Lab 3 Part 1

1. $f(t) = 5t + 12e^{4t}$

$F(s) = \mathcal{L}[5t] + \mathcal{L}[12e^{4t}]$

$F(s) = 5\mathcal{L}[t] + 12\mathcal{L}[e^{4t}]$

$$\boxed{F(s) = \frac{5}{s^2} + \frac{12}{s-4}}$$

2. $f(t) = \cos 3t + \sin 3t$

$F(s) = \mathcal{L}[\cos 3t] + \mathcal{L}[\sin 3t]$

$$\boxed{F(s) = \frac{s}{s^2+9} + \frac{3}{s^2+9}}$$

3. $2y'' + 3y' - 2y = te^{-2t}$

$2[s^2Y(s) - s\cancel{y(0)} - \cancel{y'(0)}] + 3[sY(s) - \cancel{y(0)}] - 2Y(s) = \frac{1}{(s+2)^2}$ (-2)

$2s^2Y(s) + 3sY(s) - 2Y(s) = \frac{1}{(s+2)^2}$

$Y(s)(2s^2 + 3s - 2) = \frac{1}{(s+2)^2}$

$$\boxed{Y(s) = \frac{1}{(s+2)^2(2s^2+3s-2)}}$$

4. $y'' + 16y = 1 + t \qquad y(0) = -2 \quad y'(0) = 2$

$[s^2Y(s) - s\overset{(-2)}{y(0)} - \overset{(2)}{y'(0)}] + 16[sY(s) - \overset{(-2)}{y(0)}] = \mathcal{L}[1] + \mathcal{L}[t]$

$s^2Y(s) + 2s - 2 + 16sY(s) + 32 = \frac{1}{s} + \frac{1}{s^2}$

$s^2Y(s) + 16sY(s) + 2s + 30 = \frac{1}{s} + \frac{1}{s^2}$

$Y(s)(s^2 + 16s) = -2s - \frac{1}{s} + \frac{1}{s^2} - 30$

$$\boxed{Y(s) = \frac{-2s - \frac{1}{s} + \frac{1}{s^2} - 30}{s^2 + 16s}}$$

* 5. $y''' - y'' - 4y' + 4y = f(t) \qquad y(0) = y'(0) = 1 \quad y''(0) = 0$ 1

$\mathcal{L}[y'''] - \mathcal{L}[y''] - 4\mathcal{L}[y'] + 4\mathcal{L}[y] = F(s)$

$[s^3Y(s) - s^2\overset{(1)}{\cancel{y(0)}} - s\overset{(1)}{\cancel{y'(0)}} - \cancel{y''(0)}] - [s^2Y(s) - s\overset{(1)}{\cancel{y(0)}} - \overset{(1)}{\cancel{y'(0)}}] - 4[sY(s) - \overset{(1)}{\cancel{y(0)}}] + 4Y(s) = F(s)$

$s^3Y(s) - s^2 - \cancel{s} - s^2Y(s) + \cancel{s} + 1 - 4sY(s) + 4 + 4Y(s) = F(s)$

$s^3Y(s) - s^2Y(s) - 4sY(s) + 4Y(s) - s^2 + 5 = F(s)$

$Y(s)[s^3 - s^2 - 4s + 4] - s^2 + 5 = F(s)$

$Y(s)[(s-1)(s-2)(s+2)] = F(s) + s^2 - 5 \Rightarrow \boxed{Y(s)[(s-1)(s-2)(s+2)] - s^2 + 5 = F(s)}$

