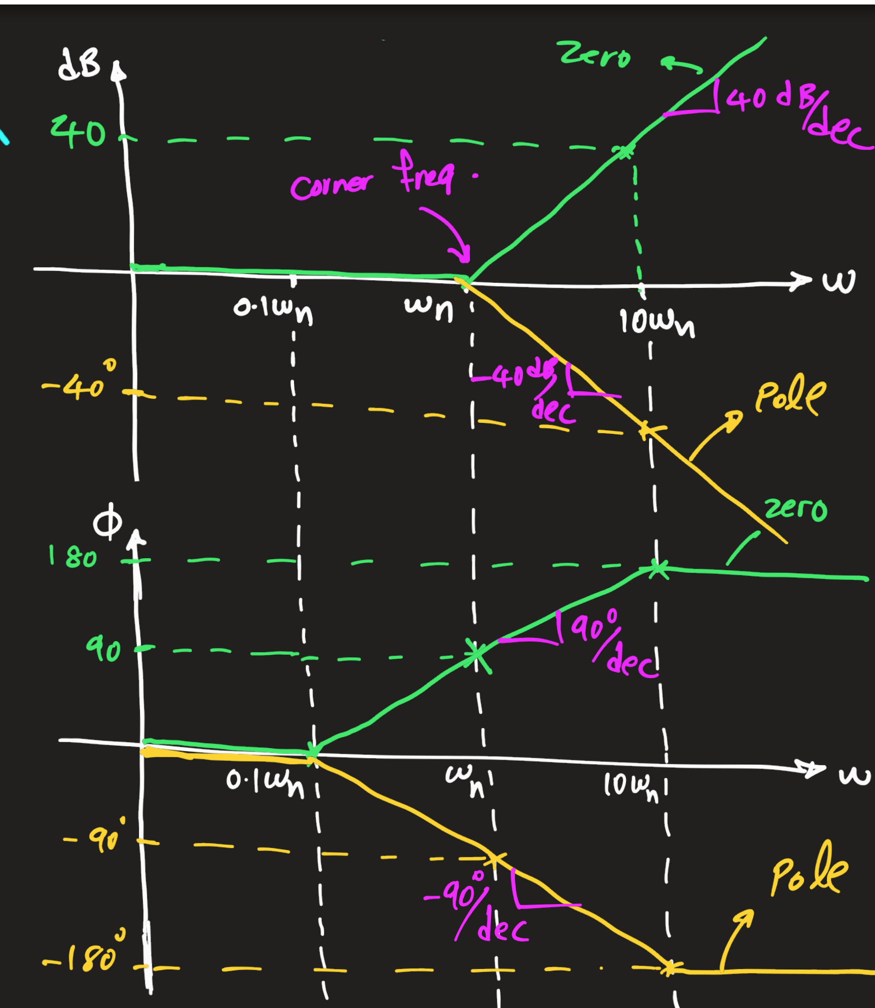
S=0 (zero/pole)



first-order (zero/pole)

