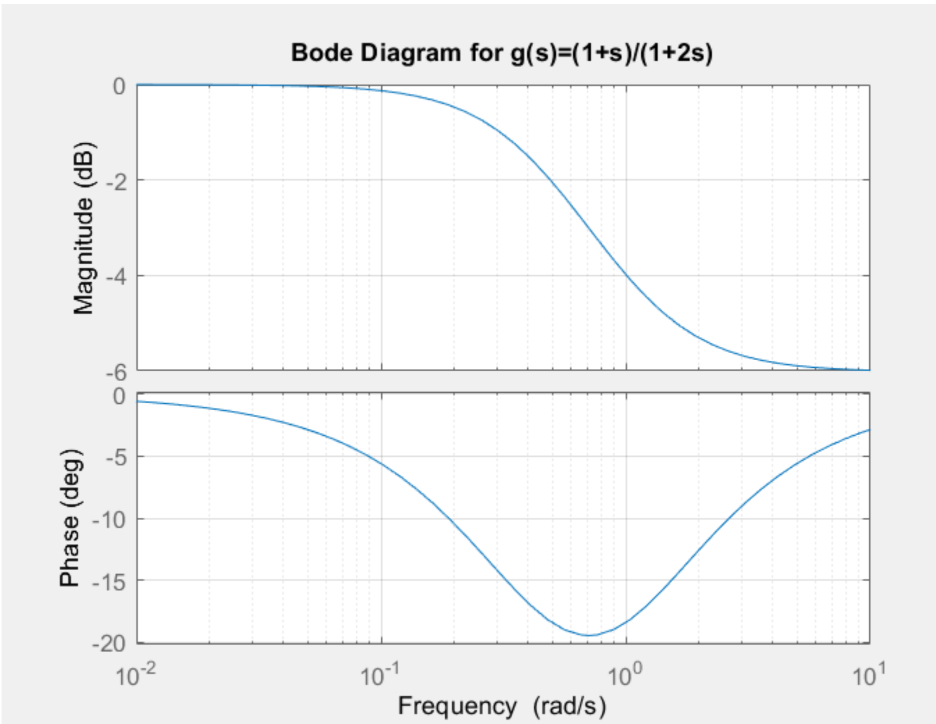
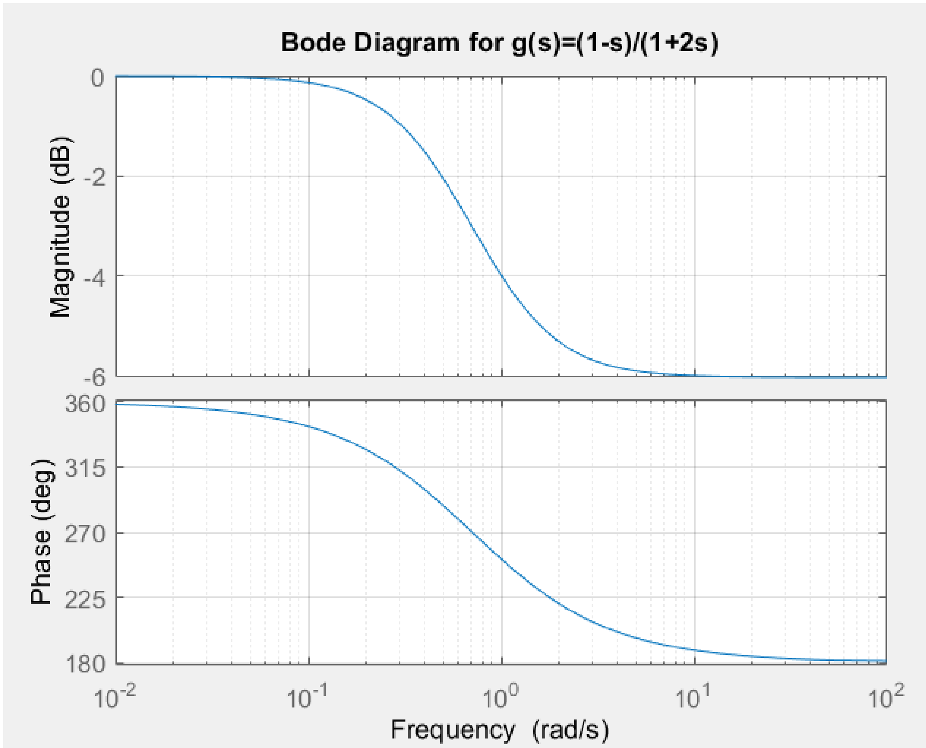
2. (note problem 1 done last)

```
>> num = [1 1];
>> den = [2 1];
>> bode(num, den)
grid
>> title('Bode Diagram for g(s)=(1+s)/(1+2s)')
```



```
>> num = [-1 1];
den = [2 1];
bode(num, den)
grid
title('Bode Diagram for g(s)=(1-s)/(1+2s)')
```


