Name: ______________________________ ID: ______________________

1.) Please consider the system described by the following differential equation:

$$\frac{1}{25}\frac{d^2y}{dt^2}+\frac{4}{25}\frac{dy}{dt}+y=u(t)$$

a) Write the open-loop transfer function of this system, and identify its poles and zeroes.

b) Find the rise time, settling time (2%), and maximum peak overshoot of this system.

c) Consider a standard proportional feedback controller for this system, draw a block-diagram, and calculate the proportional control gain (K_p) to achieve a steady state error of 0.25 for a unit step reference for the closed-loop system. .

d) Using the same K_p value in (c), calculate the rise time and settling time (2%) for the closed-loop system.

1) a) $\frac{1}{25}\ddot{y} + \frac{4}{25}\dot{y} + y = u(t)$

$\mathcal{L} \Rightarrow \frac{1}{25}s^2 Y(s) + \frac{4}{25} s Y(s) + Y(s) = U(s)$

$\Rightarrow Y(s)\left(\frac{s^2}{25} + \frac{4s}{25} + 1\right) = U(s)$

$$\boxed{\frac{Y(s)}{U(s)} = \frac{25}{s^2 + 4s + 25}}$$

$TF = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$

No zeroes! poles: $s^2 + \underbrace{4}_{2\xi\omega_n}s + \underbrace{25}_{\omega_n^2} = 0$

$P_{1,2} = -\alpha \pm j\omega_d$

$\boxed{P_{1,2} = -2 \pm j\,4.58}$

$P_{1,2} = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2} = -2 \pm j\,4.58$

