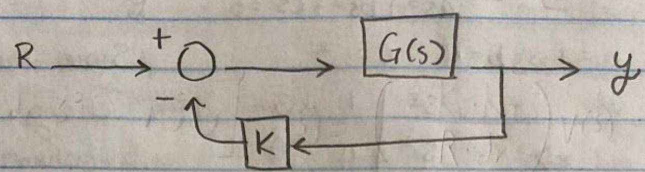
Homework 5

1. $G(s) = \dfrac{s-2}{(s+1)(s^2+6s+24)}$



$$\frac{Y}{R} = \frac{G(s)}{1+K\,G(s)}$$

Characteristic equation : $1 + K G(s) = 0$

$$1 + K\left[\frac{s-2}{(s+1)(s^2+6s+24)}\right]$$

$$(s+1)(s^2+6s+24) + K(s-2) = 0$$

$$s^3 + 6s^2 + 24s + s^2 + 6s + 24 + Ks - 2K = 0$$

$$s^3 + 7s^2 + s[30+K] + 24 - 2K = 0$$

$$\begin{array}{c|cc}
s^3 & 1 & 30+K \\
s^2 & 7 & 24-2K \\
s^1 & \dfrac{7(30+K)-24+2K}{7} & \\
s^0 & 24-2K &
\end{array}$$

Stable system $s^1 > 0$ $\quad s^0 > 0$

$7(30+K) - 24 + 2K > 0$	$24 - 2K > 0$
$9K > -186$	$-2K > -24$
$K > -20.6$	$K < 12$

Range of K: $\boxed{-20.6 < K < 12}$

2. System 1:

A) $G(s) = \frac{1}{s(s+1)(s+3)}$

Characteristic equation: $(s^3 + 4s^2 + 3s) = 0$

$as^3 + bs^2 + cs = 0$

$a = 1 \quad b = 4 \quad c = 3$

$$\begin{array}{c|cc} s^3 & a & c \\ s^2 & b & 0 \\ s^1 & d & 0 \\ s^0 & e & \end{array} \Rightarrow \begin{array}{c|cc} s^3 & 1 & 3 \\ s^2 & 4 & 0 \\ s^1 & 3 & 0 \\ s^0 & 0 & \end{array}$$

Since there is no sign change in the 1st column, the system is $\boxed{\text{stable}}$

$d = \frac{4(3) - 1(0)}{4} = 3$

$e = \frac{(d)0 - 4(0)}{d} = \frac{(3)0 - 4(0)}{3} = 0$

B) steady-state error:

unit step input

$e_{ss} = \frac{A}{1+k_p}$ ★ A is always 1

$k_p = \lim_{s\to 0} G(s) = \lim_{s\to 0} \left[\frac{1}{s^3+4s^2+3s}\right] = \frac{1}{0} = \infty$

$e_{ss} = \frac{1}{1+\infty} = \boxed{0}$

Unit ramp input

$e_{ss} = \frac{A}{k_v}$ ★ A is always 1

$k_v = \lim_{s\to 0} s \cdot G(s) = \lim_{s\to 0} s \left[\frac{1}{s^3+4s^2+3s}\right]$

$k_v = \lim_{s\to 0} \cancel{s} \cdot \frac{1}{\cancel{s}} \cdot \frac{1}{s^2+4s+3} = \frac{1}{3}$

$e_{ss} = \frac{1}{1/3} = \boxed{3}$

System 2:

A) $G(s) = \frac{s+5}{s^2-7s+12}$ Char equ: $s^2 - 7s + 12 = 0$

$$\begin{array}{c|cc} s^2 & 1 & 12 \\ s^1 & -7 & \\ s^0 & 12 & \end{array}$$

$\frac{-7(12) - 0}{-7} = 12$

$\boxed{\text{system is not stable}}$

Homework 5

2 Cont.

System 2:

B) Unit Step Input:

unit step input:

$$e_{ss} = \frac{A}{1+k_p}$$

$$k_p = \lim_{s\to 0}\left[\frac{s+5}{s^2+7s+12}\right] = \frac{5}{12}$$

$$e_{ss} = \frac{1}{1+\frac{5}{12}} = \boxed{.7058}$$

Unit ramp input:

$$e_{ss} = \frac{A}{k_v}$$

$$k_v = \lim_{s\to 0} s\left[\frac{s+5}{s^2+7s+12}\right] = 0$$

$$e_{ss} = \frac{1}{0} = \boxed{\infty}$$

3. system 1:

OLTF: $G(s) = \dfrac{5}{s(5s+1)}$

CLTF: $\dfrac{C(s)}{R(s)} = \dfrac{G(s)}{1+G(s)H(s)}$

$$= \frac{\frac{5}{s(5s+1)}}{1+\frac{5}{s(5s+1)}[1]}$$

$$= \frac{5}{5s^2+s+5}$$

System 2:

OLTF: $G(s) = \dfrac{5(1+.8s)}{s(5s+1)}$

CLTF: $\dfrac{G(s)}{R(s)} = \dfrac{G(s)}{1+G(s)H(s)}$

$$= \frac{\frac{5(1+.8s)}{s(5s+1)}}{1+\frac{5(1+.8s)}{s(5s+1)}}$$

$$= \frac{.8s+1}{s^2+s+1}$$

3 cont.

System 3:

OLTF:

Inner loop $= G_1(s) = \dfrac{\dfrac{5}{5s+1}}{1+\left[\dfrac{5}{s+1}\right](.8)} = \dfrac{1}{s+1}$

$G(s) = \dfrac{1}{s+1}\left[\dfrac{1}{s}\right] = \dfrac{1}{s(s+1)}$

CLTF: $\dfrac{C_{III}(s)}{R(s)} = \dfrac{G(s)}{1+G(s)H(s)}$

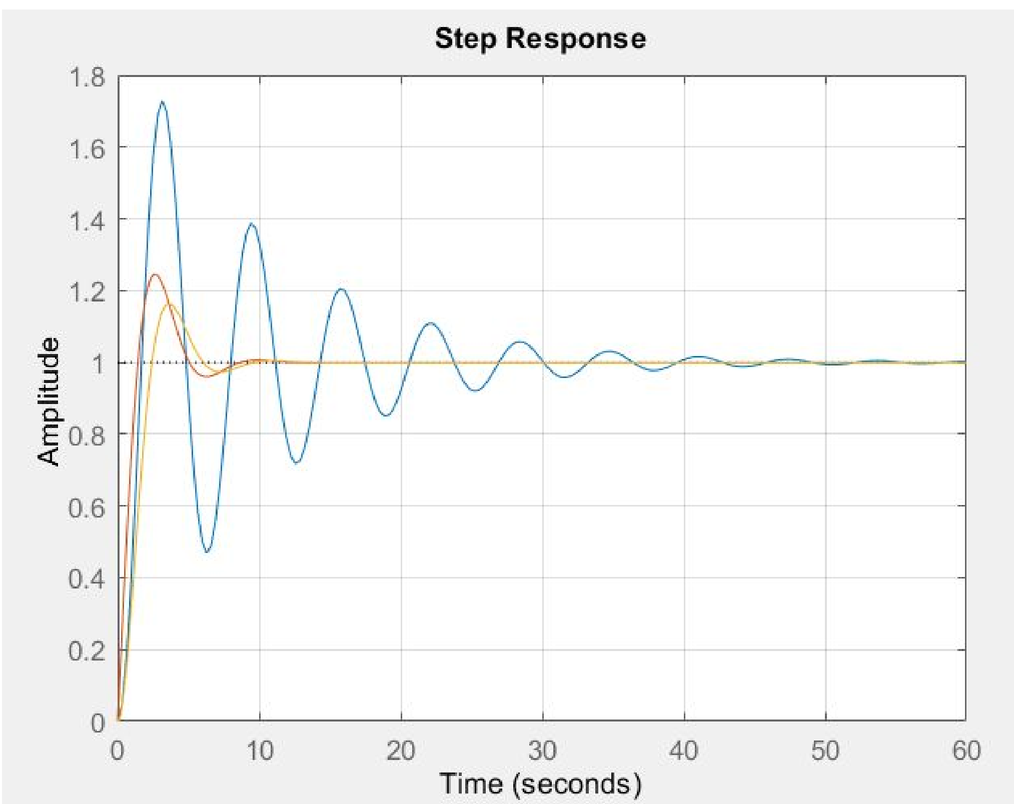
$= \dfrac{\dfrac{1}{s(s+1)}}{1+\dfrac{1}{s(s+1)}(1)}$