Bode Diagram for g(s)=(1-s)/(1+2s)
Magnitude (dB)
0
-2
-4
-6
Phase (deg)
360
315
270
225
180
10-2
10-1
100
101
102
Frequency (rad/s)

3.

$$G(s) = \frac{10(s^2+.4s+1)}{s(s^2+.8s+9)}$$

$s^2+.4s+1$ $\qquad$ $s^2+2\xi\omega_n s+\omega_n^2$

$$\left.\begin{array}{l} \underline{\omega_n = 1} \\ \\ s^2+.8s+9 \\ \underline{\omega_n = 3} \end{array}\right\} \text{corner frequency}$$

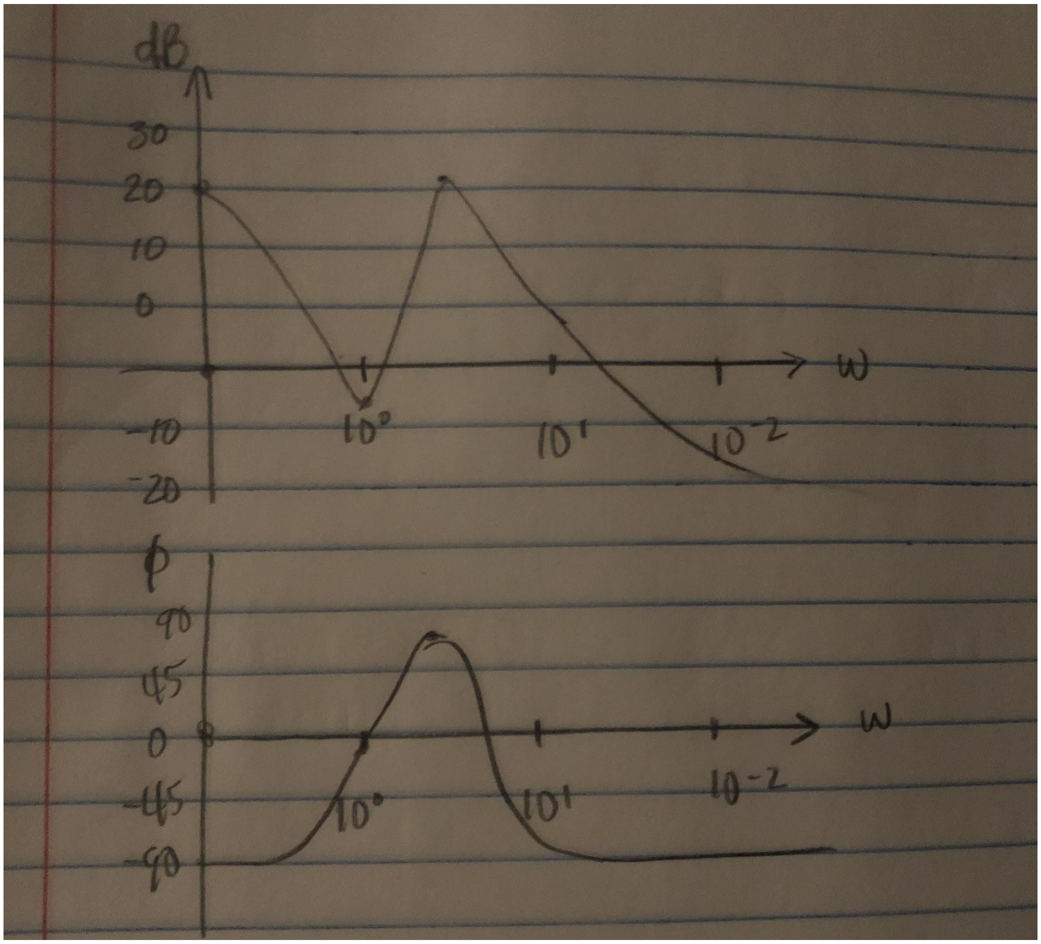
$$G(j\omega) = \frac{10\left((j\omega)^2+.4(j\omega)+1\right)}{(j\omega)(j\omega)^2+.8j\omega+9} = \frac{10(-\omega^2+j(.4)\omega+1)}{j\omega[-\omega^2+j(.8)\omega+9]}$$

