Part 1:

<u>Mass Spring Damper</u>

1. $F(t) - b\dot{x} - kx = m\ddot{x}$

$$\mathcal{L}[F(t)] = [b\dot{x} + kx + m\ddot{x}]\mathcal{L}$$

$$F(s) = Ms^2x(s) + bsx(s) + kx(s)$$

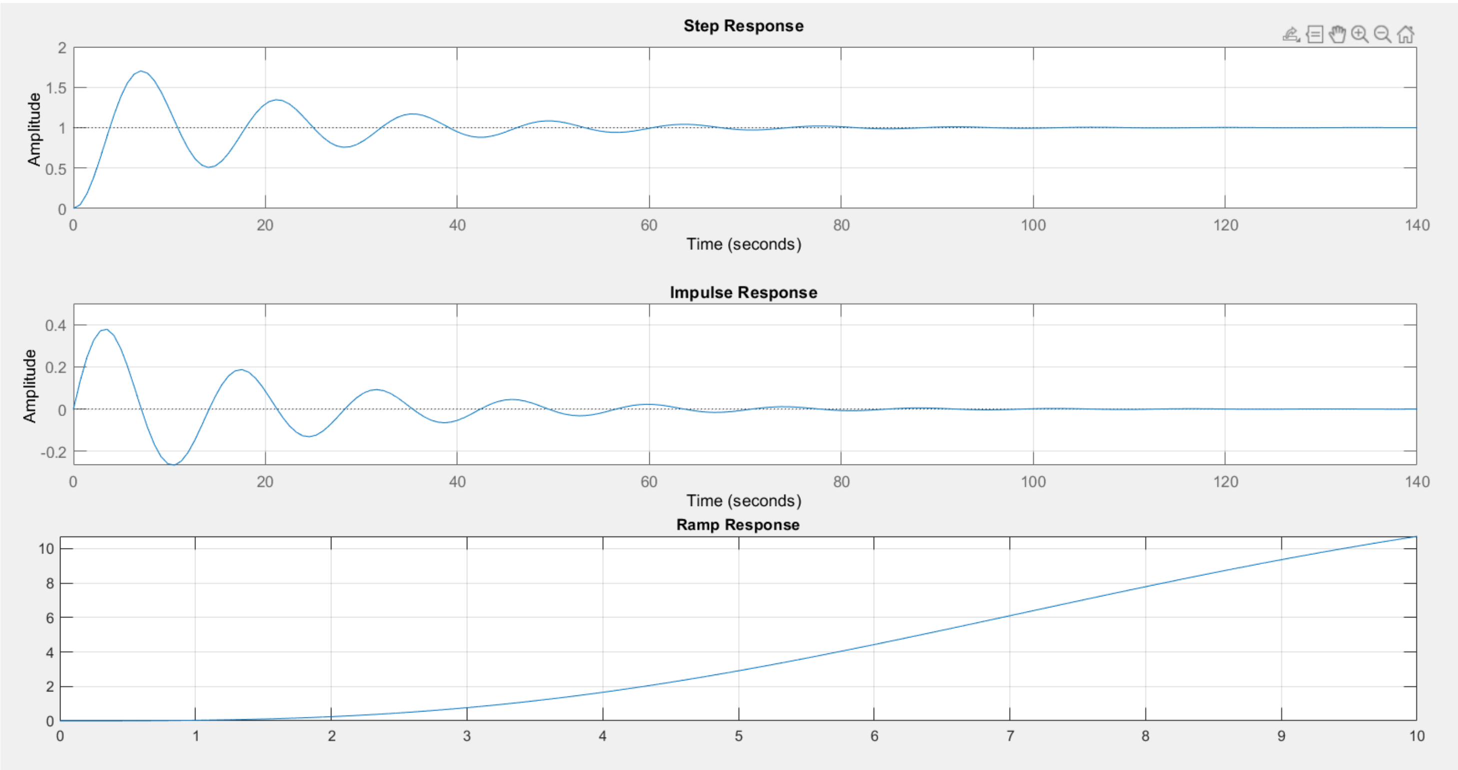
$$F(s) = X(s)[Ms^2 + bs + k]$$

Transfer Function: $\frac{X(s)}{F(s)} = \frac{1}{Ms^2+bs+k} = \frac{1}{5s^2+.5s+1}$

