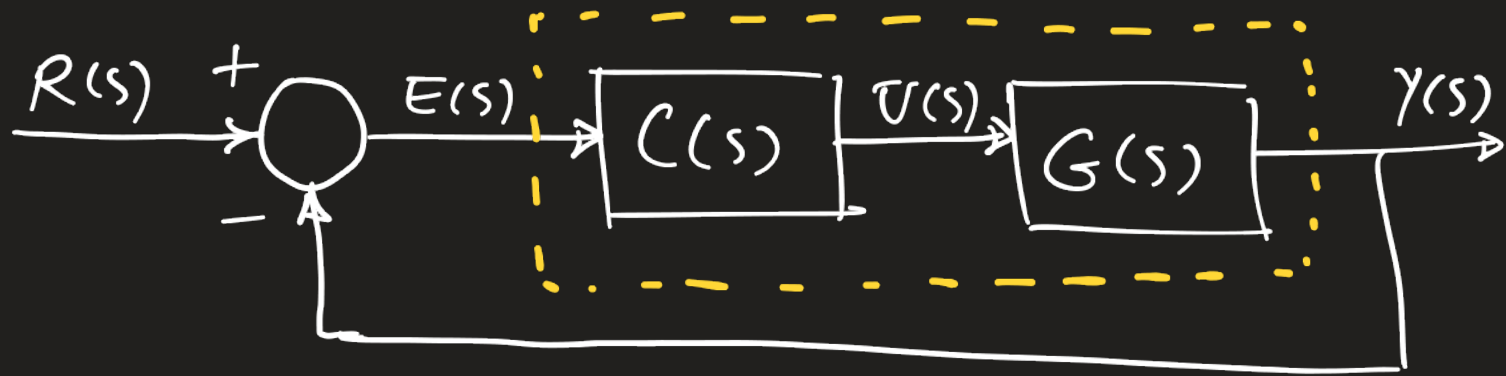
Controller Analysis: (PD and Lead compensator)

* PD controller: $C(s) = K_P + K_D \cdot s$ $\Rightarrow C(s) = K_P\left(1 + \frac{K_D}{K_P} \cdot s\right)$