Term	corner freq	slope	change in slope
$10/s$		-20	
$s^2+.4s+1$	$\omega_n = 1$	20	0
$\frac{1}{s^2+.8s+9}$	$\omega_n = 3$	-20	-20

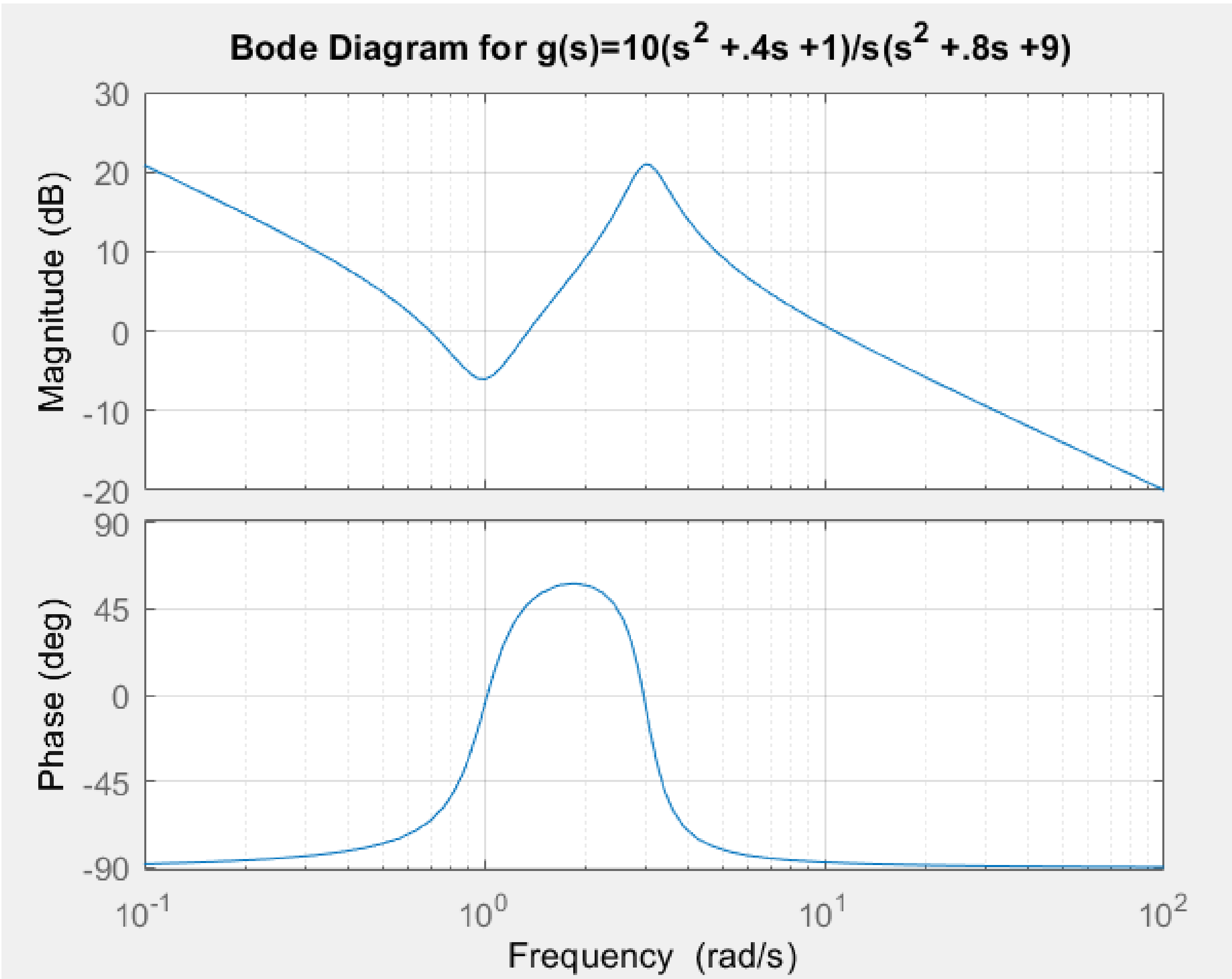
$$\angle G(j\omega) = \frac{0 + \tan^{-1}\left(\frac{.4\omega}{1-\omega^2}\right)}{90 + \tan^{-1}\left(\frac{.8\omega}{8-\omega^2}\right)}$$

$$\angle\phi = -90 - \tan^{-1}\left(\frac{.8\omega}{8-\omega^2}\right) + \tan^{-1}\left(\frac{.4\omega}{1-\omega^2}\right)$$

