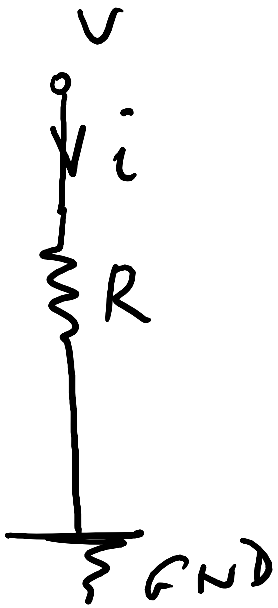
Modeling Electrical Systems:

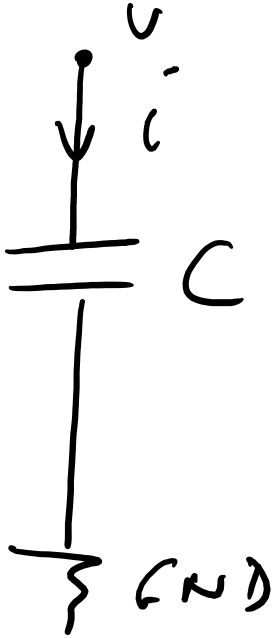
3 passive components:



Resistance

$$V = iR$$

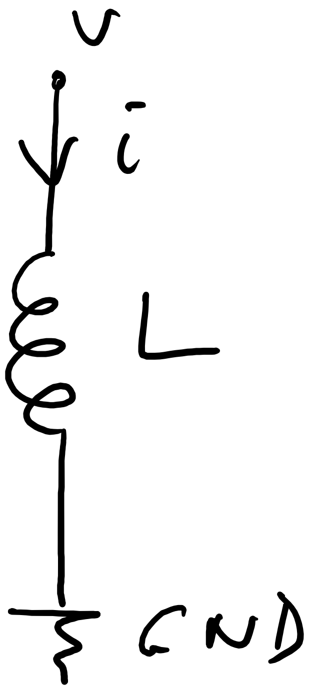
$$i = \frac{V}{R}$$



Capacitance

$$i = C\frac{dV}{dt}$$

$$V = \frac{1}{C}\int i\,dt$$

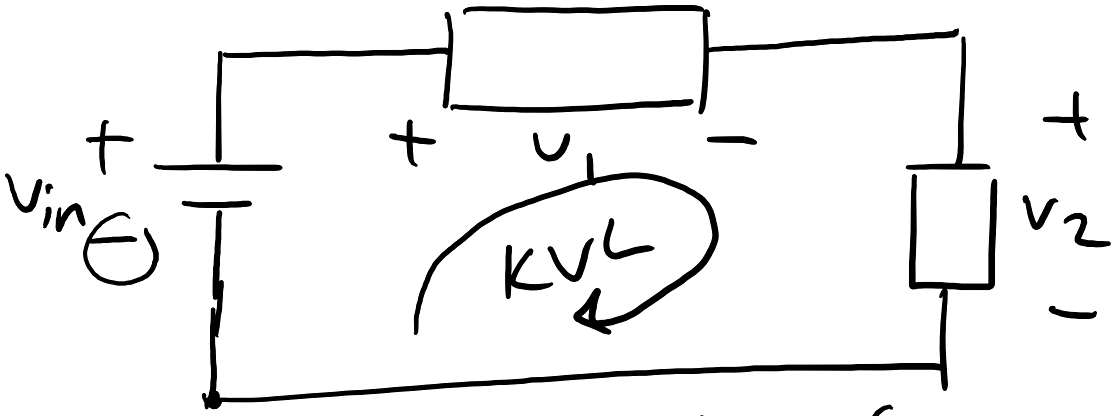


Inductance

$$V = L\frac{di}{dt}$$

$$i = \frac{1}{L}\int v\,dt$$

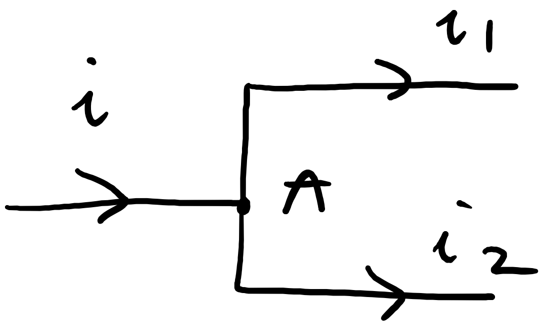
Kirchoff's Voltage Law (KVL)