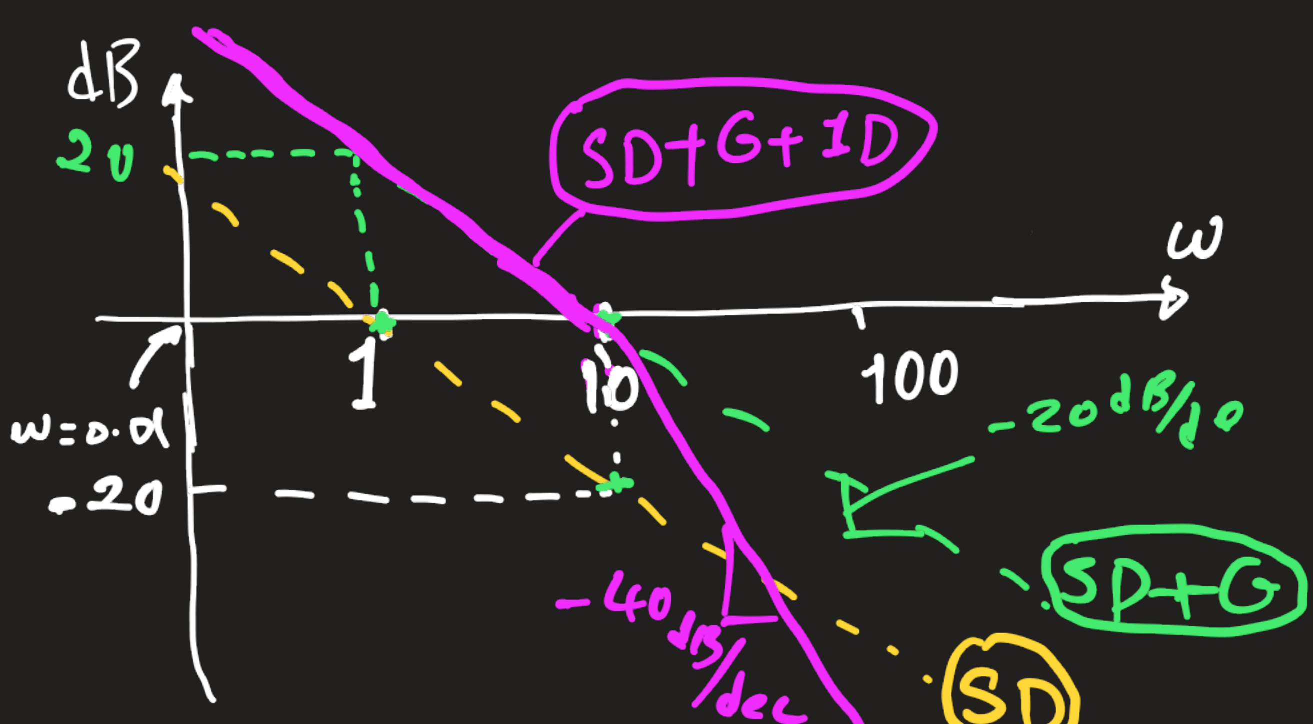


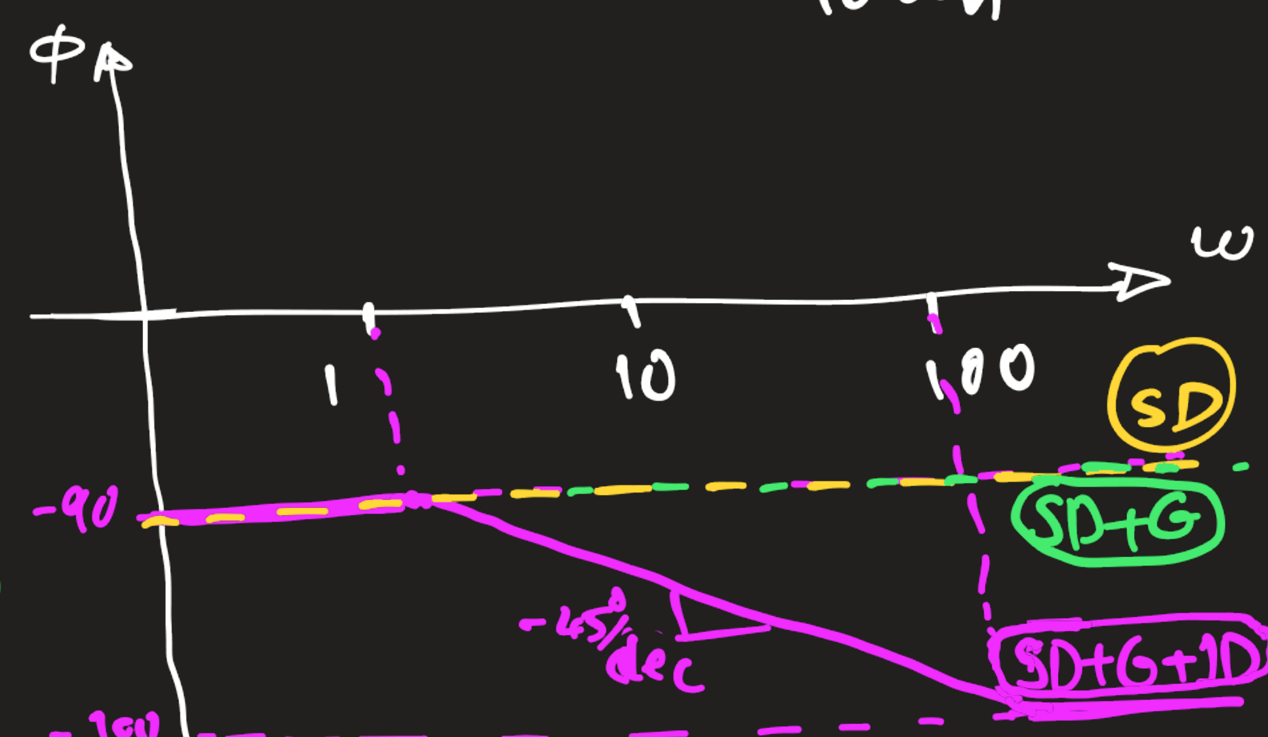
Second-order (zero/pole)