$2\xi\omega_n = 4 \Rightarrow \boxed{\xi = 0.4}$

$\omega_n^2 = 25 \Rightarrow \boxed{\omega_n = 5}$

b) rise time : 0.433 s

$t_r = \frac{\pi - \beta}{\omega_d}$ $\omega_d = \omega_n\sqrt{1-\xi^2} = 4.582$

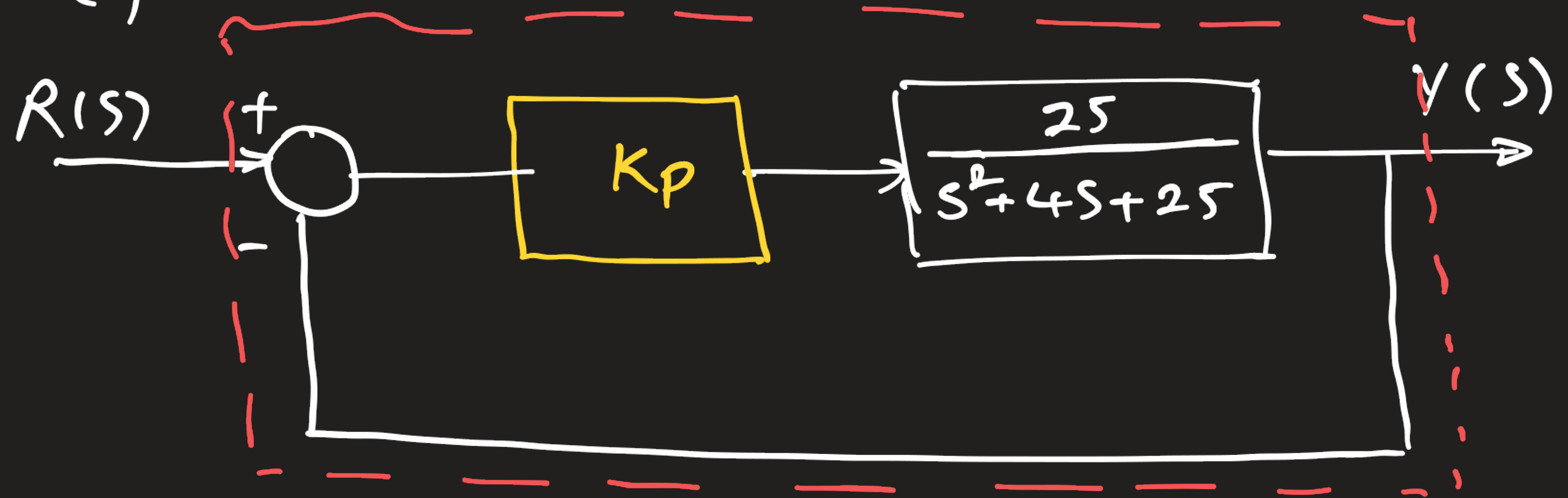
$\beta = \cos^{-1}(\xi) = 1.1593$ rad

$\Rightarrow \boxed{t_r = 0.433\ s}$

$t_s = \frac{4}{\alpha} = \frac{4}{\xi\omega_n} = \boxed{2\,\text{sec} = t_s}$

$M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} = 0.25 \Rightarrow \boxed{M_p = 25\%}$

c)



$$E(s) = \frac{R(s)}{1 + C(s)G(s)} = \frac{1/s}{1 + \dfrac{25K_p}{s^2+4s+25}}$$

$$\underline{E(s)} = \left(\frac{1}{s}\right) \frac{s^2+4s+25}{s^2+4s+(25+25K_p)}$$

$\Rightarrow \underline{1}$ pole at $s=0 \Rightarrow \underline{FV = cte}$

$e(\infty) = \lim_{t \to \infty} e(t)$

$$* \boxed{e_{ss} = \lim_{s \to 0} s \cdot E(s)} = \lim_{s \to 0} \cancel{s} \cdot \frac{1}{\cancel{s}} \cdot \frac{\cancel{s^2} + 4\cancel{s} + 25}{\cancel{s^2} + 4\cancel{s} + (25+25K_p)}$$

$$e_{ss} = \frac{25}{25+25K_p} \Rightarrow \boxed{e_{ss} = \frac{1}{1+K_p}}$$

$$e_{ss} = 0.25 \Rightarrow \frac{1}{1+K_p} = 0.25 \Rightarrow \boxed{K_p = 3}$$

d) $\frac{Y(s)}{R(s)} = \frac{75}{s^2 + 4s + \underbrace{100}}$

$2\xi\omega_n$

$20\xi = 4 \Rightarrow \boxed{\xi = 0.2}$

$\omega_n^2 \Rightarrow \boxed{\omega_n = 10}$

$t_r = \frac{\pi - \beta}{\omega_d}$

$\beta = \cos^{-1}(\xi) = 1.37$ rad

$\omega_d = \omega_n\sqrt{1-\xi^2} = 9.798$ rad/sec

$\Downarrow$

$\boxed{t_r = 0.18\,s}$

$t_s = \frac{4}{\xi\omega_n} = \frac{4}{(0.2)(10)} = \boxed{2s \quad 2\%}$

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2.) Please find the 2% settling time for the following system:

$$5\frac{dx}{dt} + 6x = u$$

problem 2

$$5\dot{x} + 6x = u$$

$$\mathcal{L} \Rightarrow 5\,s\,X(s) + 6\,X(s) = U(s)$$

$$\Rightarrow \frac{X(s)}{U(s)} = \frac{1/6}{\frac{5}{6}s + 1} = \frac{K_p}{\tau s + 1}$$

$$2\% \Rightarrow t_s = 4\tau = 4 \cdot \frac{5}{6} = \frac{20}{6} = \frac{10}{3}$$