6. $\mathcal{L}^{-1}\left[\frac{7}{s^2-9}\right]$

$$\frac{7}{s^2-9} = \frac{A}{(s+3)} + \frac{B}{(s-3)}$$

$$7 = A(s-3) + B(s+3)$$

$$A = -\frac{7}{6} \qquad B = \frac{7}{6}$$

$$\mathcal{L}^{-1}\left[\frac{-7}{6(s+3)} + \frac{7}{6(s-3)}\right]$$

$$-\frac{7}{6}\mathcal{L}^{-1}\left[\frac{1}{s+3}\right] + \frac{7}{6}\mathcal{L}^{-1}\left[\frac{1}{s-3}\right]$$

$$\boxed{-\frac{7}{6}e^{-3t} + \frac{7}{6}e^{3t}}$$

7. $\mathcal{L}^{-1}\left[\frac{s}{s^2+64}\right]$

$$\mathcal{L}^{-1} \frac{s}{s^2+64} = \boxed{\cos(8t)}$$

8. $\mathcal{L}^{-1}\left[\frac{1}{s+42} - \frac{1}{(s+3)^4}\right]$

$$\mathcal{L}^{-1}\left[\frac{1}{s+42}\right] - \mathcal{L}^{-1}\left[\frac{1}{(s+3)^4}\right]$$

$$\boxed{e^{-42t} - \frac{e^{-3t}t^3}{6}}$$

9. $\mathcal{L}^{-1}\left[\frac{s}{s^2+64}\right] = \boxed{\cos(8t)}$

10. $\mathcal{L}^{-1}\left[\frac{1}{(s^2+4)(s^2-4)}\right]$

$$\frac{1}{(s+2)(s-2)(s^2+4)} = \frac{A}{(s+2)} + \frac{B}{(s-2)} + \frac{C}{(s^2+4)}$$

$$1 = A(s-2)(s^2+4) + B(s+2)(s^2+4) + C(s+2)(s-2)$$

$$A = \frac{-1}{8} \qquad B = \frac{-1}{32} \qquad C = \frac{1}{32}$$

10 Cont. $\mathcal{L}^{-1}\left[\frac{-1}{8(s^2+4)} - \frac{1}{32(s+2)} + \frac{1}{32(s-2)}\right]$

$$\mathcal{L}^{-1}\left[\frac{-1}{-8(s^2+4)}\right] - \frac{1}{32}\mathcal{L}^{-1}\left[\frac{1}{s+2}\right] + \frac{1}{32}\mathcal{L}^{-1}\left[\frac{1}{s-2}\right]$$

$$\boxed{-\frac{1}{16}\sin(2t) - \frac{1}{32}e^{-2t} + \frac{1}{32}e^{2t}}$$

11. Mass-Spring-Damper

$$F(t) - b\dot{x} - kx = m\ddot{x}$$

$$\mathcal{L}^{-1}[F(t)] = [m\ddot{x} + b\dot{x} + kx]\mathcal{L}^{-1}$$

$$F(s) = [ms^2 + bs + k]X(s)$$

$$\frac{X(s)}{F(s)} = \frac{1}{ms^2 + bs + k}$$

Motor Position

$$J\ddot{\theta} + b\dot{\theta} = ki$$

$$\theta(s)(Js^2 + bs) = k\,I(s)$$

$$\theta(s) = \frac{k\,I(s)}{Js^2 + bs}$$

$$L\frac{di}{dt} + Ri = V - k\dot{\theta}$$

$$L\dot{i} + Ri = V - k\dot{\theta}$$

$$I(s)(Ls + R) = \frac{1}{s^2} - (ks)\,\theta(s)$$

$$I(s)[Ls + R] = \frac{1}{s^2} - ks\left[\frac{kI(s)}{Js^2 + bs}\right]$$

$$I(s)[Ls + R] = \frac{1}{s^2} - \frac{k^2 \cancel{s}\, I(s)}{J\cancel{s^2}_{s} + b\cancel{s}}$$

$$I(s)[Ls + R] = \frac{1}{s^2} - \frac{k^2 I(s)}{Js + b}$$

$$I(s)[Ls + R] + \frac{k^2 I(s)}{Js + b} = \frac{1}{s^2}$$

$$I(s)\left[(Ls + R) + \frac{k^2}{Js + b}\right] = \frac{1}{s^2}$$

$$I(s) = \frac{1}{s^2\left[(Ls + R) + \frac{k^2}{Js + b}\right]}$$

Motor Position Cont.