ω	$\tan^{-1}\left(\frac{.8\omega}{1-\omega^2}\right)$	$\tan^{-1}\left(\frac{.4\omega}{1-\omega^2}\right)$	$\angle\phi$
.1	4.61	2.313	-92.287
.2	9.46	4.76	-94.7
.3	14.77	7.56	-87.21
.4	20.85	10.78	-100.07




```
>>  num = [0 10 4 10];
den = [1 .8 9 0];
bode(num, den)
grid
title('Bode Diagram for g(s)=10(s^2 +.4s +1)/s(s^2 +.8s +9)')
```


Bode Diagram for g(s)=10(s2 +.4s +1)/s(s2 +.8s +9)
Magnitude (dB)
30
20
10
0
-10
-20
Phase (deg)
90
45
0
-45
-90
10-1
100
101
102
Frequency (rad/s)

Homework 6

4. $$G(s) = \frac{\omega_n^2}{s^2 + 2\xi_\ell \omega_n s + \omega_n^2}$$

$$G(j\omega) = \frac{\omega_n^2}{(j\omega)^2 + 2\xi_\ell \omega_n (j\omega) + \omega_n^2}$$

$$G(j\omega) = \frac{\omega_n^2}{-\omega^2 + 2j\omega \xi_\ell \omega_n + \omega_n^2}$$

$$= \frac{1}{\frac{-\omega^2}{\omega_n^2} + \frac{2j\omega \xi_\ell}{\omega_n} + 1}$$

$$G(j\omega)\Big|_{\omega = \omega_n} = \frac{1}{\frac{-\cancel{\omega_n^2}}{\cancel{\omega_n^2}} + \frac{2j\cancel{\omega_n}\xi_\ell}{\cancel{\omega_n}} + 1}$$

$$= \frac{1}{\left| \cancel{-1} + 2j\xi_\ell + \cancel{1} \right|}$$

$$= \frac{1}{2\xi_\ell}$$

1. a)

a) transfer function:

$$G(s) = \frac{10(s+1)}{(s+4)(s^2+6s+81)} = \frac{10s+10}{s^3+10s^2+105s+324}$$

Sinusoidal input $V_{sin/m}(t) = A\sin(w_0 t)$

gain @ output side for sinusoidal input response

$$\left.\frac{(10)\sqrt{1+w^2}}{\sqrt{4+w^2}\cdot\sqrt{(81-w^2)^2+(6w)^2}}\right|_{w=w_0} \quad \text{when } A=1$$

phase shift for sinusoidal input responses:

for $w = w_0$

$$G(jw) = \frac{10(jw+1)}{(jw+4)(6jw-w^2+81)}$$

$$G(jw) = \frac{10(1+jw)}{(4+jw)((81-w^2)+6jw)}$$

for $81 - w^2 > 0$

$w^2 < 81$

$w < 9$

$$\boxed{\phi = \angle G(jw) = \cancel{\tan^{-1}(-w)} + \tan^{-1}\left(-\frac{w}{4}\right) - \tan^{-1}\left(\frac{-6w}{81-w^2}\right)}$$

for $w \geq 9$

$$\cancel{\phi = \angle G(jw) = \tan^{-1}(-w) - \tan^{-1}\left(-\frac{w}{4}\right) + \tan^{-1}\left(\frac{-6w}{w^2-81}\right) - 180}$$

$$G(s) = \frac{10(s+1)}{\frac{4(s+4)}{4}\,\frac{(s^2+6s+81)}{81}\,81} = \frac{\frac{10}{4(81)}(s+1)}{\left(\frac{s}{4}+1\right)\left(\frac{s^2}{81}+\frac{6s}{81}+1\right)} = \frac{\frac{10}{324}(s+1)}{\left(\frac{s}{4}+1\right)\left(\frac{s^2}{81}+\frac{6s}{81}+1\right)}$$