$$\Rightarrow \boxed{t_s = 3.\bar{3}\ \text{se}} \quad (2\%)$$

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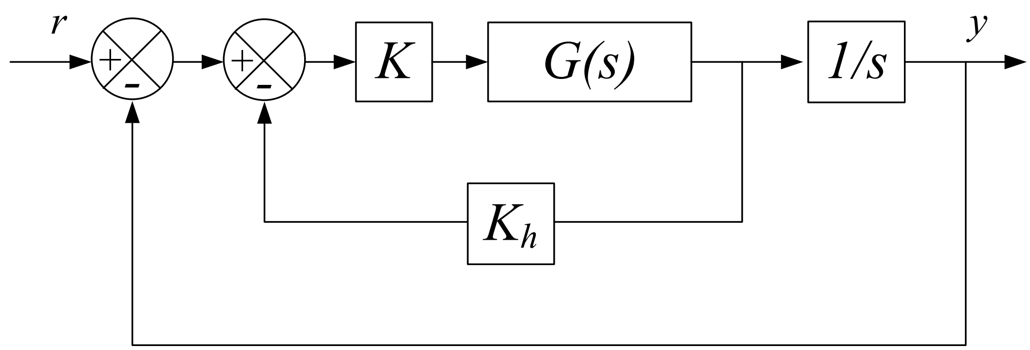
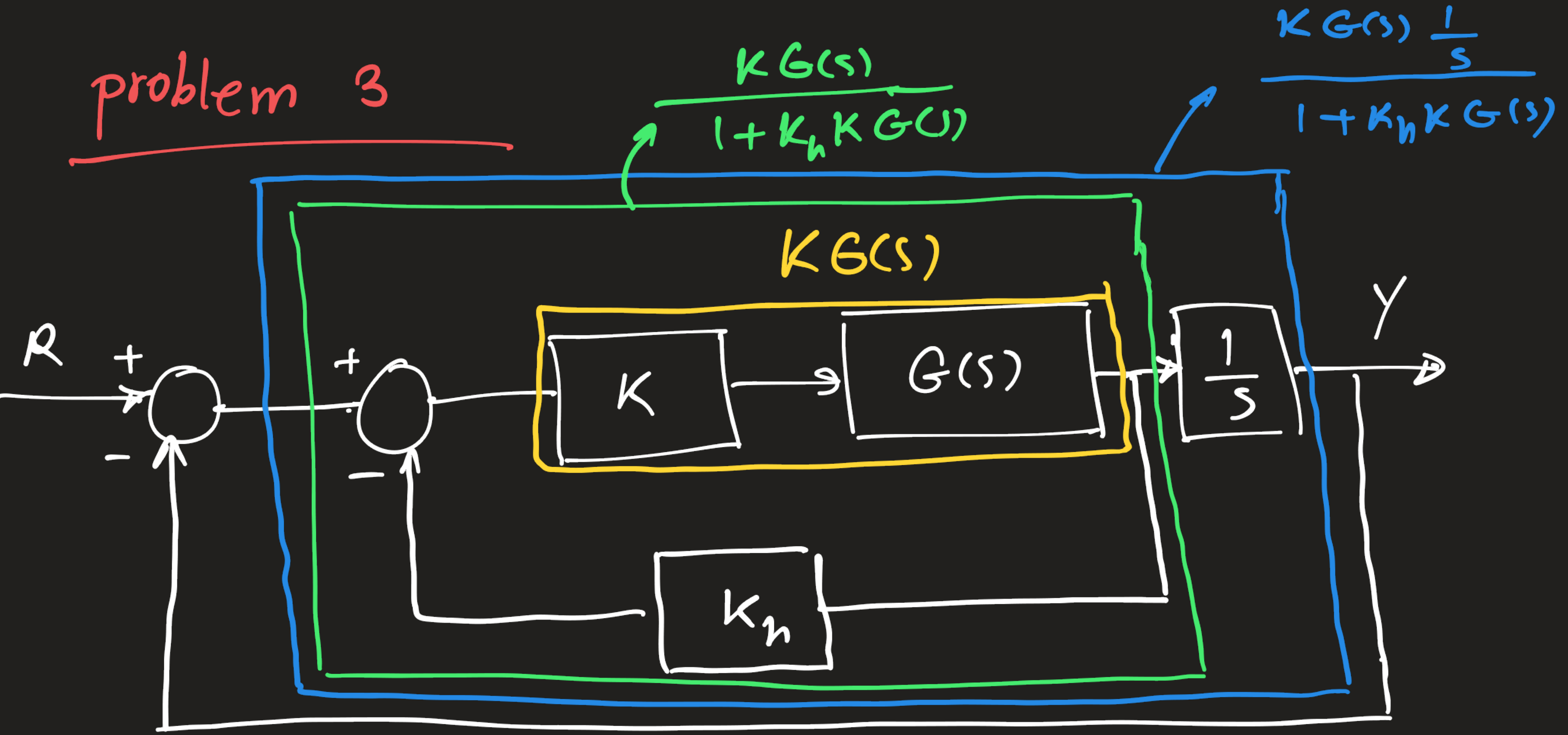
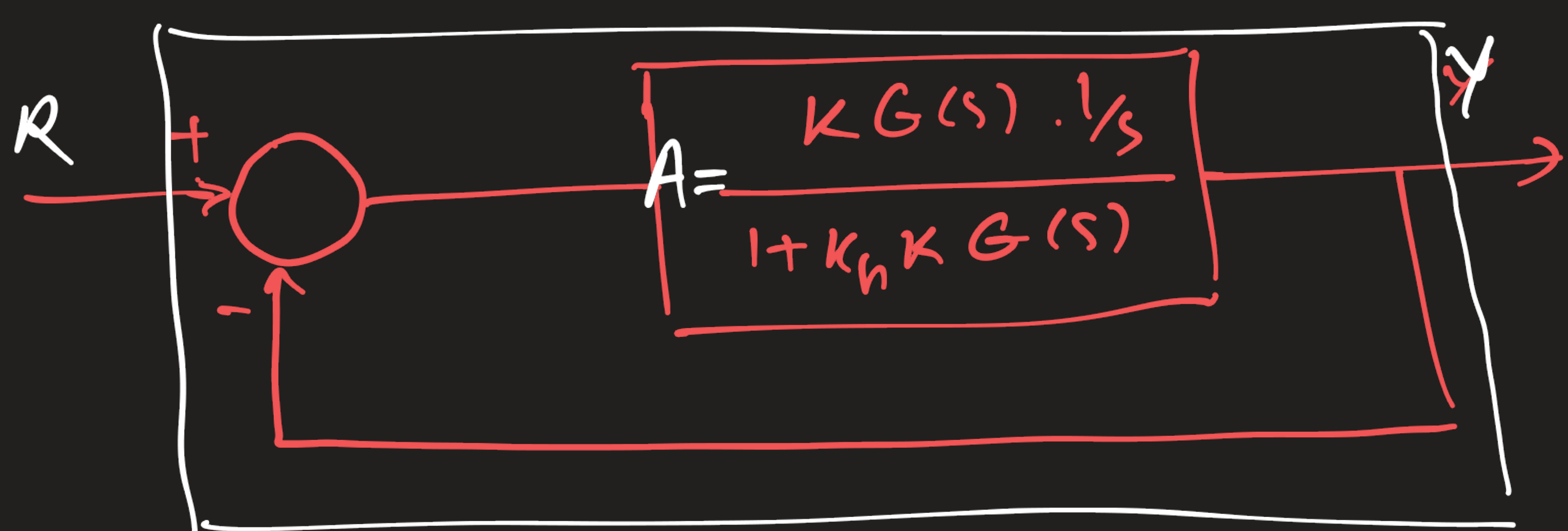
3.) Please determine the range of stable K values, for $K_h = 0.25$ and $G(s) = \frac{1}{s^2+2s+2}$ in the block diagram given below.