$$\boxed{I(s) = \frac{Js+b}{s^2(Ls(Js+b)+R(Js+b)+K^2)}}$$

$$\theta(s) = \frac{K I(s)}{Js^2+bs}$$

$$\boxed{\theta(s) = \frac{K\left[\frac{Js+b}{s^2(Ls(Js+b)+R(Js+b)+K^2)}\right]}{Js^2+bs}}$$

5.1 $\theta(s)\left[s^2 \frac{I_x}{d_g} - mgd_r\right] = \underbrace{\frac{k_T}{Rd_r}\left[v_R + v_L - \frac{2k_v}{d_r} V_x(s)\right]}$

$$\frac{V_R + V_L}{2} = V_y$$

$$V_R + V_L = \frac{2V_y}{d_r}$$

$$\underbrace{\frac{k_T}{Rd_r}\left[\frac{2V_y(s)}{d_r} - \frac{2k_v}{d_r} V_y(s)\right]}$$

$$\frac{k_T}{Rd_r}\left[\frac{2V_y(s) - 2k_v V_y(s)}{d_r}\right]$$

$$\theta(s)\left[s^2 \frac{I_x}{d_g} - mgd_r\right] = \left[\frac{k_T(2-k_v)}{Rd_r^2}\right] V_y(s)$$

$$\boxed{\theta(s) = \frac{\left[\frac{k_T(2-k_v)}{Rd_r^2}\right]}{\left[s^2 \frac{I_x}{d_g} - mgd_r\right]} V_y(s)}$$

$\boxed{5.2}$:

$$d_r m \dot{v}_y = \frac{k_T}{R}\left(v_L + v_R - k_v\left(\frac{2v_y}{d_r}\right)\right)$$

$$d_r m v_y(s) = \frac{k_T}{R}\left(V_L(s) + V_R(s) - \frac{2k_v}{d_r} V_y(s)\right)$$

$$d_r m V_y(s) + \frac{2k_v k_T}{R d_r} V_y(s) = \frac{k_T}{R}\left(V_L(s) + V_R(s)\right)$$

$$V_y(s)\left[d_r m + \frac{2k_v k_T}{R d_r}\right] = \frac{k_T}{R}\left(V_L(s) + V_R(s)\right)$$

$$\boxed{V_y(s) = \frac{\frac{k_T}{R}\left(V_L(s) + V_R(s)\right)}{d_r m + \frac{2k_v k_T}{R d_r}}}$$

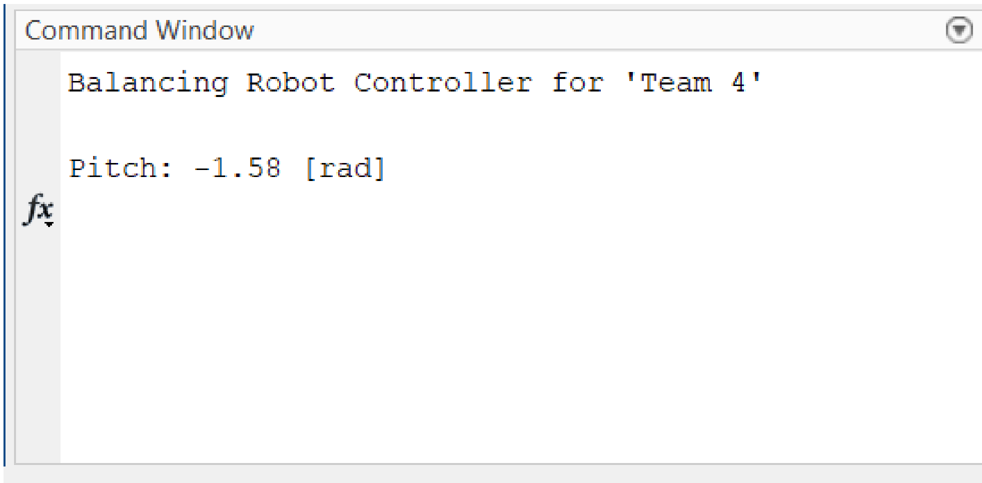
$\boxed{5.3}$:

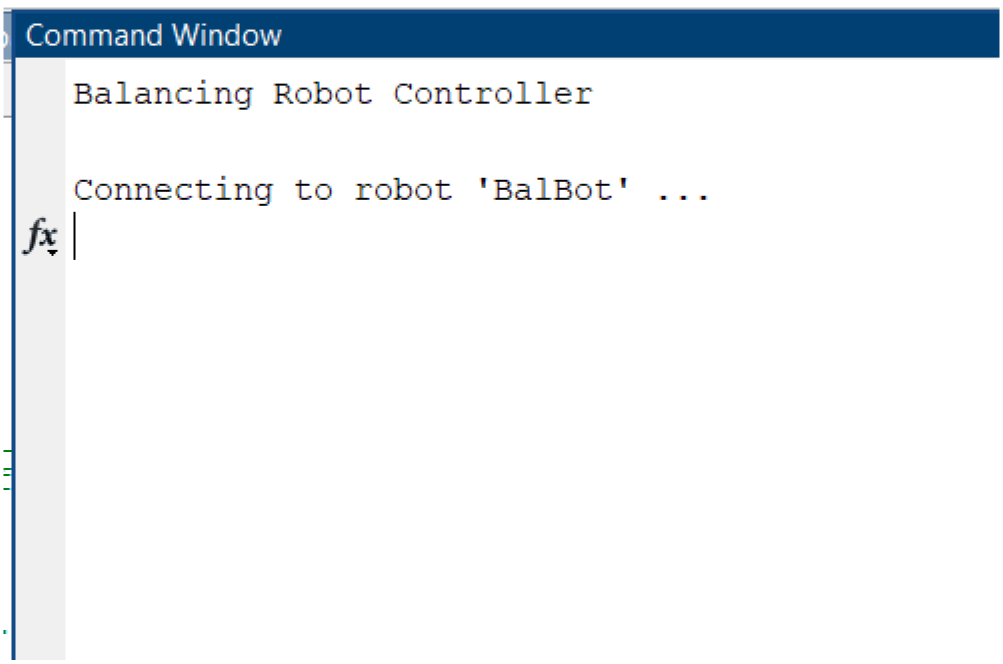
$$d_r(I_z \dot{w}_z) = \frac{k_T}{R}\left(v_R - v_L - k_v\left(\frac{2w_z d_w}{d_r}\right)\right)$$

