# EXAM 2

**February 19, 2021**

**Do All Problems – Show All Work**
**This exam is open notes and open book.**

**You have 24 hours to complete all problems.**

**Student ID:** ______________________________

**Student Printed Name:** ______________________________

**Student Signature:** ______________________________

Name: ____________________________________ ID: ___________________________

**1.) (30 pts)** Please find the free response $y(t)$ of the following differential equation using Laplace transform. Initial conditions are $y(0) = 1,\ \dot{y}(0) = -2$.

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 1y = u(t)$$

Can you imagine a mechanical or electrical system that would have a differential equation model of this form? What does the free response found above represent based on this physical analogy?

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**2.) (20 pts)** If the response of a system with zero initial conditions to a unit impulse input (i.e. $u(t) = \delta(t)$) is given as:

$$y(t) = 3te^{-2t} - 2e^{-3t}, \qquad t > 0$$

Please determine the system transfer function G(s) = Y(s)/U(s).

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**3.) (20 pts)** Please determine the Inverse Laplace Transform for the systems listed below:

$$\mathcal{L}^{-1}\left\{\frac{5}{s^2+16}\right\} =$$

$$\mathcal{L}^{-1}\left\{\frac{5}{s^2-16}\right\} =$$

$$\mathcal{L}^{-1}\left\{\frac{5}{s^2+4s+13}\right\} =$$

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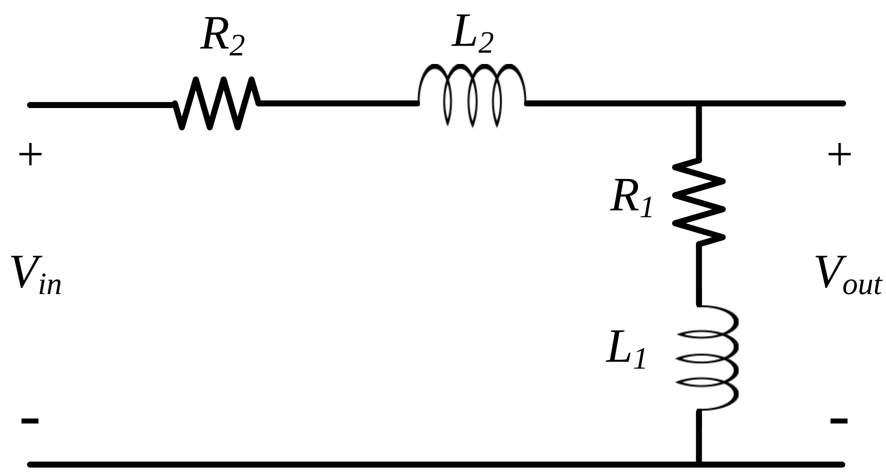
**4.) (30 pts)** Please consider the electrical system shown in the figure below for $R_2$ = 3, $L_2$ = 0.8, $R_1$ = 1, $L_1$ = 0.2. Initial conditions are $V_{out}(0) = 0,\ \dot{V}_{out}(0) = 0$. If $V_{in}(t) = 5e^{-5t}$, $V_{out}(t)$ = ?



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| Time Function | LaPlace Transform |
|---|---|
| Unit Impulse, $\delta(t)$ | $1$ |
| Unit step, $u_s(t)$ | $\frac{1}{s}$ |
| $t$ | $\frac{1}{s^2}$ |
| $\frac{t^2}{2}$ | $\frac{1}{s^3}$ |
| $\frac{t^n}{n!}$ | $\frac{1}{s^{n+1}}$ |
| $e^{-\alpha t}$ | $\frac{1}{s+\alpha}$ |
| $te^{-\alpha}$ | $\frac{1}{(s+\alpha)^2}$ |
| $1-e^{-\alpha t}$ | $\frac{\alpha}{s(s+\alpha)}$ |
| $\sin(\omega t)$ | $\frac{\omega}{s^2+\omega^2}$ |
| $e^{-\alpha t}\sin(\omega t)$ | $\frac{\omega}{(s+\alpha)^2+\omega^2}$ |
| $\cos(\omega t)$ | $\frac{s}{s^2+\omega^2}$ |
| $e^{-\alpha t}\cos(\omega t)$ | $\frac{s+\alpha}{(s+\alpha)^2+\omega^2}$ |

Name: ______________________________________ ID: ____________________________

Extra Page