Controls

Exam 2

$y(0) = 1 \quad y'(0) = -2$

1. $\mathcal{L}[y'' + 2y' + y] = [u(t)]\mathcal{L}$

$[s^2Y(s) - sy(0) - y'(0)] + 2[sY(s) - y(0)] + Y(s) = U(s)$

$[s^2Y(s) - s(1) - (-2)] + 2[sY(s) - 1] + Y(s) = 0$

$(s^2 + 2s + 1)Y(s) - s + 2 - 2 = 0$

$(s^2 + 2s + 1)\,Y(s) = s$

$$Y(s) = \frac{s}{s^2 + 2s + 1} = \frac{s + 1 - 1}{(s+1)^2}$$

$$Y(s) = \frac{1}{s+1} - \frac{1}{(s+1)^2}$$

$$y(t) = \mathcal{L}^{-1}[Y(s)] = \mathcal{L}^{-1}\left[\frac{1}{s+1} - \frac{1}{(s+1)^2}\right]$$

$$\boxed{y(t) = e^{-t} - te^{-t}}$$

This can represent a spring-mass-damper system with mass = 1, damping = 2, spring stiffness = 1. The initial position & velocities are also given. The first part of the above solution is a negative exponential. This part approaches zero as t increases. The second part also approaches zero as t increases but at a faster rate since it is multiplied by t.

2. $\mathcal{L}[y(t)] = \mathcal{L}[3te^{-2t} - 2e^{-3t}]$

$Y(s) = \mathcal{L}[3te^{-2t} - 2e^{-3t}]$

$Y(s) = 3\mathcal{L}[te^{-2t}] - 2\mathcal{L}[e^{-3t}]$

$Y(s) = \left[3\left[-\frac{d}{ds}\left[\frac{1}{s+2}\right]\right] - 2\left[\frac{1}{s+3}\right]\right] U(s)$

$$Y(s) = \left[3\left[\frac{1}{(s+2)^2}\right] - \frac{2}{s+3}\right] U(s)$$

$$Y(s) = \left[\frac{3}{s^2 + 4s + 4} - \frac{2}{s+3}\right] U(s)$$

$$Y(s) = \left[\frac{3s + 9 - 2s^2 - 8s - 8}{(s+3)(s^2 + 4s + 4)}\right] U(s)$$

$$Y(s) = \left[\frac{-2s^2 - 5s + 1}{s^3 + 7s^2 + 16s + 12}\right] U(s)$$

$$\boxed{\frac{Y(s)}{U(s)} = \frac{-2s^2 - 5s + 1}{s^3 + 7s^2 + 16s + 12}}$$

$$\cancel{G(s) = \frac{Y(s)}{U(s)} \;\&\; U(s) = \frac{1}{s}}$$

$$\cancel{G(s) \cdot \frac{1}{s} = Y(s)}$$

$$\cancel{G(s) = Y(s) \cdot s = \frac{Y(s)}{U(s)}}$$

$$\cancel{\boxed{\frac{Y(s)}{U(s)} = \frac{s(-2s^2 - 5s + 1)}{s^3 + 7s^2 + 16s + 12}}}$$

3. a) $\mathcal{L}^{-1}\left[\frac{5}{s^2+16}\right]$

$= \mathcal{L}^{-1}\left[\frac{5}{4} \cdot \frac{4}{s^2+4^2}\right]$

$= \frac{5}{4}\underbrace{\mathcal{L}^{-1}\left[\frac{4}{s^2+4^2}\right]}_{\sin(4t)}$

$= \boxed{\frac{5}{4}\sin(4t)}$

b) $\mathcal{L}^{-1}\left[\frac{5}{s^2-16}\right]$

$\frac{5}{s^2-16} = \frac{A}{s+4} + \frac{B}{s-4}$

$5 = A(s-4) + B(s+4)$

$5 = As - 4A + Bs + 4B$

$5 = -4A + 4B$ $\qquad 5 = 4B + 4B$

$0 = A + B$ $\qquad B = \frac{5}{8}$

$A = -B$

$\mathcal{L}^{-1}\left\{\frac{-5}{8(s+4)} + \frac{5}{8(s-4)}\right\}$

$-\underbrace{\mathcal{L}^{-1}\left[\frac{5}{8(s+4)}\right] + \left[\frac{5}{8(s-4)}\right]\mathcal{L}^{-1}}_{\frac{5}{8}e^{4t}}$

$= \boxed{\frac{-5}{8}e^{-4t} + \frac{5}{8}e^{4t}}$

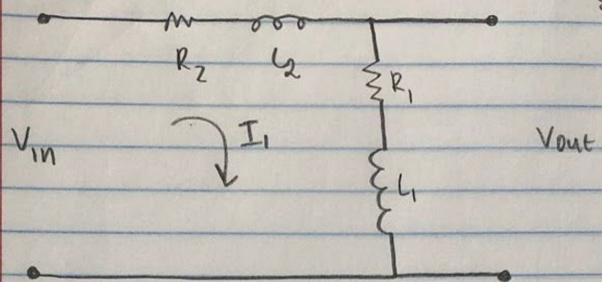
c) $\mathcal{L}^{-1}\left[\frac{5}{s^2+4s+13}\right] = \mathcal{L}^{-1}\left[5 \cdot \frac{1}{(s+2)^2+9}\right] = 5\underbrace{\mathcal{L}^{-1}\left[\frac{1}{(s+2)^2+9}\right]}_{e^{-2t}\frac{1}{3}\sin(3t)}$

$= \boxed{\frac{5}{3}e^{-2t}\sin(3t)}$

4. $R_1 = 1\,\Omega$ $R_2 = 3\,\Omega$ $L_1 = .2H$ $L_2 = .8H$

$V_{out}(0) = 0$ $\dot{V}_{out}(0) = 0$ $V_{in}(t) = 5e^{-5t}$

$$\mathcal{L}[V_{in}(t)] = V(s) = \mathcal{L}[5e^{-5t}] = \frac{5}{s+5}$$



$$\mathcal{L}\left[V_{in} = I_1(R_1+R_2) + (L_1+L_2)\frac{dI_1}{dt}\right]$$

$$\mathcal{L}\left[V_{out} = R_1 I_1 + L_1 \frac{dI_1}{dt}\right]$$

$$V_{in}(s) = 4I_1(s) + 1s\,I_1(s)$$

$$V_{out}(s) = I_1(s) + .2s\,I_1(s)$$

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{.2s+1}{s+4}$$

$$V_{out}(s) = \frac{(.2s+1)}{s+4} \cdot \frac{5}{s+5} = \frac{\cancel{s+5}}{(s+4)\cancel{(s+5)}}$$

$$V_{out}(s) = \frac{1}{s+4}$$

$$\mathcal{L}^{-1}[V_{out}(s)] = \mathcal{L}^{-1}\left[\frac{1}{s+4}\right] = e^{-4t}u(t)$$

$$\boxed{V_{out}(t) = e^{-4t}u(t)}$$