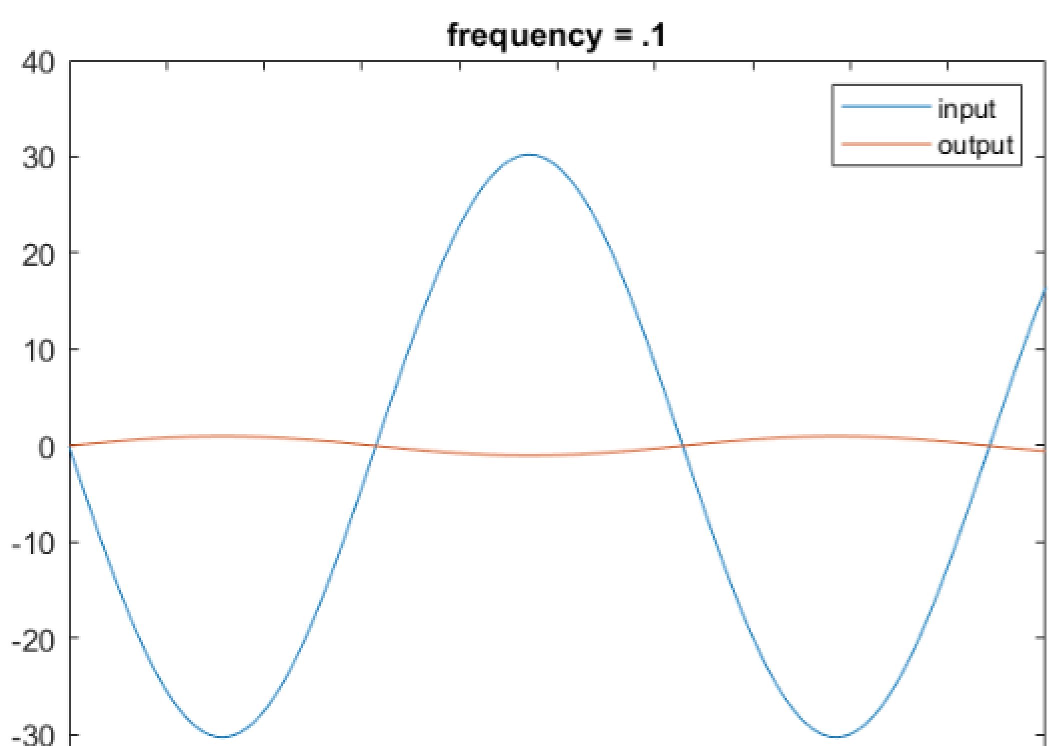
$$\cancel{|G(jw)| = 20\log\left(\frac{10}{324}\right) + 20\log\left(\sqrt{1^2+w^2}\right) - 20\log\left(\sqrt{1^2+\left(\frac{w}{4}\right)^2}\right) - 20\log\left(\sqrt{\left(1-\left(\frac{w}{9}\right)^2\right)^2+\left(\frac{6w}{9}\right)^2}\right)}$$