$$\boxed{X(s) = \frac{2}{5s^2+.5s+1}}$$

2.

```
>> numerator = 0.2;
denominator = [1,0.1,0.2];
sys = tf(numerator,denominator);
subplot(3,1,1),step(sys),grid on;
subplot(3,1,2),impulse(sys),grid on;
t=0:0.001:10;
u=t;
[y,x]=lsim(sys,u,t);
subplot(3,1,3),plot(t,y),grid on,title('Ramp Response');
disp(stepinfo(sys));
```



3.

```
     RiseTime: 2.5448
 SettlingTime: 78.1524
  SettlingMin: 0.5072
  SettlingMax: 1.7021
    Overshoot: 70.2118
   Undershoot: 0
         Peak: 1.7021
     PeakTime: 7.0248
```

4. $\frac{X(s)}{F(s)} = \frac{1}{5s^2+.5s+1} = \frac{.2}{s^2+.1s+.2}$

$s^2 + 2\xi_e \omega_n s + \omega_n^2$

$\omega_n^2 = .2 \Rightarrow \omega_n = .4472$ rad/sec

$2\xi_e\omega_n = .1$

$\xi_e = \frac{.1}{2(.2)} = .25$

Peak Response $= \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\xi_e^2}} = \frac{\pi}{.432} = \boxed{7.2 \text{ sec.}}$

Setting time: $T = \frac{1}{\xi_e \omega_n} = \frac{1}{.1118} = 8.94$

for 2% error:

$t_s = 4T = 4(8.94) = \boxed{35.76 \text{ sec}}$

Rise time $= \frac{\pi - \theta}{\omega_d} = \frac{\pi - 1.31}{.432} = \boxed{4.32 \text{ sec}}$

$\theta = \tan^{-1}\left[\frac{\sqrt{1-\xi_e^2}}{\xi_e}\right] = \tan^{-1}\left[\frac{\sqrt{1-.25^2}}{.25}\right] = 1.31$