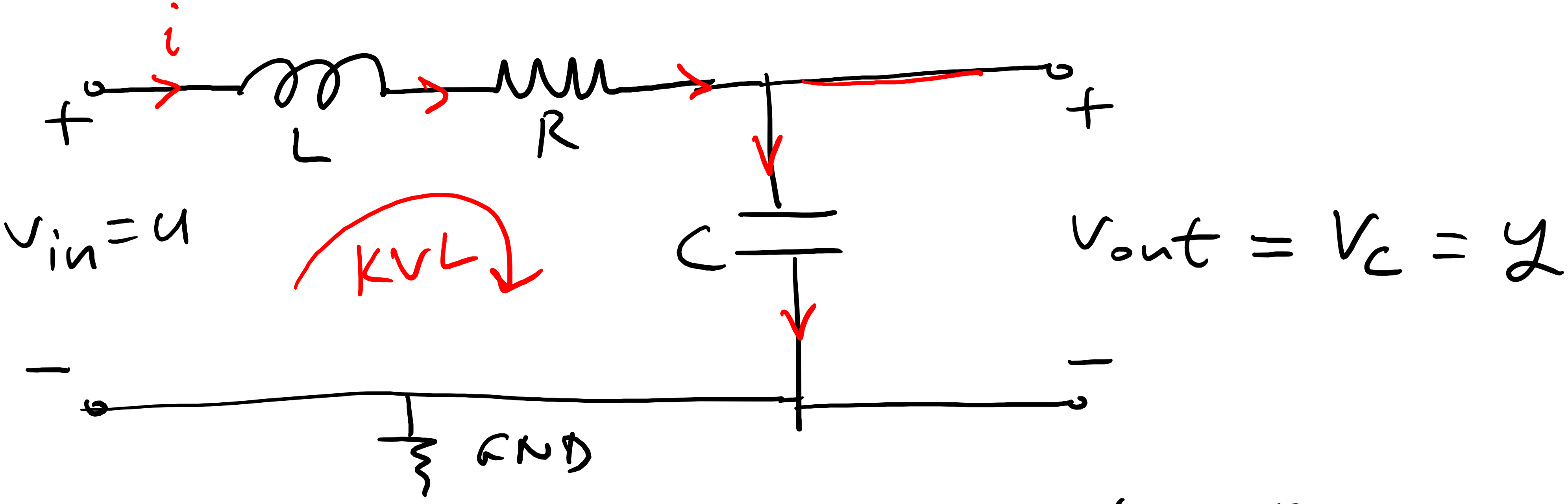
$$-V_{in} + V_1 + V_2 = 0$$

$$\boxed{V_{in} = V_1 + V_2}$$

Kirchoff's Current Law (KCL)



$$\boxed{i = i_1 + i_2}$$



KVL: $-V_{in} + V_L + V_R + V_C = 0$

L: $V_L = L\dfrac{di}{dt}$

R: $V_R = R\,i$

C: $V_C = \dfrac{1}{C}\displaystyle\int i\,dt \quad \Rightarrow \quad \boxed{i = C\dfrac{dV_C}{dt}}$

$$V_{in} = L\frac{di}{dt} + Ri + V_c$$

$u \to V_{in}$, $\quad V_c \to y$

$$\boxed{i = C\frac{dV_c}{dt}} = C\frac{dy}{dt}$$

$$L\frac{di}{dt} = L\frac{d\left(C\frac{dy}{dt}\right)}{dt} = LC\frac{d^2y}{dt^2}$$