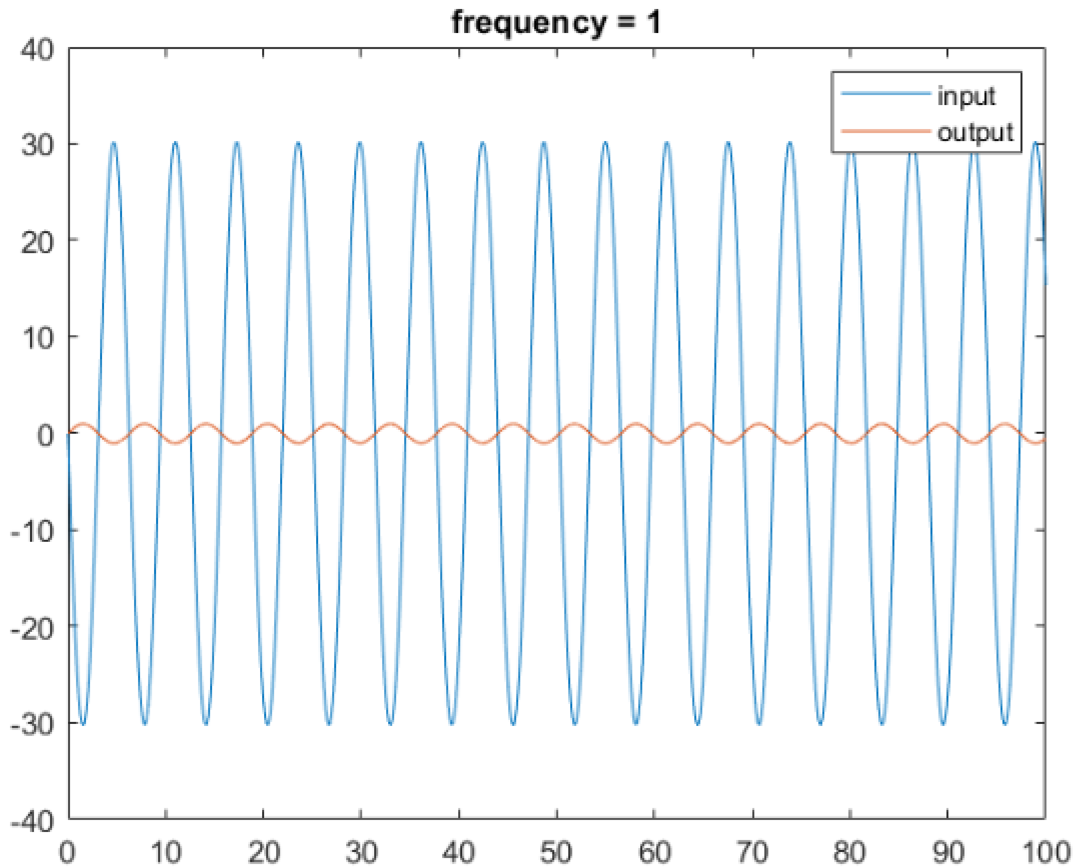
$$\boxed{|G(jw)| = 20\log\left(\frac{10}{324}\right) + 10\log(1+w^2) - 10\log\left(1+\left(\frac{w}{4}\right)^2\right) - 10\log\left(\left(1-\left(\frac{w}{9}\right)^2\right)^2+\left(\frac{6w}{9}\right)^2\right)}$$

$$y_{ss} = A\,|G(jw)|\sin(wT + \angle G(jw))$$

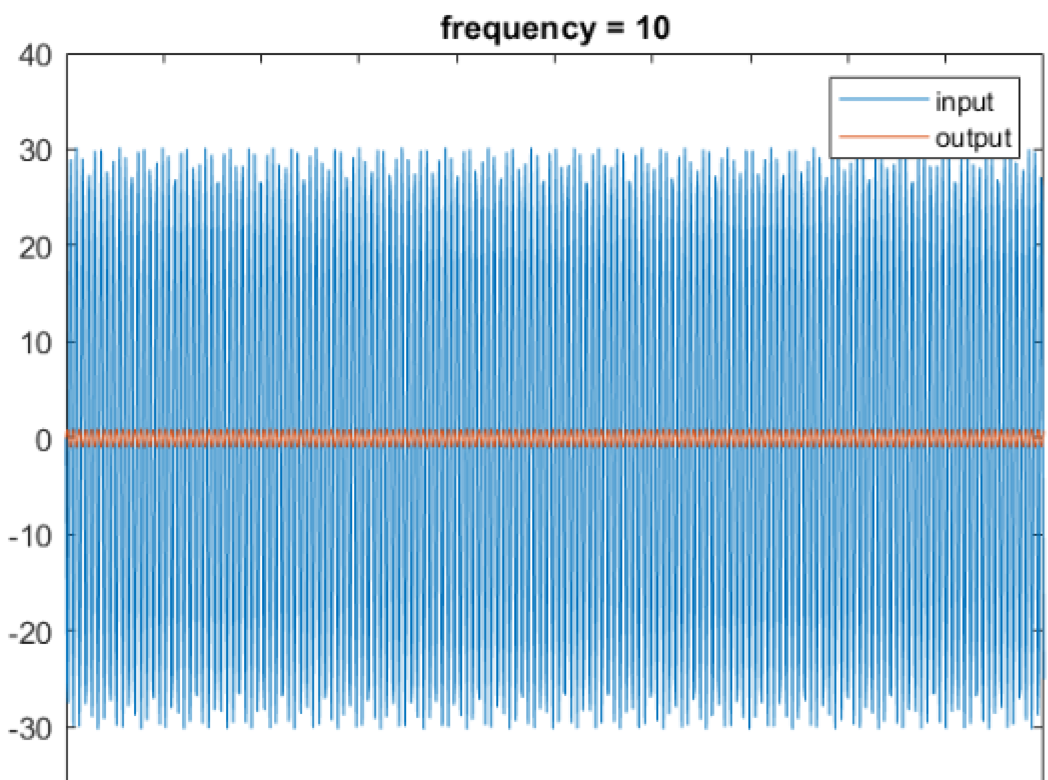
b)

```
t = 0:.1:100;

w = 0;

gain = 20*log10(10/324)+10*log10(1+w^2)-10*log10(1+(w/4)^2)-10*log10((1-(w/9)^2)^2 +(6*w/9)^2)

phase = atan(w/1)-atan(.25*w/1)-atan((6*w/9)/(1-(w/9)^2))

w = .1;
u = sin(w*t);
Yss = gain*sin(w*t+phase);
plot(t,Yss, t, u)
title('frequency = .1')
legend('input', 'output')

w = 1;
u = sin(w*t);
Yss = gain*sin(w*t+phase);
plot(t,Yss, t, u)
title('frequency = 1')
legend('input', 'output')

w = 10;
u = sin(w*t);
Yss = gain*sin(w*t+phase);
plot(t,Yss, t, u)
title('frequency = .1')
legend('input', 'output')

w = 100;
u = sin(w*t);
Yss = gain*sin(w*t+phase);
plot(t,Yss, t, u)
title('frequency = .1')
legend('input', 'output')
```