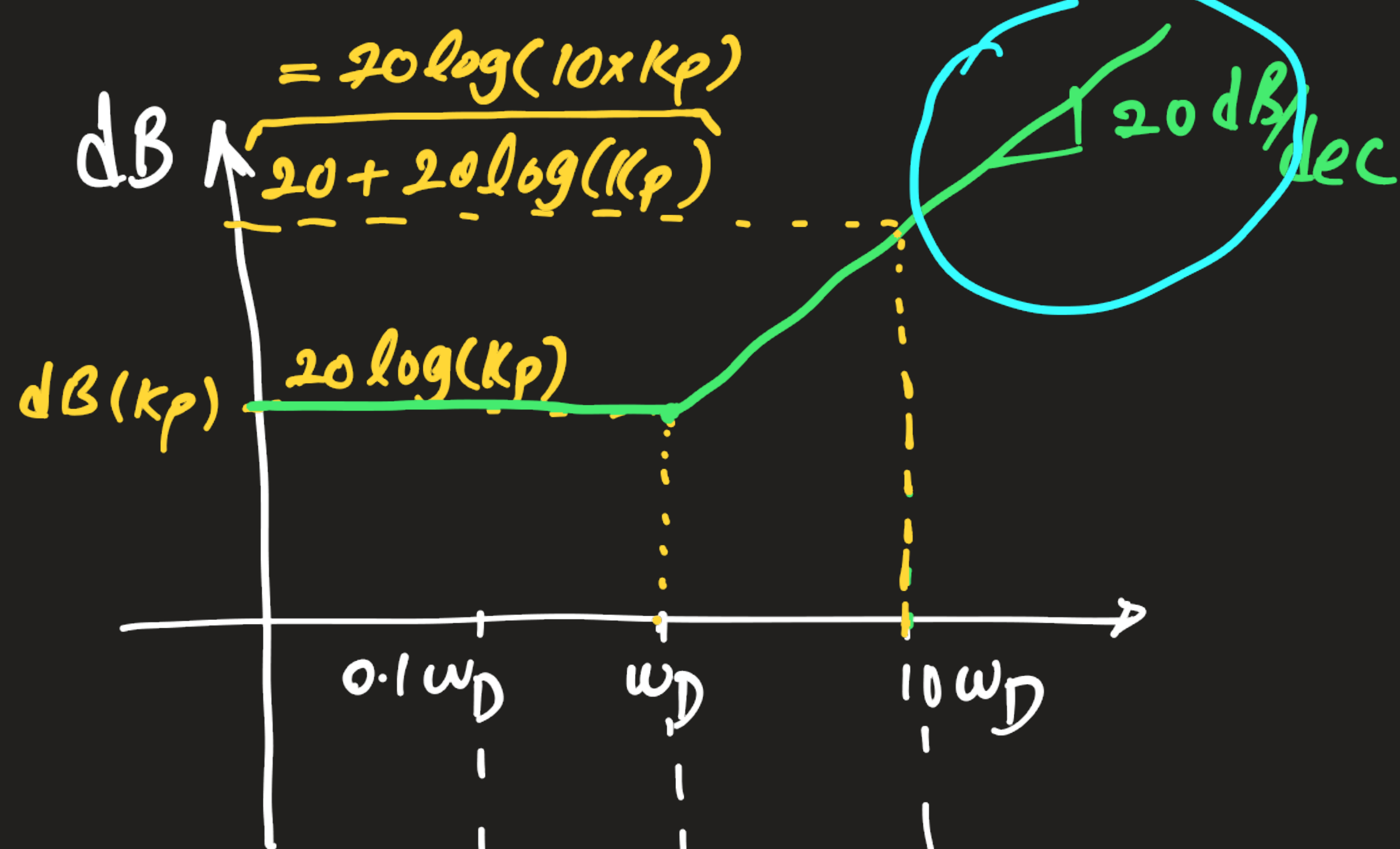


$T_D = \frac{1}{\omega_D}$

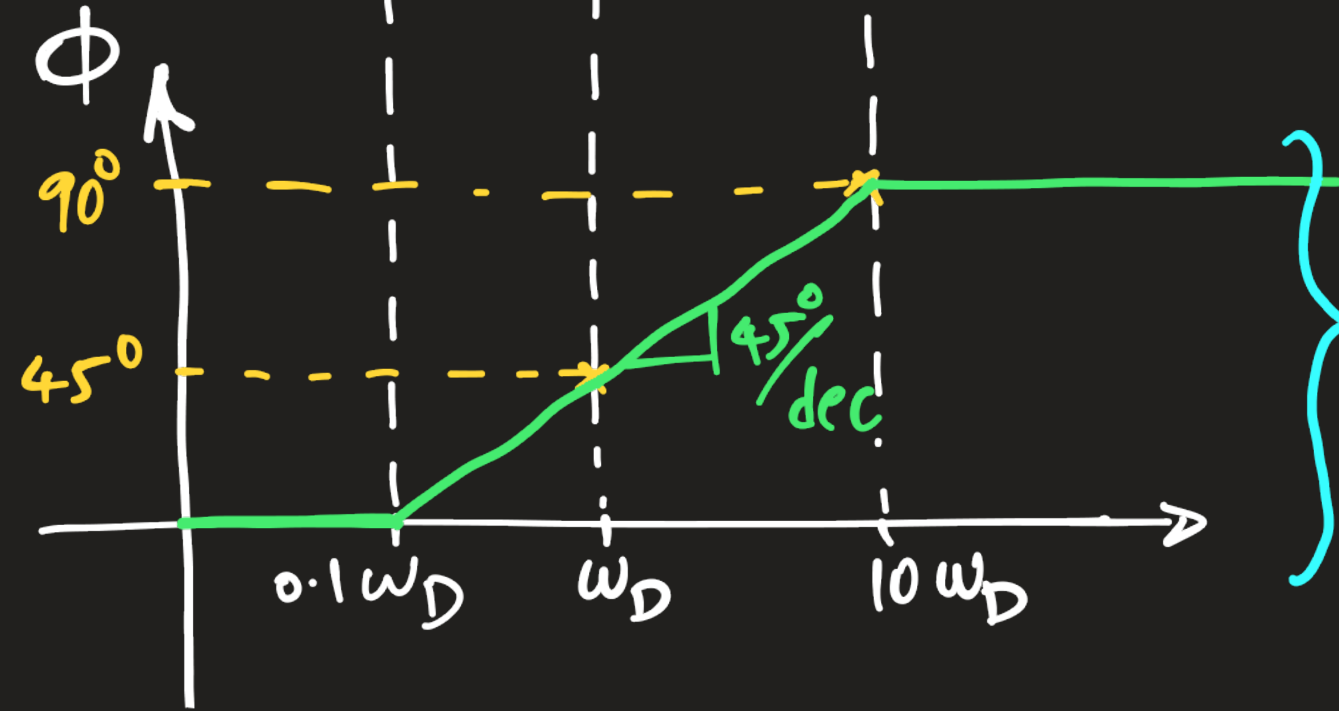
$$\Rightarrow \boxed{C(s) = K_P\left(\frac{s}{\omega_D} + 1\right)}$$

