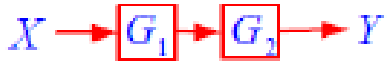
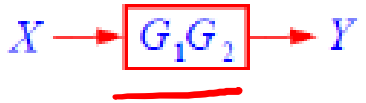
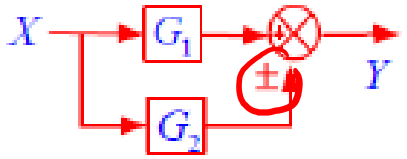
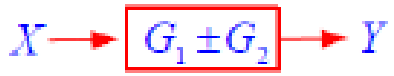
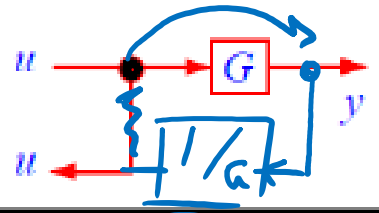
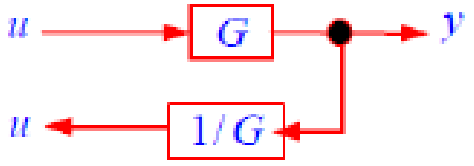
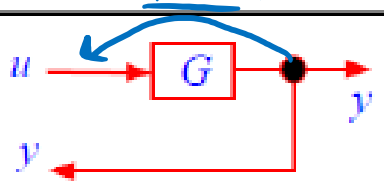