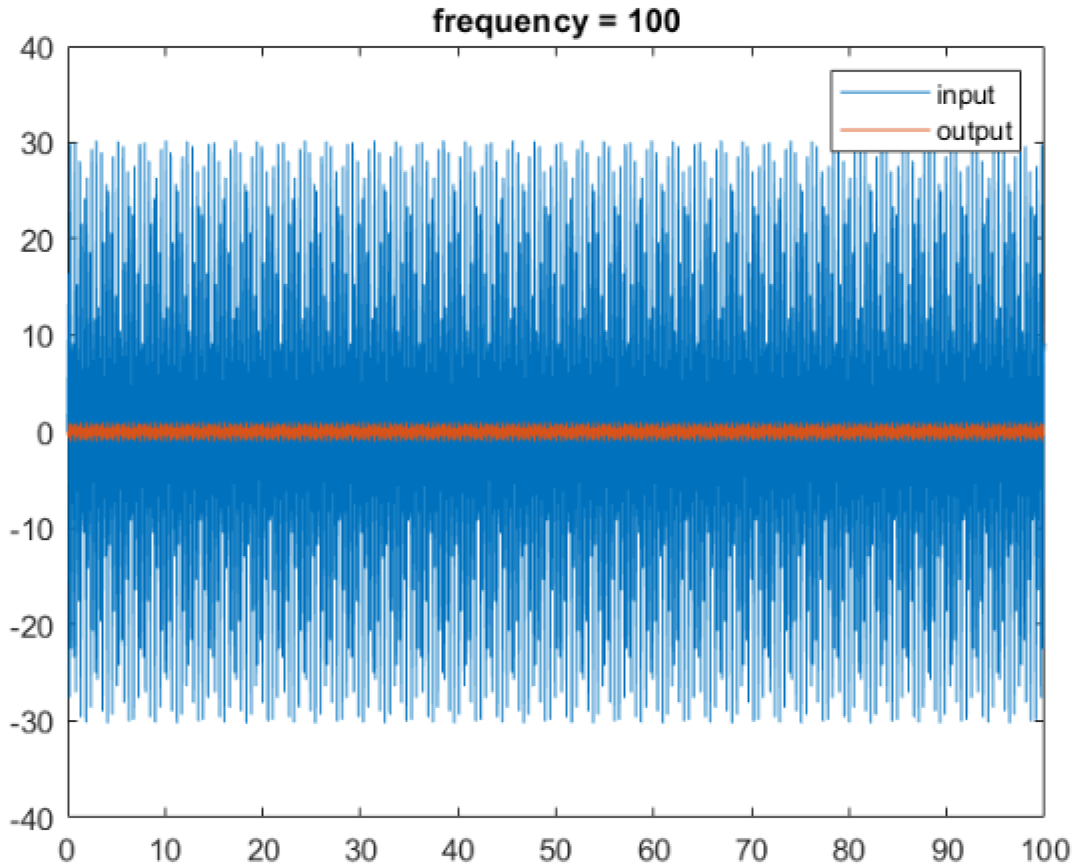




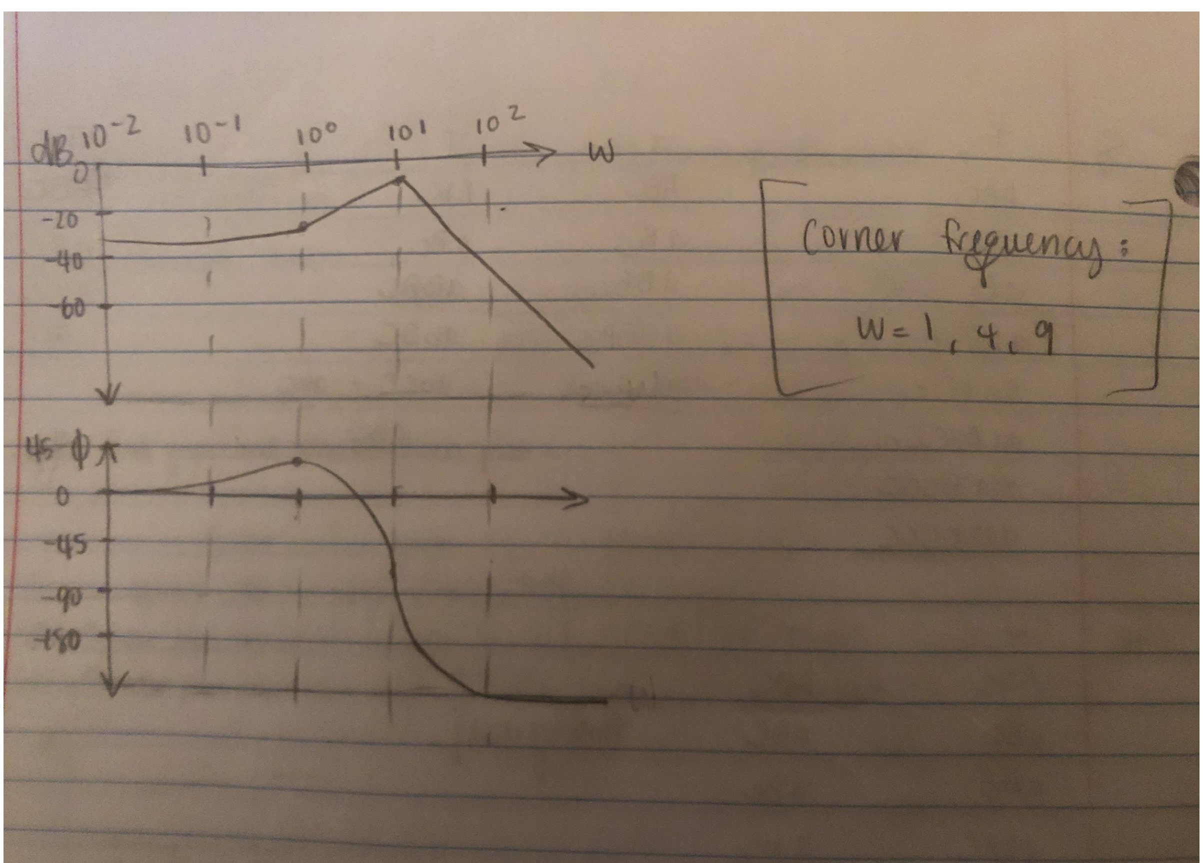
frequency = 1
input
output
40
30
20
10
0
-10
-20
-30
-40
0
10
20
30
40
50
60
70
80
90
100



frequency = 10
input
output
40
30
20
10
0
-10
-20
-30


frequency = 100
input
output
40
30
20
10
0
-10
-20
-30
-40
0
10
20
30
40
50
60
70
80
90
100

c)


dB
10^-2
10^-1
10^0
10^1
10^2
w
0
-20
-40
-60
45
ϕ
0
-45
-90
-180
Corner frequency:
w = 1, 4, 9

d)

```
  10 s + 10
  ---------
    s + 4

Continuous-time transfer function.

>> f2=tf([1],[1 6 81])

f2 =

         1
  --------------
  s^2 + 6 s + 81

Continuous-time transfer function.

>> f=f1*f2

f =

          10 s + 10
  --------------------------
  s^3 + 10 s^2 + 105 s + 324

Continuous-time transfer function.

>> bode(f)
```