OLTF: $C(s)G(s) = K_P\left(\frac{s}{\omega_D} + 1\right) G(s)$

$\Rightarrow$ PD controller adds a gain & 1-st order term to the numerator of OLTF



$\Rightarrow$ amplifies high-frequency signals (BAD X)



shifts the phase up.

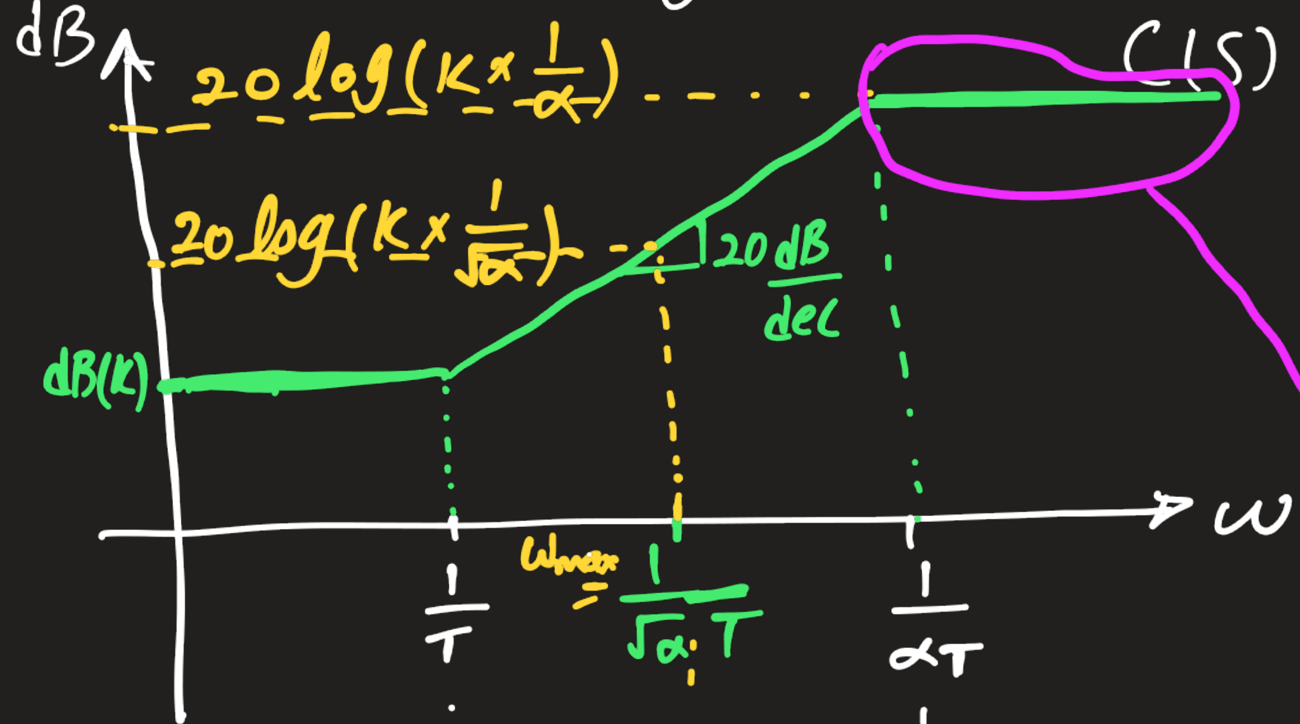
when $\omega_D < \omega_G \Rightarrow$

PM will be increased (GOOD ✓)

