# What you need to know

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We are **analyzing** the response of 1st and 2nd order systems in terms of **transient response ($\boldsymbol{y}(\boldsymbol{t})$)** and **steady-state response ($\boldsymbol{y_{ss}}$)** to different inputs where $\boldsymbol{y_{ss}} = \lim_{\boldsymbol{t}\to\infty} \boldsymbol{y}(\boldsymbol{t}) = \lim_{\boldsymbol{s}\to\boldsymbol{0}} \boldsymbol{s}.\boldsymbol{Y}(\boldsymbol{s})$.

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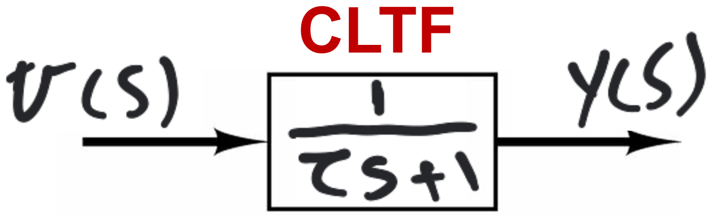
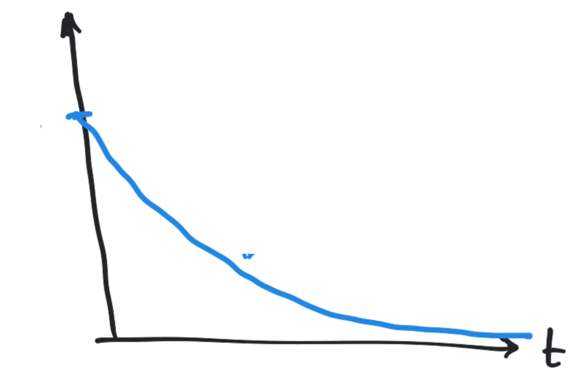
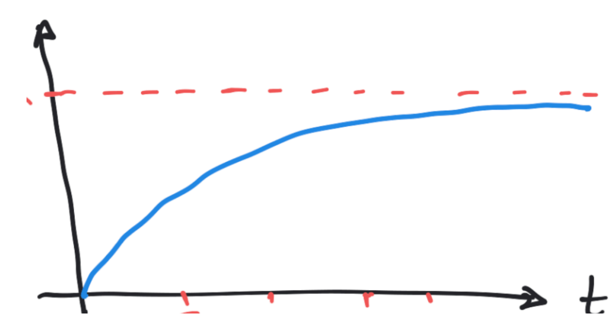
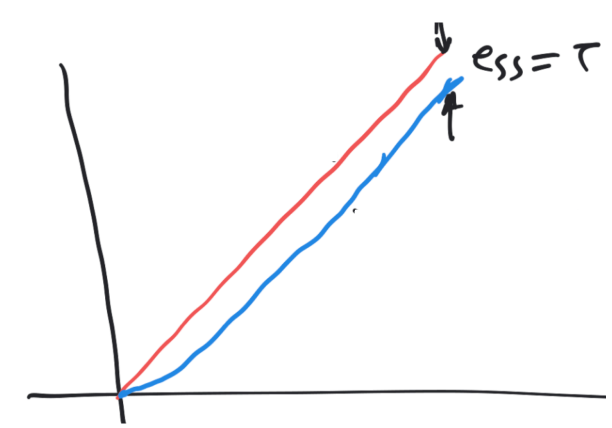