Example: $G(s) = \dfrac{100}{s(s+10)} = \dfrac{100}{s\,\underline{(s+10)}\,\frac{10}{10}} = \dfrac{10}{s\left(\frac{s}{10}+1\right)}$

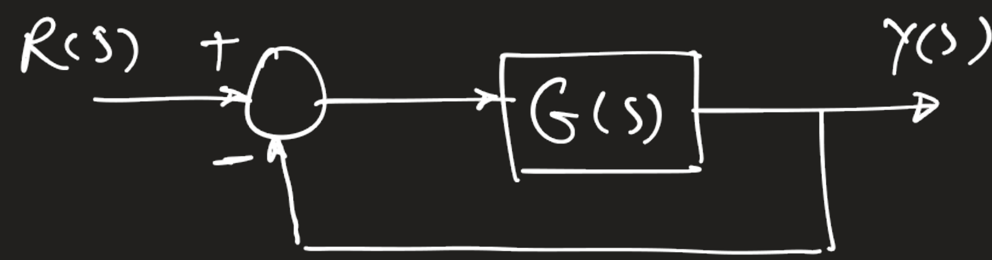
Factor	Gain	Phase
SD $s=0$ pole	$0 \to \infty$: -20 dB/dec 0 dB @ $\omega=1$	-90° : $0<\omega<\infty$
G 10	$0 \to \infty$: $20\log 10 = 20$ dB slope: 0 dB/dec	0° : $0<\omega<\infty$
1D first-order $\omega_n=10$	$10 \to \infty$: -20 dB/dec	-45°/dec : $1<\omega<100$ $1 \to 0.1\omega_n$, $100 \to 10\,\omega_n$





stability and performance analysis in Frequency Domain:

unity-feedback control system:



$$CLTF = \frac{Y(s)}{R(s)} = \frac{G(s)}{1+G(s)}$$

* For the closed loop system to be on the verge of instability:

Chan. eq. must have a root at $\underline{\underline{s = j\omega_d}}$

marginally stable
critically stable

$\Rightarrow 1 + G(s) = 0 \Rightarrow 1 + G(j\omega_d) = 0$

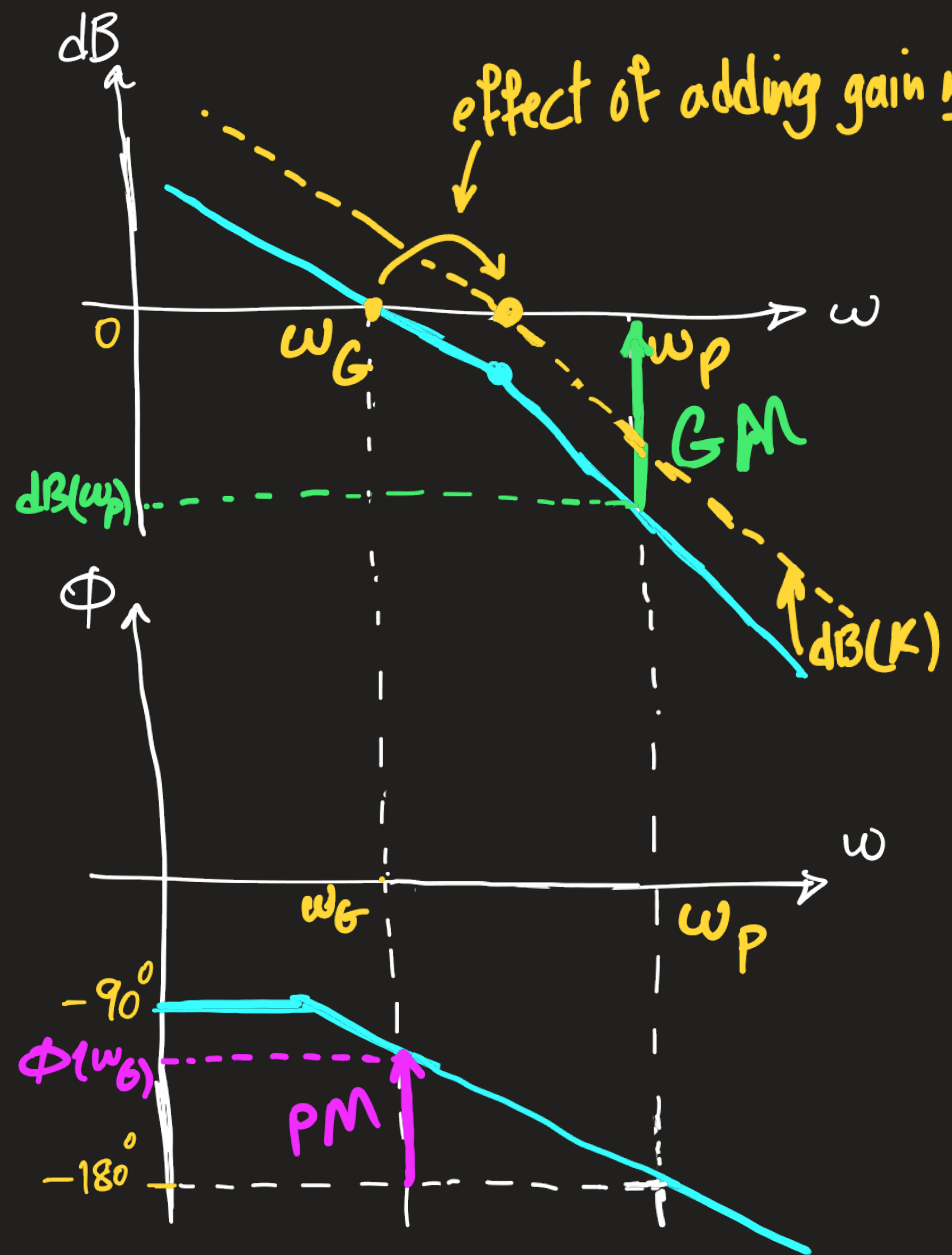
$\Rightarrow \boxed{G(j\omega_d) = -1}$

$\Rightarrow |G(j\omega_d)| = \underline{\underline{1}} \Rightarrow 20\log(1) = \boxed{0\ dB}$ → can be used to analyze stability of the closed-loop system

$\Rightarrow \angle G(j\omega_d) = \pm 180^0 \Rightarrow \boxed{\phi = -180^0}$

Note: G(s) is the open-loop TF.