$\Rightarrow$ we can pick ω_D based on how much additional phase shift is desired.

* LEAD compensator : $C(S) = K\left[\frac{TS+1}{\alpha TS+1}\right]$, $0<\alpha<1$

A controller that maintains the good parts of PD control, without amplifying noise.



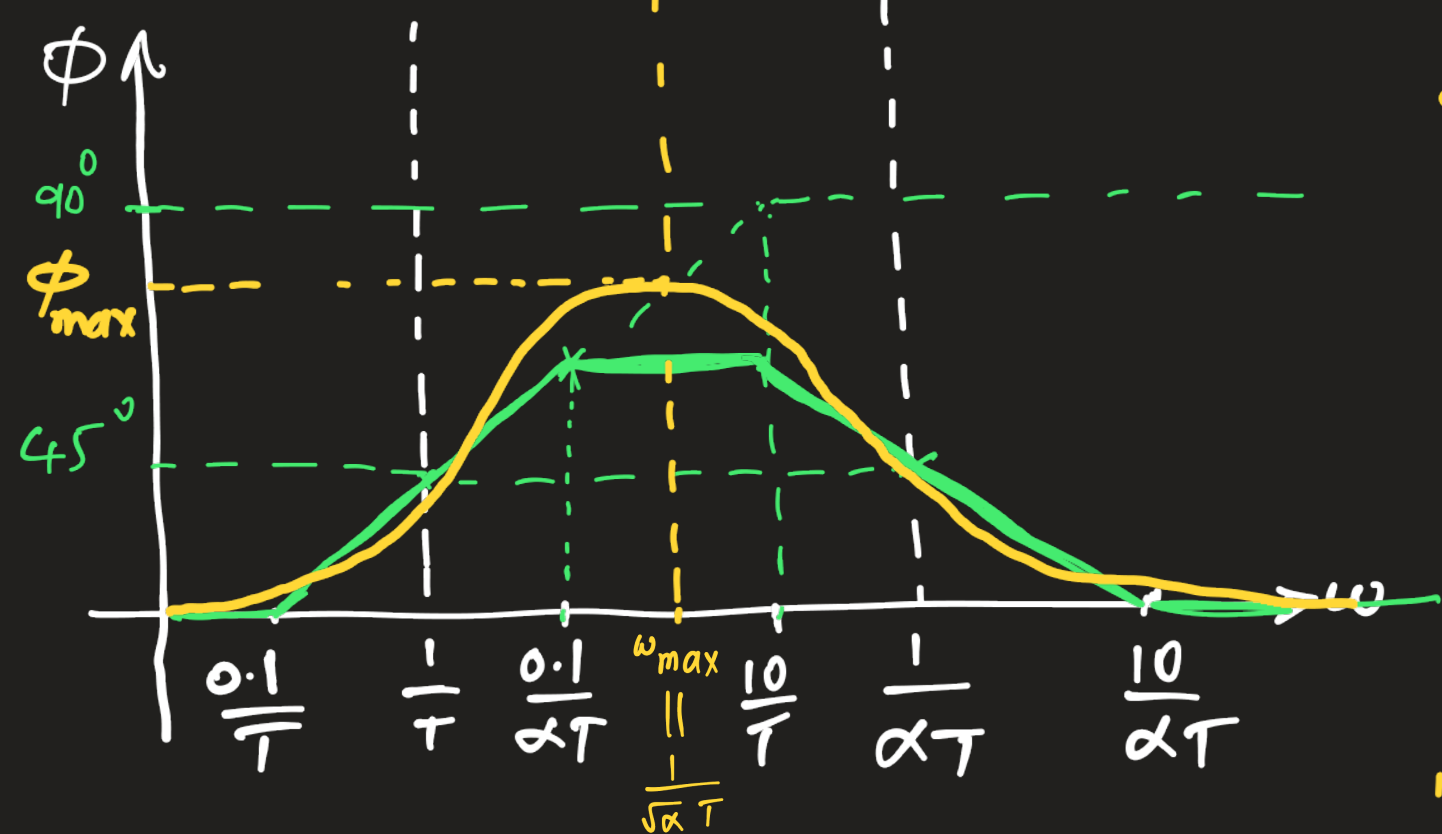
$$C(S) = \underset{G}{K} \frac{\overset{1N}{\left(\frac{S}{1/T}+1\right)}}{\underset{1D}{\left(\frac{S}{1/\alpha T}+1\right)}}$$