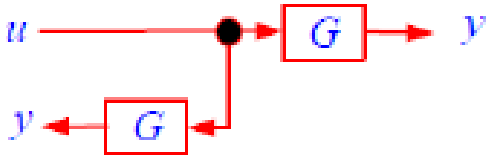
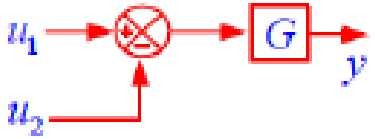
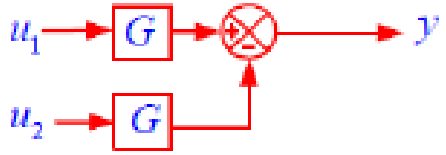
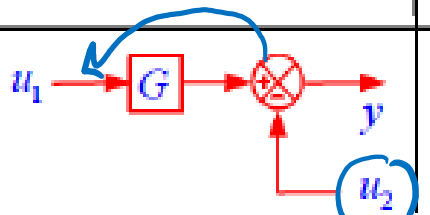
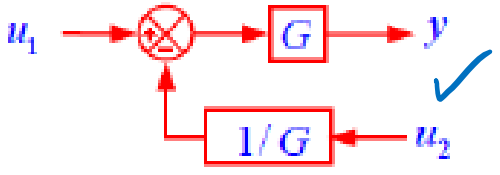
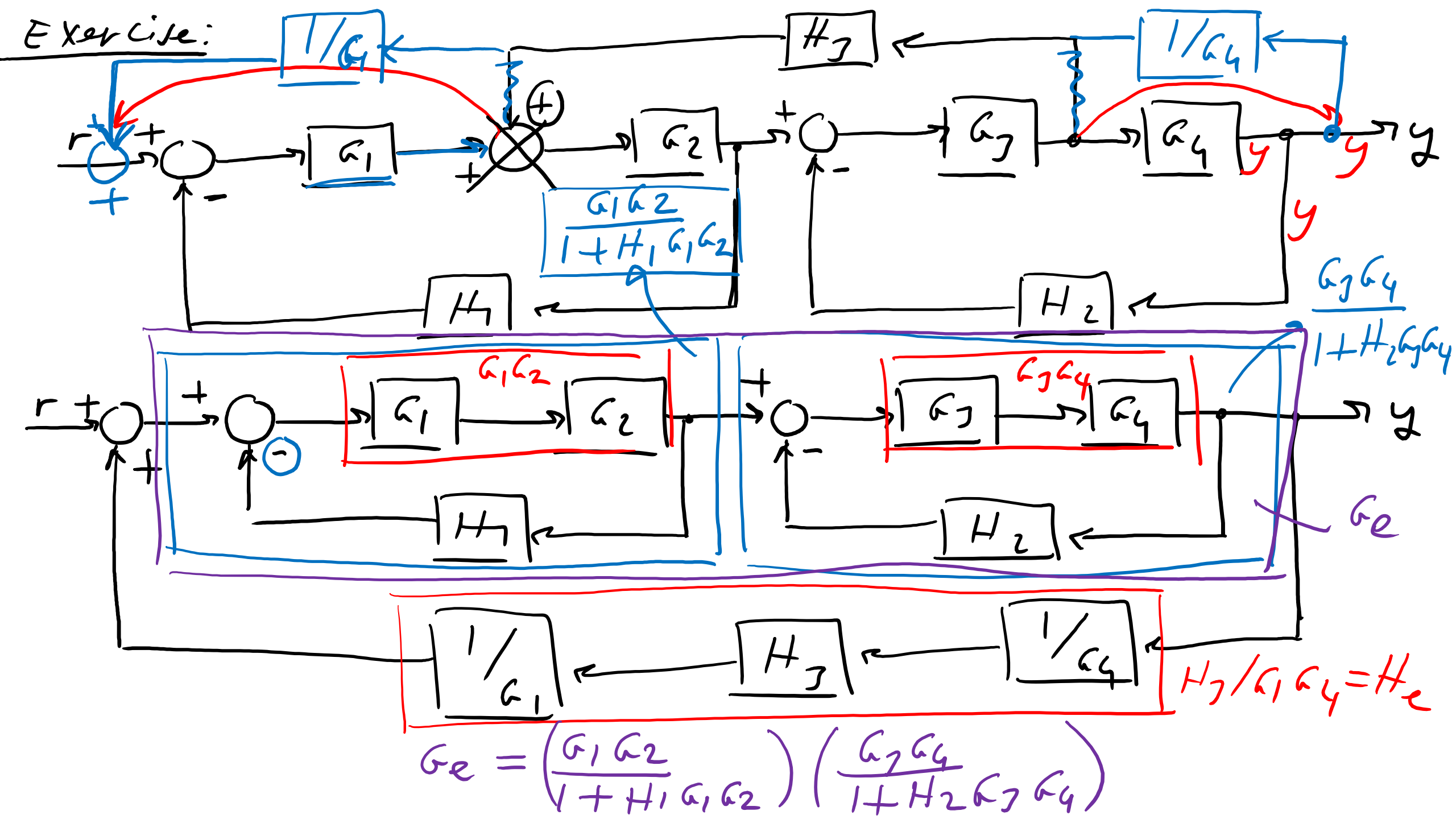
Reducing Complicated Block Diagrams:

0. Move summation and take-off points to desirable positions. (to avoid staggered loops)
1. Combine all cascade blocks (multiply) $(G_1 \cdot G_2)$
2. Combine all parallel blocks (add) $(G_1 + G_2)$
3. Eliminate all feedback loops $\left(\frac{G}{1 \mp HG}\right)$

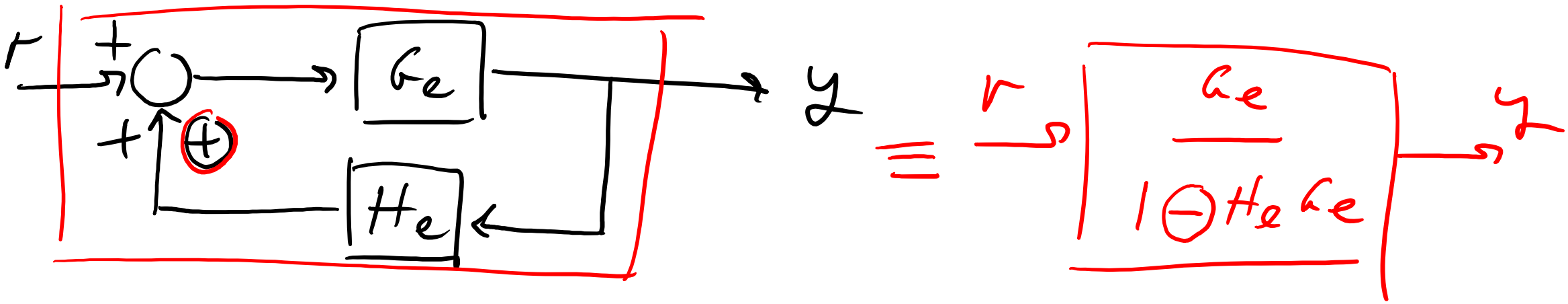
Block Diagram Transformations

	Manipulation	Original Block Diagram	Equivalent Block Diagram	Equation
1	Combining Blocks in Cascade			$Y = (G_1 G_2) X$
2	Combining Blocks in Parallel; or Eliminating a Forward Loop			$Y = (G_1 \pm G_2) X$
3	Moving a pickoff point behind a block			$y = G\,u$ $u = \frac{1}{G} y$
4	Moving a pickoff point ahead of a block			$y = Gu$
5	Moving a summing point behind a block			$y = G(u_1 - u_2)$
6	Moving a summing point ahead of a block			$y = Gu_1 - u_2$

Exercise:



$$G_e = \left(\frac{G_1 G_2}{1 + H_1 G_1 G_2}\right)\left(\frac{G_3 G_4}{1 + H_2 G_3 G_4}\right)$$


r
+
+
+
G_e
H_e
y
≡
r
$\frac{G_e}{1 \ominus H_e G_e}$
y

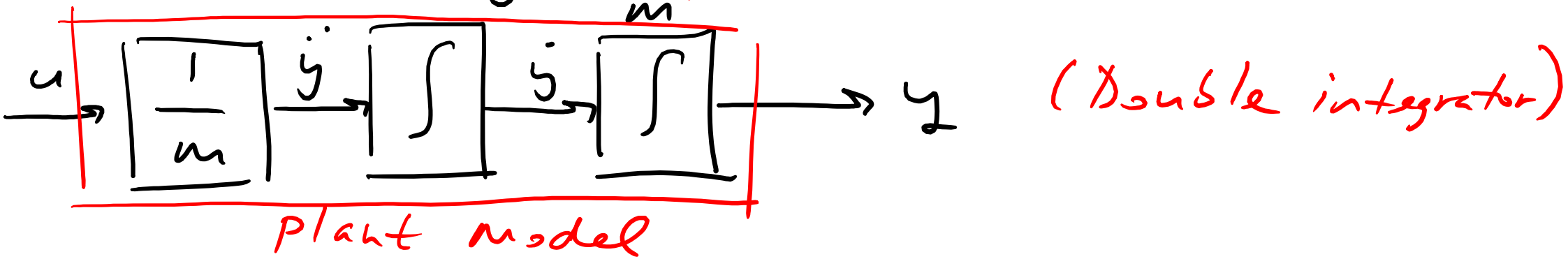
Modeling Mechanical Systems:

* "ALL MODELS ARE WRONG, BUT SOME ARE USEFUL!" G. Box

- Newton's 2nd Law: $\sum F = ma$; $\sum M = J\ddot{\theta}$ (2nd order system)

$$F = m\ddot{y}$$

$$\ddot{y} = \frac{1}{m}F$$



(Double integrator)

A common method to standardize: **state-space form**

State-Space Form

$$\dot{\vec{x}} = A\vec{x} + B\vec{u} \quad \text{(state eqn.)}$$

$$\vec{y} = C\vec{x} + D\vec{u} \quad \text{(output eqn.)}$$

$$\vec{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$\dot{\vec{x}} = \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix}$$

$x_1 = y \Rightarrow \dot{x}_1 = \dot{y} = x_2$ ←

$x_2 = \dot{y} \Rightarrow \dot{x}_2 = \ddot{y} = \frac{1}{m}u$ ←

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}}_{A} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \underbrace{\begin{bmatrix} 0 \\ 1/m \end{bmatrix}}_{B} u$$

$$y = x_1 \Rightarrow y = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \underbrace{\begin{bmatrix} 0 \end{bmatrix}}_{D} u$$