Stability Margins : (Distance to instability)



ω_G : Gain crossover frequency

ω_P : Phase crossover frequency

GM: Gain Margin :

the additional gain required to bring the system To instability

$$\boxed{GM = 0 - dB(\omega_P)}$$

$GM \geq 0$ for stability

PM : Phase Margin :

the additional phase required to bring the system to instability

$$PM = \Phi(\omega_G) - (-180)$$

$$\Rightarrow \boxed{PM = \Phi(\omega_G) + 180}$$

$PM \geq 0$ for stability

* open-loop Bode Diagram :

  ✓ Four parameters : ω_G, ω_P, G_M, PM

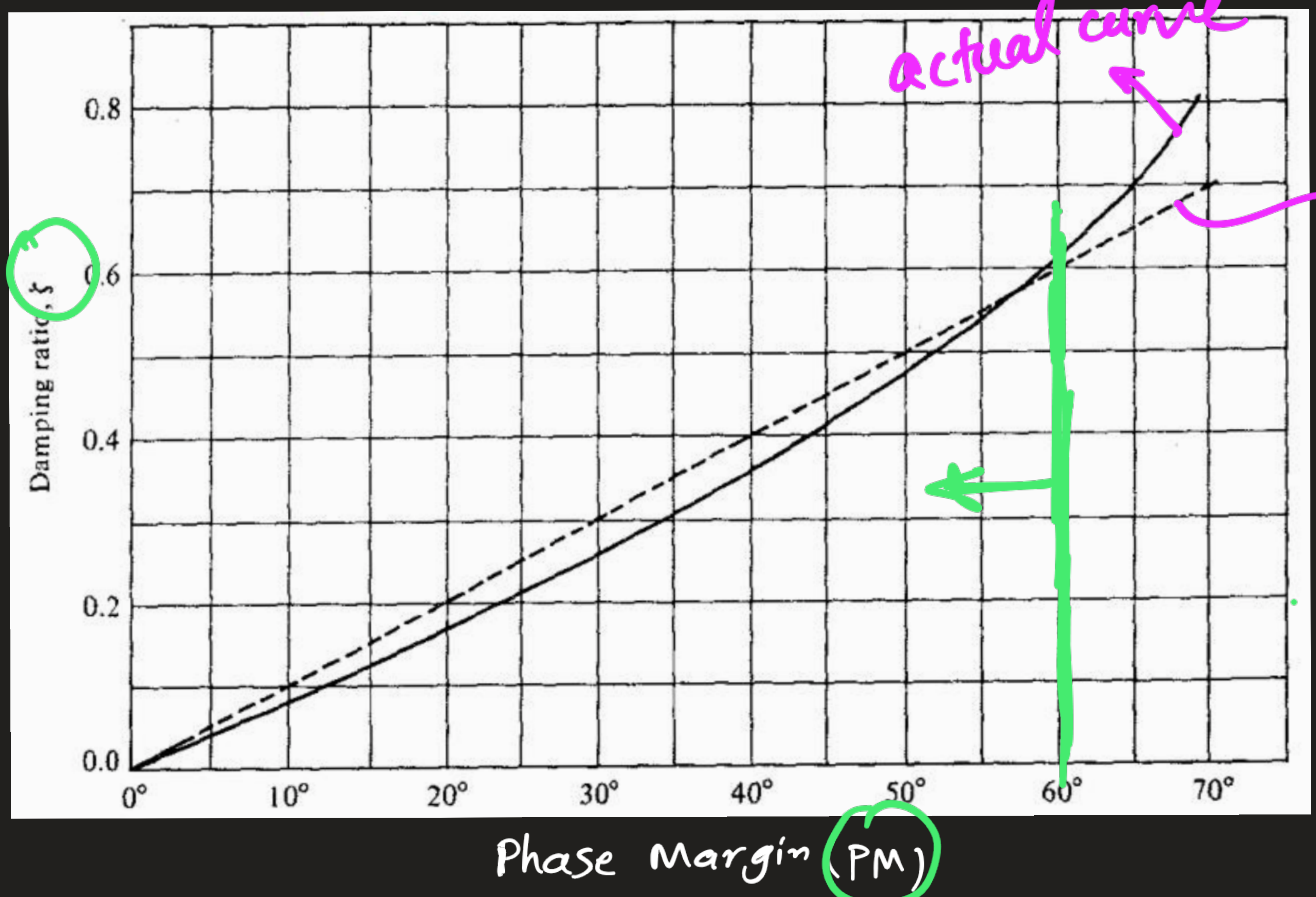
* Increasing the constant gain ⇒ decreases both GM & PM