$= \dfrac{1}{s^2+s+1}$

3 cont.

Unit step responses:

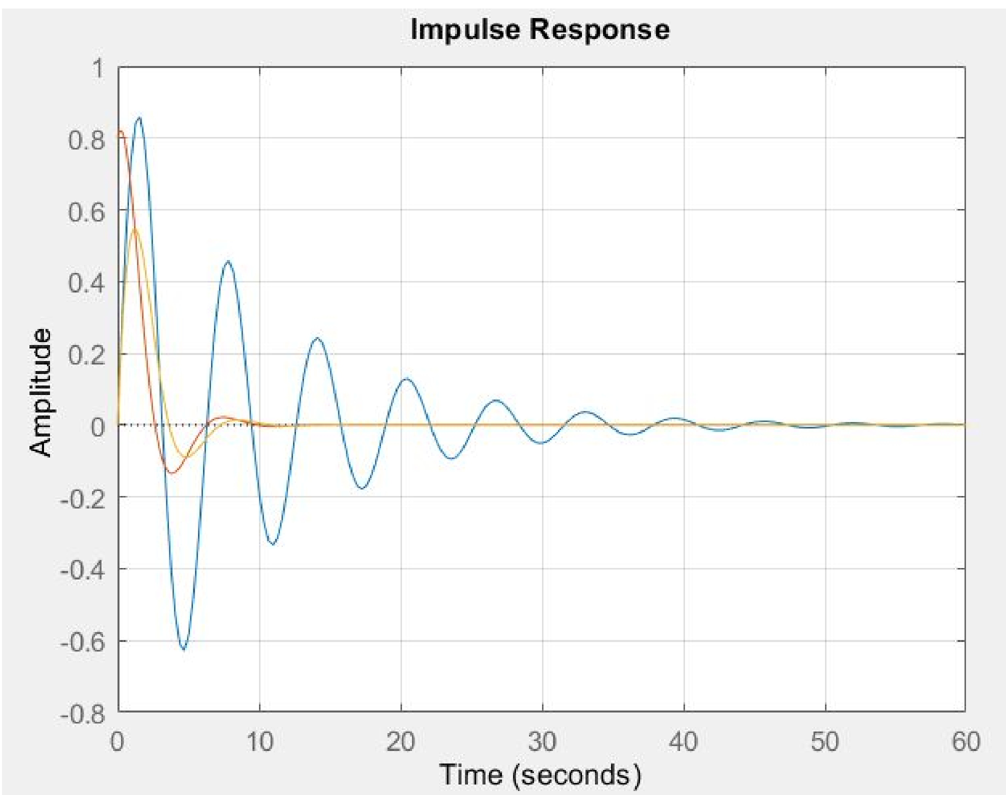
```
>> num = 5;
den = [5 1 5];
sys=tf(num,den);
step(sys)
hold on
num = [.8 1];
den = [1 1 1];
sys=tf(num,den);
step(sys)
num =1;
den = [1 1 1]; sys=tf(num,den);
step(sys)
grid
```



System three has a maximum overshoot less than that of the other systems and reaches a steady-state in less time than the other systems, making it the best.