Maximum overshoot $= e^{-\xi_e\pi/\sqrt{1-\xi_e^2}}$

$= e^{-.78/.96}$

$= .443$

$\boxed{44.3\%}$

Motor Posnon

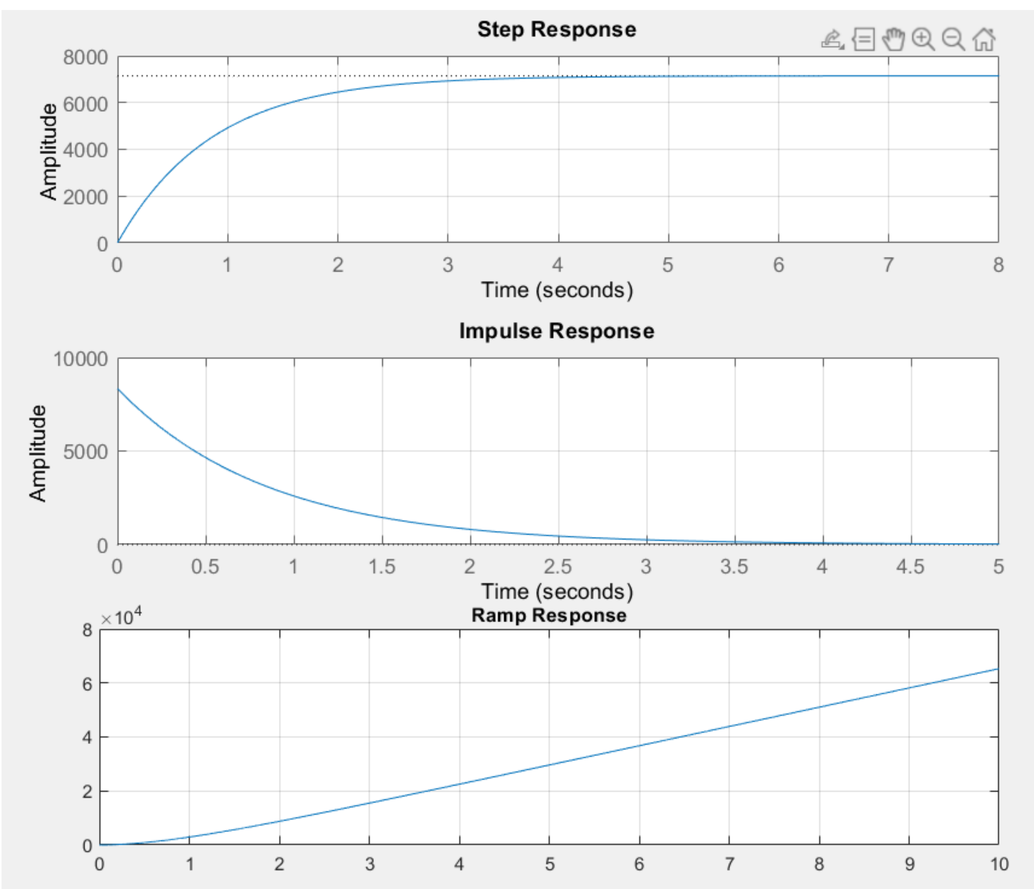
1. $\mathcal{L}[J\ddot{\theta} + b\dot{\theta}] = [Ki]\mathcal{L}$ $\quad$ $\mathcal{L}[L\frac{di}{dt} + Ri] = [V - K\dot{\theta}]\mathcal{L}$

$Js^2\Theta(s) + bs\Theta(s) = KI(s)$ $\quad$ $LsI(s) + RI(s) = V - K\theta(s)$

Transfer function $= \frac{\theta(s)}{I(s)} = \frac{K}{Js^2+bs} = \boxed{\frac{.025}{3\times10^{-6}s^2 + 3.5\times10^{-6}s}}$

$\frac{\theta(s)\,[Js^2+bs]}{K} = I(s) \Rightarrow \frac{\theta(s)}{I(s)} = \frac{K}{Js^2+bs}$

2.



3.

```
>>  numerator = 0.025;
denominator = [3*10^-6,3.5*10^-6];
sys = tf(numerator,denominator);
subplot(3,1,1),step(sys),grid on;
subplot(3,1,2),impulse(sys),grid on;
t=0:0.001:10;
u=t;
[y,x]=lsim(sys,u,t);
subplot(3,1,3),plot(t,y),grid on,title('Ramp Response');
disp(stepinfo(sys));
        RiseTime: 1.8831
    SettlingTime: 3.3532
     SettlingMin: 6.4607e+03
     SettlingMax: 7.1427e+03
       Overshoot: 0
      Undershoot: 0
            Peak: 7.1427e+03
        PeakTime: 9.0393
```

Part 2:

1. 5.1:

$$\frac{I_x \ddot{\theta}_x d_r}{d_g} = \frac{k_T}{R}\left[V_R + V_L - k_v\frac{(2v_y)}{d_r}\right] + mg d_r \theta_x$$

$$M_x \ddot{\theta}_x = G_T\left[V_+ - G_v(2v_y)\right] + T_g \theta_x$$

5.2:

$$d_r m \dot{v}_y = \frac{k_T}{R}\left[V_L + V_R - k_v\frac{(2v_y)}{d_r}\right]$$

$$M_y \dot{v}_y = G_T\left[V_+ - G_v(2v_y)\right]$$

5.3:

$$\frac{d_r(I_z \dot{w}_z)}{d_w} = \frac{k_T}{R}\left[V_R - V_L - k_v\frac{(2w_z d_w)}{d_r}\right]$$

$$\frac{d_r I_z \dot{w}_z}{d_w} = \frac{k_T}{R}\left[V_R - V_L - k_v\frac{(2w_z d_w)}{d_r}\right]$$

$$M_z \dot{w}_z = G_T\left[V_- - G_w(2w_z)\right]$$

Part 2

2. $M_x \ddot{\theta}_b = T_g \theta_b - 2G_T G_V V_y + G_T V$

$M_x \ddot{\theta}_b - T_g \theta_b = G_T(-2G_V V_y + V)$

$\frac{I_x d_r}{d_g} \ddot{\theta}_b - m g d_r \theta_b = \frac{k_T}{R}\left(-2\left(\frac{k_v}{d_r}\right) V_y + V\right)$

$\frac{.00215(.034)}{.062} \ddot{\theta}_b - .955(9.81)(.034)\theta_b = \frac{.2025}{5.4}\left(-2\frac{(.3253)}{.034} V_y + V\right)$

$.001179 \ddot{\theta}_b - .31853\theta_b = .0375 V - .71757 V_y$

$x_1 = \theta_b$ $\qquad \dot{x}_1 = \dot{\theta}_b = x_2$

$x_2 = \dot{\theta}_b \Rightarrow \dot{x}_2 = \ddot{\theta}_b$

$x_3 = V_y$

$\dot{x}_2 = \frac{.31853\theta_b + .0375 V - .71757 V_y}{.001179} = 270.17x_1 + 31.81 V - 608.63x_3$

$M_y \dot{V_y} = -2G_T G_V V_y + G_T V$

$m d_r \dot{V_y} = -2\left(\frac{k_T}{R}\right)\left(\frac{k_v}{d_r}\right) V_y + \frac{k_T}{R} V$

$.955(.034)\dot{V_y} = -2\frac{(.2025)}{5.4}\frac{(.3253)}{.034} V_y + \frac{.2025}{5.4} V$