$\omega_n = \frac{1}{T}$

$\omega_n = \frac{1}{\alpha T}$

⇒ Better noise response

⇒ Approximates PD control until $\omega = \frac{1}{\alpha T}$



ϕ_{max} : maximum phase shift occurs at $\boxed{\omega_{max} = \frac{1}{\sqrt{\alpha}\,T}}$

Geometric mean of $\frac{1}{T}$ & $\frac{1}{\alpha T}$

$$\phi_{max} = \angle C(j\omega_{max}) = \sin^{-1}\left(\frac{1-\alpha}{1+\alpha}\right)$$

invert $\Rightarrow \alpha = \frac{1-\sin(\phi_{max})}{1+\sin(\phi_{max})}$

How to design a Lead Compensator:

$C(s) = K \frac{TS+1}{\alpha TS+1}$ $\Rightarrow$ Need to determine 3 parameters (K, T, α)

Design Specs: $\rightarrow$ Desired ω_{G_d} (usually selected at system's ω_G)

$\rightarrow$ Desired PM_d (found from ζ or M_p)

Calculate the gain and phase of plant $G(s)$ at ω_{G_d}.

$K_G = |G(j\omega_{G_d})|$ $\rightarrow$ This is not the dB value $\quad dB = 20 \log \underline{\underline{K_G}}$

we need this value

$\phi_G = \angle G(j\omega_{G_d})$

Phase lead required: $(PM_d = \phi_G + 180 + \phi_{max}) \Rightarrow$ 1 $\boxed{\phi_{max} = PM_d - 180^\circ - \phi_G}$

2 $\Rightarrow \boxed{\alpha = \frac{1-\sin(\phi_{max})}{1+\sin(\phi_{max})}}$

$\omega_{max} = \frac{1}{\sqrt{\alpha}\,T} = \omega_{G_d} \Rightarrow$ 3 $\boxed{T = \frac{1}{\sqrt{\alpha}\ \omega_{G_d}}}$ $\quad \omega_{max} = \omega_{G_d}$

Gain at ω_{max} should cancel out system gain $K_G \Rightarrow$ because $dB\ @\omega_{G_d} = 0$

$\Rightarrow 20 \log\left(\frac{K}{\sqrt{\alpha}}\right) + 20 \log(K_G) = 0 \Rightarrow 20 \log\left(\frac{K}{\sqrt{\alpha}}\right) = -20 \log K_G^{-1}$

$\Rightarrow 20 \log\left(\frac{K}{\sqrt{\alpha}}\right) = 20 \log\left(\frac{1}{K_G}\right) \Rightarrow$ 4 $\boxed{K = \frac{\sqrt{\alpha}}{K_G}}$