  ✓ Easy to evaluate the effect of gain

Closed loop system performance : (Calculating Maximum Overshoot (M_p))

For a second-order system, ξ is related to PM through the following eq:

$$PM = \tan^{-1}\left(\frac{2\xi}{\sqrt{\sqrt{1+4\xi^4}-2\xi^2}}\right) \rightarrow \text{plot}$$

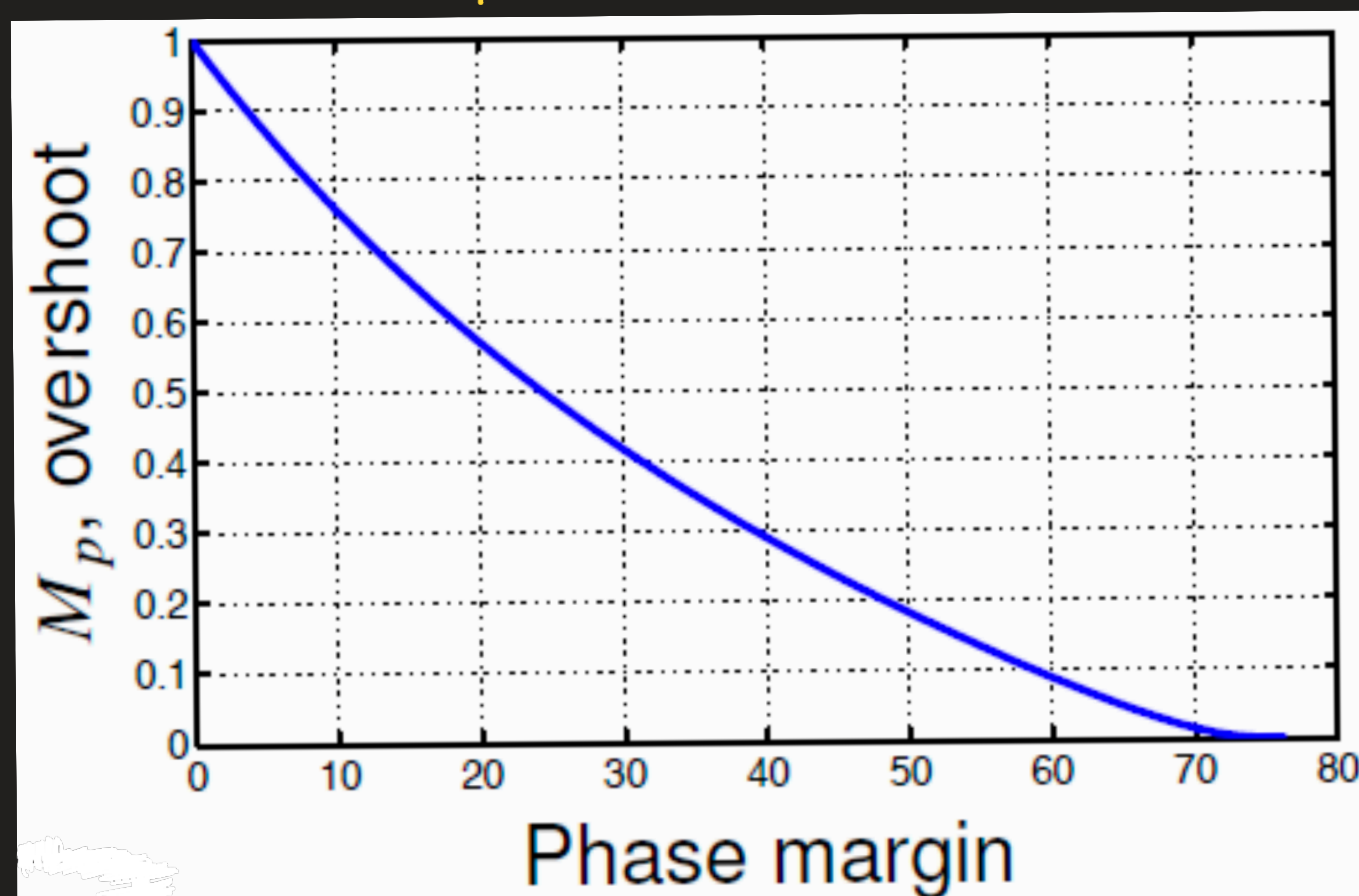


For $PM \leq 60 \Rightarrow \boxed{\xi \simeq \frac{PM\ (\text{degrees})}{100}}$

Procedure of Calculating M_p from PM:

If $PM \leq 60^\circ \Rightarrow$ Calculate $\zeta \cong \frac{PM(\text{degrees})}{100} \Rightarrow \boxed{M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}}}$ ✓