Impulse Response:

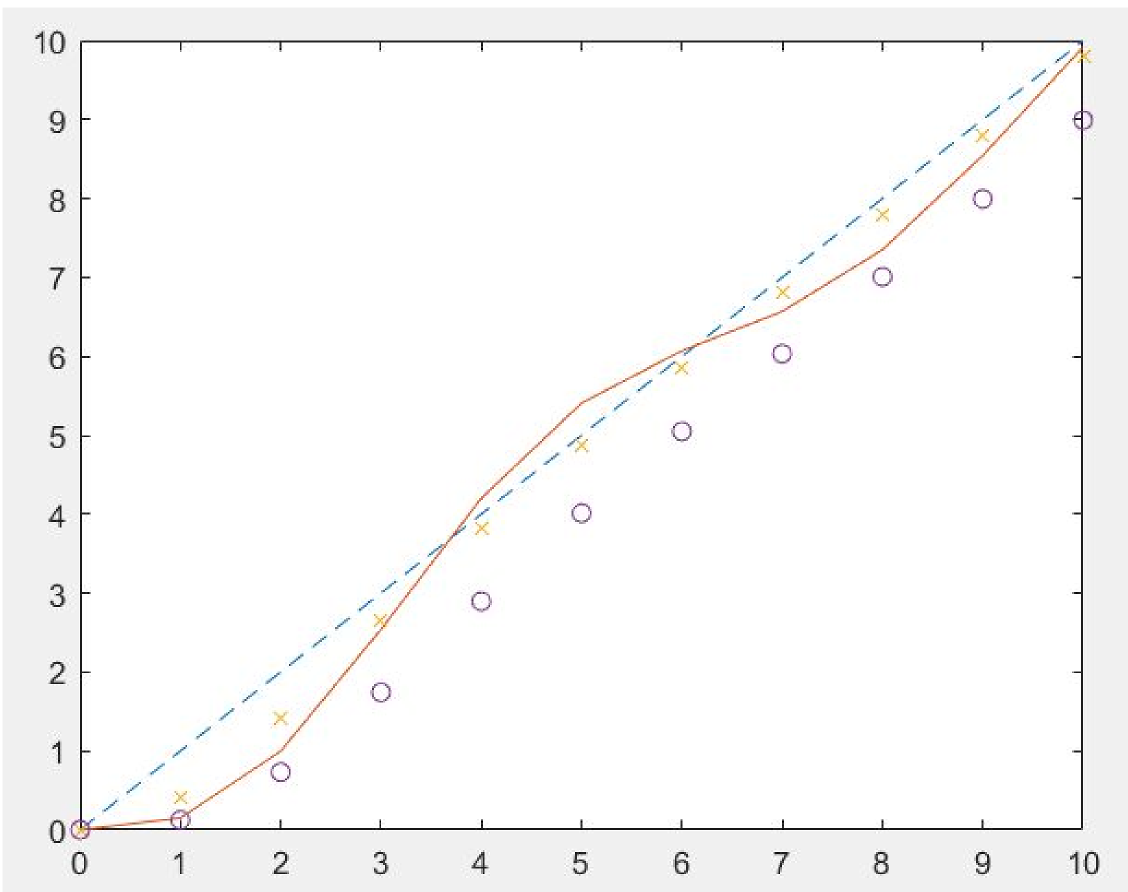
```
>> num = 5;
den = [5 1 5];
sys=tf(num,den);
impulse(sys)
hold on
num = [.8 1];
den = [1 1 1];
sys=tf(num,den);
impulse(sys)
num =1;
den = [1 1 1]; sys=tf(num,den);
impulse(sys)
grid
```



System three is the best, as the maximum overshoot is less compared to the other systems and reaches the steady-state faster than the other systems.