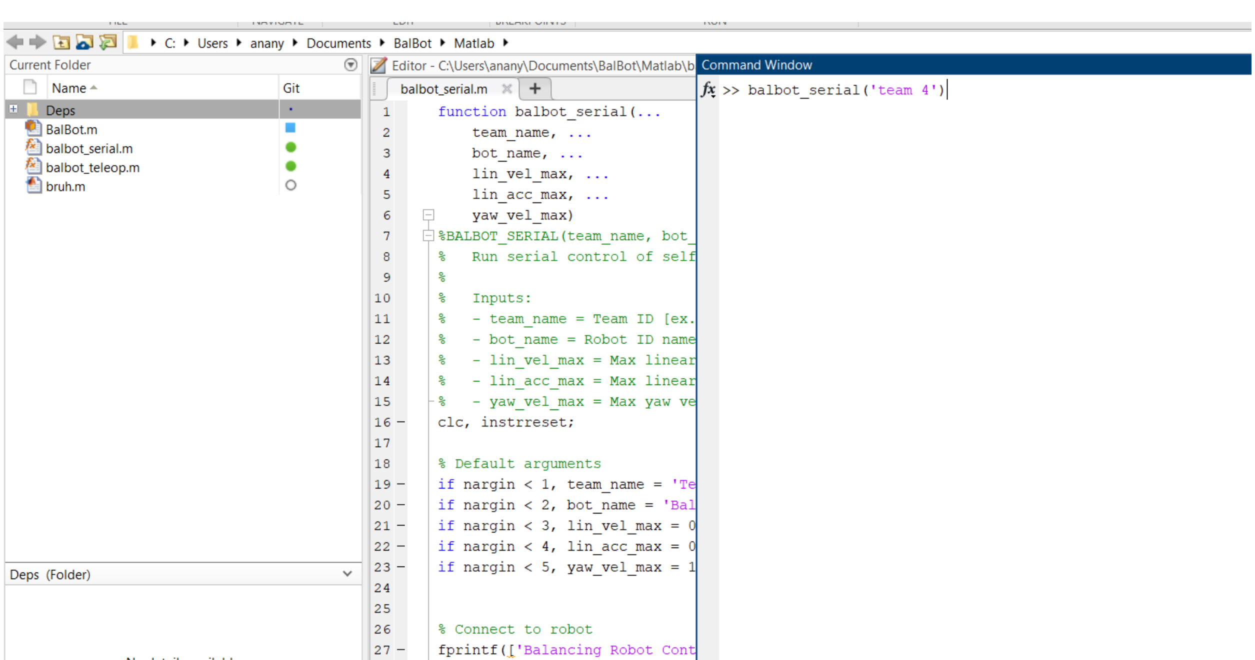
$$d_r I_z W_z(s) = \frac{k_T}{R}\left(V_R(s) - V_L(s)\right) - \frac{2k_v k_T d_w}{d_r R}\left(W_z(s)\right)$$

$$\left[d_r I_z + \frac{2k_v k_T d_w}{R d_r}\right] W_z(s) = \frac{k_T}{R}\left[V_R(s) - V_L(s)\right]$$

$$\boxed{W_z(s) = \frac{\frac{k_T}{R}\left[V_R(s) - V_L(s)\right]}{d_r I_z + \frac{2k_v k_T d_w}{R d_r}}}$$


Command Window
Balancing Robot Controller for 'Team 4'
Pitch: -1.58 [rad]


Command Window
Balancing Robot Controller
Connecting to robot 'BalBot' ...


C: ▸ Users ▸ anany ▸ Documents ▸ BalBot ▸ Matlab ▸
Current Folder
Name
Git
Deps
BalBot.m
balbot_serial.m
balbot_teleop.m
bruh.m
Deps (Folder)
Editor - C:\Users\anany\Documents\BalBot\Matlab\b
balbot_serial.m
function balbot_serial(...
team_name, ...
bot_name, ...
lin_vel_max, ...
lin_acc_max, ...
yaw_vel_max)
%BALBOT_SERIAL(team_name, bot_
% Run serial control of self
%
% Inputs:
% - team_name = Team ID [ex.
% - bot_name = Robot ID name
% - lin_vel_max = Max linear
% - lin_acc_max = Max linear
% - yaw_vel_max = Max yaw ve
clc, instrreset;
% Default arguments
if nargin < 1, team_name = 'Te
if nargin < 2, bot_name = 'Bal
if nargin < 3, lin_vel_max = 0
if nargin < 4, lin_acc_max = 0
if nargin < 5, yaw_vel_max = 1
% Connect to robot
fprintf(['Balancing Robot Cont
Command Window
>> balbot_serial('team 4')

Introduction:

The focus of this lab is the Laplace Transform. We simplified our system modeling and analysis by converting our equations from time-domain (differential equations) to s-domain (algebraic equations). The three main objectives include finding the Laplace and inverse Laplace transforms for basic dynamical systems, Laplace transforms for the BalBot (self-balancing robot), and testing Balbot IMU Calibration.

Conclusion:

In this lab, we were able to build upon the derivatives found in the last lab by solidifying our understanding of the origins of those equations. We then simplified those derivatives using the Laplace transformations. In the end, it helped us achieve our goal of modeling our system.