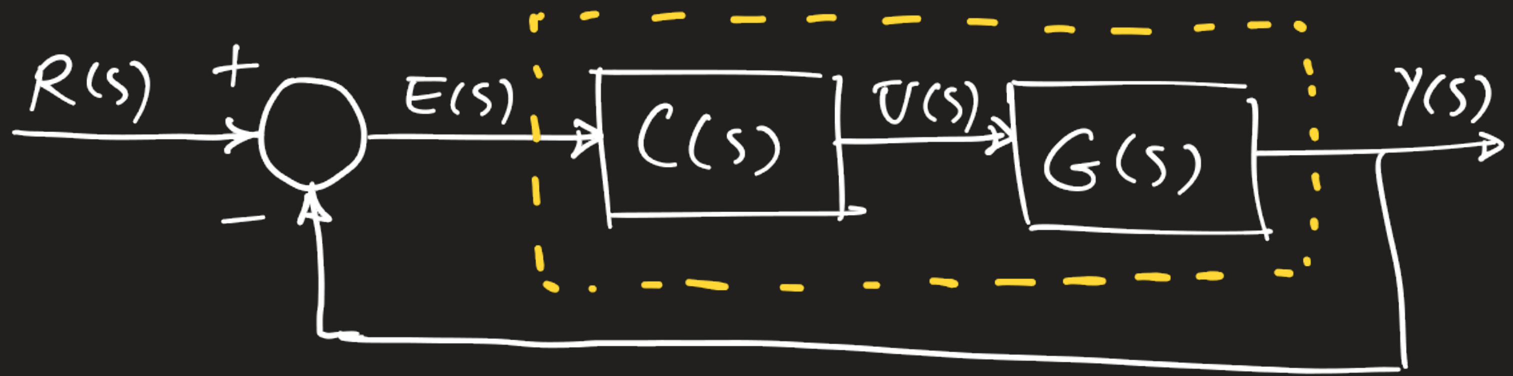
Maximum Overshoot ← M_p Vs. PM → Phase Margin



Controller Analysis: (PD and Lead compensator)

* PD controller: $C(s) = K_P + K_D \cdot s \Rightarrow C(s) = K_P\left(1 + \frac{K_D}{K_P} \cdot s\right)$

$$T_D = \frac{1}{\omega_D}$$



$$\Rightarrow \boxed{C(s) = K_P\left(\frac{s}{\omega_D} + 1\right)}$$

OLTF: $C(s)G(s) = K_P\left(\frac{s}{\omega_D} + 1\right) G(s)$

$\Rightarrow$ PD controller adds a gain & 1-st order term to the numerator of OLTF