$.03247 \dot{V_y} = -.71757 V_y + .0375 V$

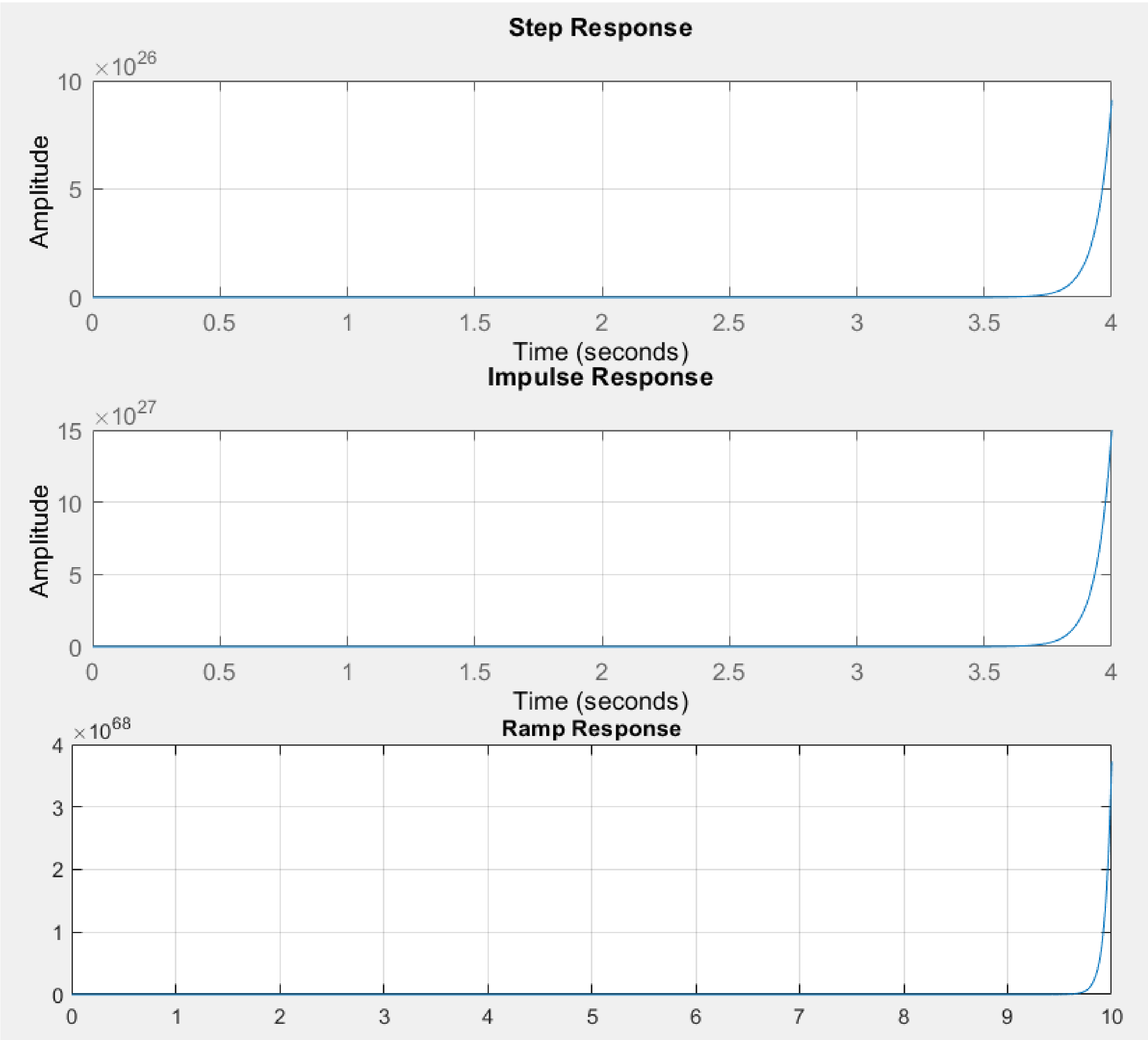
$\dot{x}_3 = \frac{-.71757x_3 + .0375 V}{.03247} = -22.10x_3 + 1.15V$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 270.17 & 0 & -608.63 \\ 0 & 0 & -22.10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 31.81 \\ 1.15 \end{bmatrix} V$$

$$\theta = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + [0] V$$

3.

```
>>  A=[0 1 0; 270.17 0 -608.63; 0 0 -22.10];
B=[0; 31.81; 1.15];  C=[1 0 0];  D=0;
SYS=ss(A,B,C,D);
[n,d]=tfdata(SYS,'v')

n =

         0         0   31.8100    3.0765



d =

   1.0e+03 *

    0.0010    0.0221   -0.2702   -5.9708

>> sys = tf(n,d);
subplot(3,1,1),step(sys),grid on;
subplot(3,1,2),impulse(sys),grid on;
t=0:0.001:10;
u=t;
[y,x]=lsim(sys,u,t);
subplot(3,1,3),plot(t,y),grid on,title('Ramp Response');
disp(stepinfo(sys));
        RiseTime: NaN
    SettlingTime: NaN
     SettlingMin: NaN
     SettlingMax: NaN
       Overshoot: NaN
      Undershoot: NaN
            Peak: Inf
        PeakTime: Inf
```



Intro: For the lab we had to simplify the equations we had regarding the balbot. Using the given values/variables, we simplified our equations relating voltages, pitch angle, and linear velocity and derived the corresponding state space. We then determined values for max overshoot, peak time, settling time, and rise time when subjected to unit impulse, unit step, and unit ramp functions.

Conclusion:
From this lab we learned how to perform transient response analyses on basic dynamical systems and perform transient response analyses for real robotic systems (the BalBot). We were able to build upon the knowledge gained from the previous labs to graphically represent our robot system.