problem 3





$$\frac{Y}{R} = \frac{A}{1+A} \Rightarrow \frac{Y}{R} = \frac{K}{s^3 + 2s^2 + (2 + KK_h)s + K}$$

$$\frac{Y}{R} = \frac{A}{1+A} \Rightarrow \frac{Y}{R} = \frac{K}{s^3 + 2s^2 + (2 + KK_h)s + K}$$

K_h: 0.25

Char. poly: $\underline{\underline{1}} \cdot s^3 + \underline{\underline{2}} \cdot s^2 + \underbrace{(2 + 0.25K)}\, s + \underbrace{K}$

$\underline{\underline{K > 0}}$

s^3	1	$2 + 0.25K$
s^2	2	K
s^1	$b_1 = \frac{4 - 0.5K}{2}$	$\underline{\underline{0}}$
s^0	$c_1 = K$	

$\underline{\underline{K > 0}}$

$$b_1 = \frac{(2)(2 + 0.25K) - K}{2}$$

$$b_1 = \frac{4 - 0.5K}{2}$$

$$c_1 = \frac{\cancel{b_1} K - 0}{\cancel{b_1}} = \underline{\underline{K}}$$

$$b_1 = \frac{4 - 0.5K}{2} > 0 \Rightarrow 4 - 0.5K > 0 \Rightarrow \underline{\underline{K < 8}}$$

$$\Rightarrow \boxed{0 < K < 8}$$