Unit Ramp Responses:

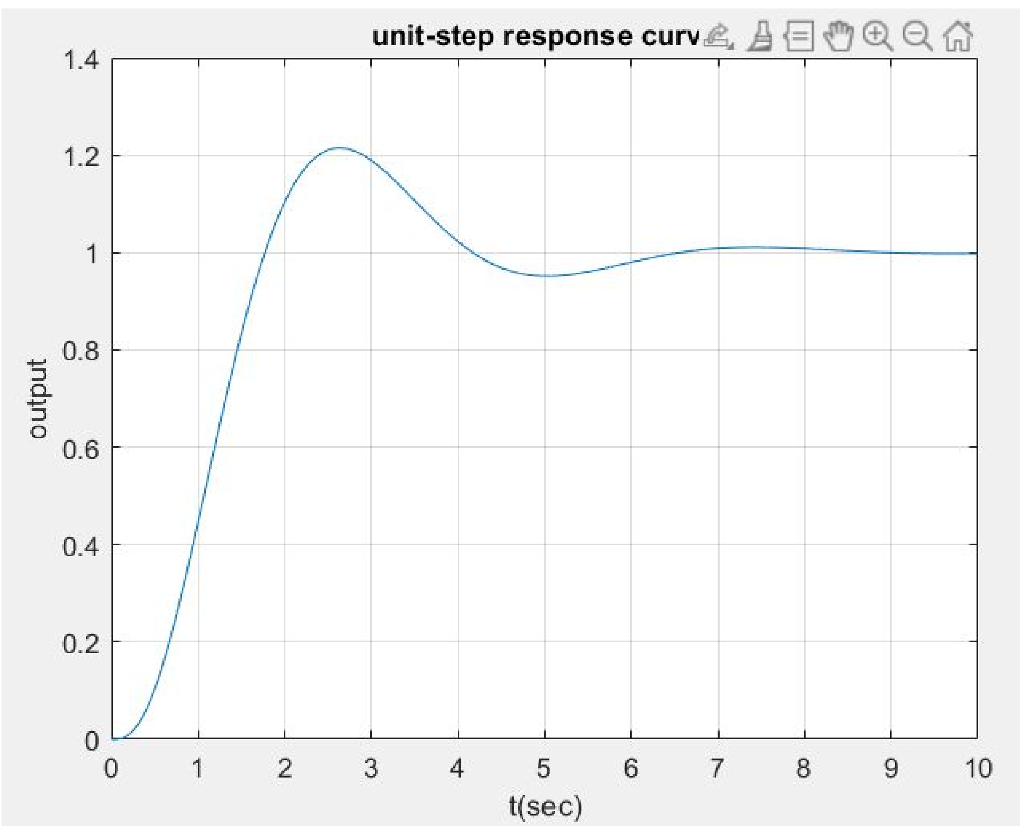
```
>> num1r = [0 0 0 1];
den1r = [1 .2 1 0];
num2r = [0 0 0.8 1];
den2r = [1 1 1 0];
num3r = [0 0 0 1];
den3r = [1 1 1 0];
t = 0:1:10;
y1 = step(num1r,den1r,t);
y2 = step(num2r,den2r,t);
y3 = step(num3r,den3r,t);
plot(t,t,'--',t,y1,'-',t,y2,'x',t,y3,'o')
```



Looking at the graphs, system 1 is the red curve, system 2 is depicted with the yellow symbol '+', and system 3 is shown by the purple symbol 'o'. System three is the best in terms of the speed of the response and the maximum overshoot.

4.

```
>>  num = [0 0 0 10];
den = [1 6 8 10];
t = 0:0.002:10;
[y,x,t] = step(num,den,t);
plot(t,y)
grid
title('unit-step response curve')
xlabel('t(sec)')
ylabel('output')
```



```
>> r=1; while y(r)<1.0001; r=r+1; end
>> rise_time = (r-1)*0.002

rise_time =

    1.7720

>> [ymax,tp] = max(y);
>> peak_time = (tp-1)*.002

peak_time =

    2.6320

>> max_overshoot=ymax-1

max_overshoot =

    0.2146

>> s = 5001; while y(s)>.98 & y(s)<1.02; s=s-1; end;
>> settling_time = (s-1)*.002

settling_time =

    5.9960
```