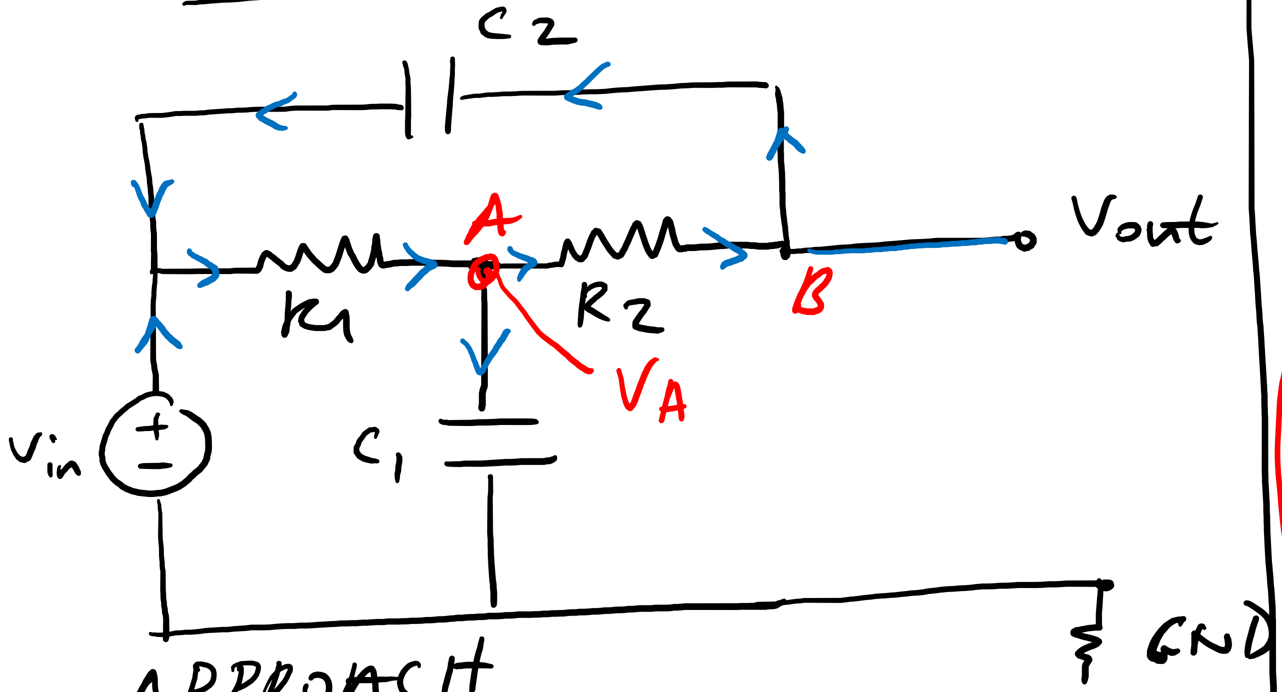
$$\frac{u}{LC} = \frac{LC\underbrace{\frac{d^2y}{dt^2}}_{\ddot{y}} + RC\frac{dy}{dt} + y}{LC}$$

$$\boxed{\ddot{y} + \frac{R}{L}\dot{y} + \frac{1}{LC}y = \frac{1}{LC}u}$$

Second-order system

Equivalent to mass-spring-damper.

Exercise:



APPROACH

1. Draw current path
2. 2 branches → 2 equations
3. Pick 2 nodes
4. Apply KCL each node
5. Combine

KCL @A:

$$\frac{V_{in}-V_A}{R_1} = C_1\frac{dV_A}{dt} + \frac{V_A - V_{out}}{R_2}$$

KCL @ B:

$$\left(\frac{V_A - V_{out}}{R_2} = C_2\frac{d(V_{out}-V_{in})}{dt}\right)R_2$$

$$V_A - V_{out} = R_2C_2(\dot{V}_{out} - \dot{V}_{in})$$

$$\boxed{V_A = V_{out} + R_2C_2(\dot{V}_{out} - \dot{V}_{in})}$$

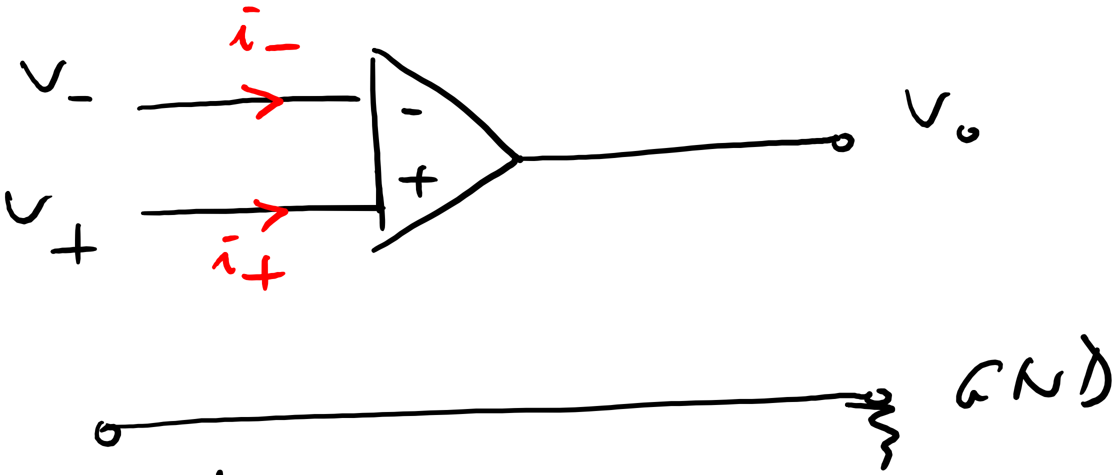
$u = V_{in}$

$y = V_{out}$

$$\boxed{(R_1R_2C_1C_2)\ddot{y} + (R_2C_2 + R_1C_2 + R_1C_1)\dot{y} + y = (R_1R_2C_1C_2)\ddot{u} + (R_2 + R_1C_2)\dot{u} + u}$$

Operational Amplifiers:

↳ 'Active' circuit component



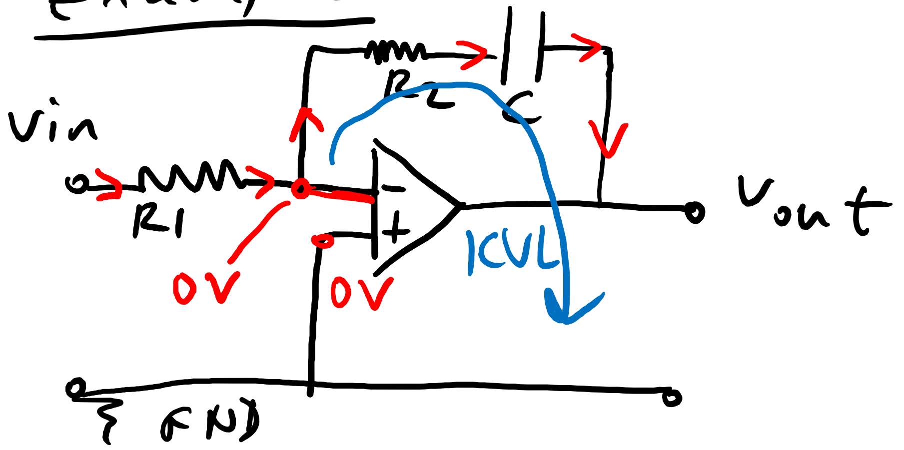
$$i_- = i_+ = 0$$

$$V_o = A_o (V_+ - V_-)$$

↳ $A_o \to \infty$

! when V_o is fed back to inputs: $(V_+ - V_-) \to 0$

Example:



R_1: $i = \frac{V_{in} - 0}{R_1}$

C: $i = C\frac{dV_c}{dt}$

R_2: $i = \frac{V_{R2}}{R_2} \Rightarrow \boxed{V_{R2}} = i R_2 = \boxed{V_{in}\frac{R_2}{R_1}}$

KVL: $0 + V_{R2} + V_c + V_{out} = 0$

$$V_{in}\frac{R_2}{R_1} + V_C + V_{out} = 0$$

v_{in}/R_1

$$\frac{1}{C}\int i\,dt$$

$$\boxed{V_{out} = \underbrace{-\frac{R_2}{R_1}V_{in}}_{P} \underbrace{- \frac{1}{R_1 C}\int V_{in}\,dt}_{I}}$$