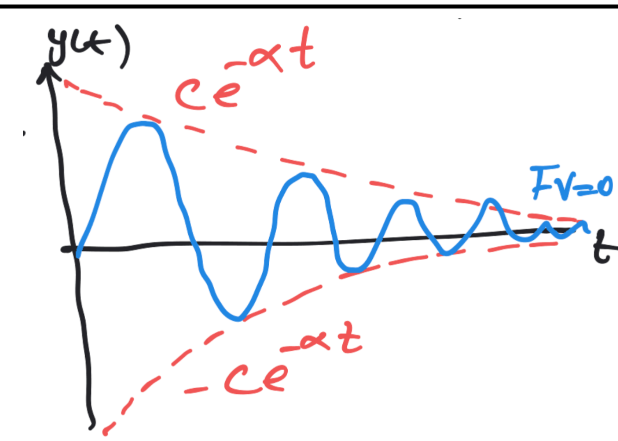
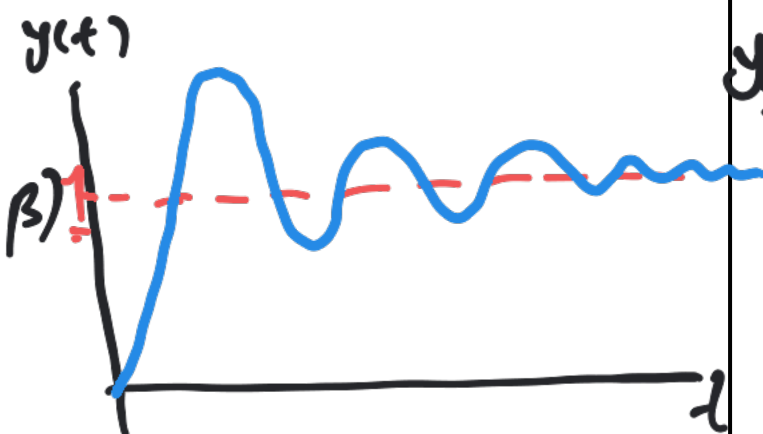

# 1st-order System Response (gain K=1)

| Input | Response (Output): Y(s) | Response (Output): y(t): Transient Resp. | Response Plot | Steady-State Response: FV |
|---|---|---|---|---|
| Unit-Impulse<br>$U(s)=1$ | $Y(s)=\frac{1}{\tau s+1}$ | $y(t)=\frac{1}{\tau}e^{-t/\tau}$ |  | $\lim_{s\to 0} s\,Y(s)$<br>$y_{ss}=FV=0$ |
| Unit-Step<br>$U(s)=\frac{1}{s}$ | $Y(s)=\frac{1}{s}\ \frac{1}{\tau s+1}$ | $y(t)=(1-e^{-t/\tau})$ |  | $\lim_{s\to 0} s\,Y(s)$<br>$y_{ss}=FV=1$ |
| Unit-Ramp<br>$U(s)=\frac{1}{s^2}$ | $Y(s)=\frac{1}{s^2(\tau s+1)}$ | $y(t)=t-\tau(1-e^{-t/\tau})$ |  | $\lim_{s\to 0} s\,Y(s)$<br>$y_{ss}=FV=\infty$ |

CLTF

$$U(s) \longrightarrow \boxed{\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}} \longrightarrow Y(s)$$

# 2nd-order System Response (gain K=1)

| Input | Response (Output): Y(s) | Response (Output): y(t): Transient Resp. | Response Plot | Steady-State Response: FV |
|---|---|---|---|---|
| Unit-Impulse $U(s)=1$ | $Y(s)=\frac{\omega_n^2}{s^2+\zeta\omega_n s+\omega_n^2}$ <br> $C$ <br> $Y(s)=\frac{\omega_n^2}{\omega_d}\frac{\omega_d}{(s+\alpha)^2+\omega_d^2}$ | $\rightarrow y(t)=C\,e^{-\alpha t}\sin\omega_d t$ |   | $y_{ss}=\lim_{s\to 0} s\,Y(s)$ <br> $y_{ss}=FV=0$ |
| Unit-Step $U(s)=\frac{1}{s}$ | $Y(s)=\frac{1}{s}\frac{\omega_n^2}{s^2+\zeta\omega_n s+\omega_n^2}$ <br> $Y(s)=\frac{C}{s}\frac{\omega_d}{(s+\alpha)^2+\omega_d^2}$ | $\rightarrow y(t)=1-\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin(\omega_d t+\beta)$ <br> $\beta=\cos^{-1}\zeta$ |   | $y_{ss}=\lim_{s\to 0} s\,Y(s)$ <br> $y_{ss}=FV=1$ |

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**Note:** deriving y(t) itself is not our focus in this module because we already learned how to do it from previous modules ([CLTF=Y(s)/U(s)] → Y(s) → PFE + Inv.L → y(t)).

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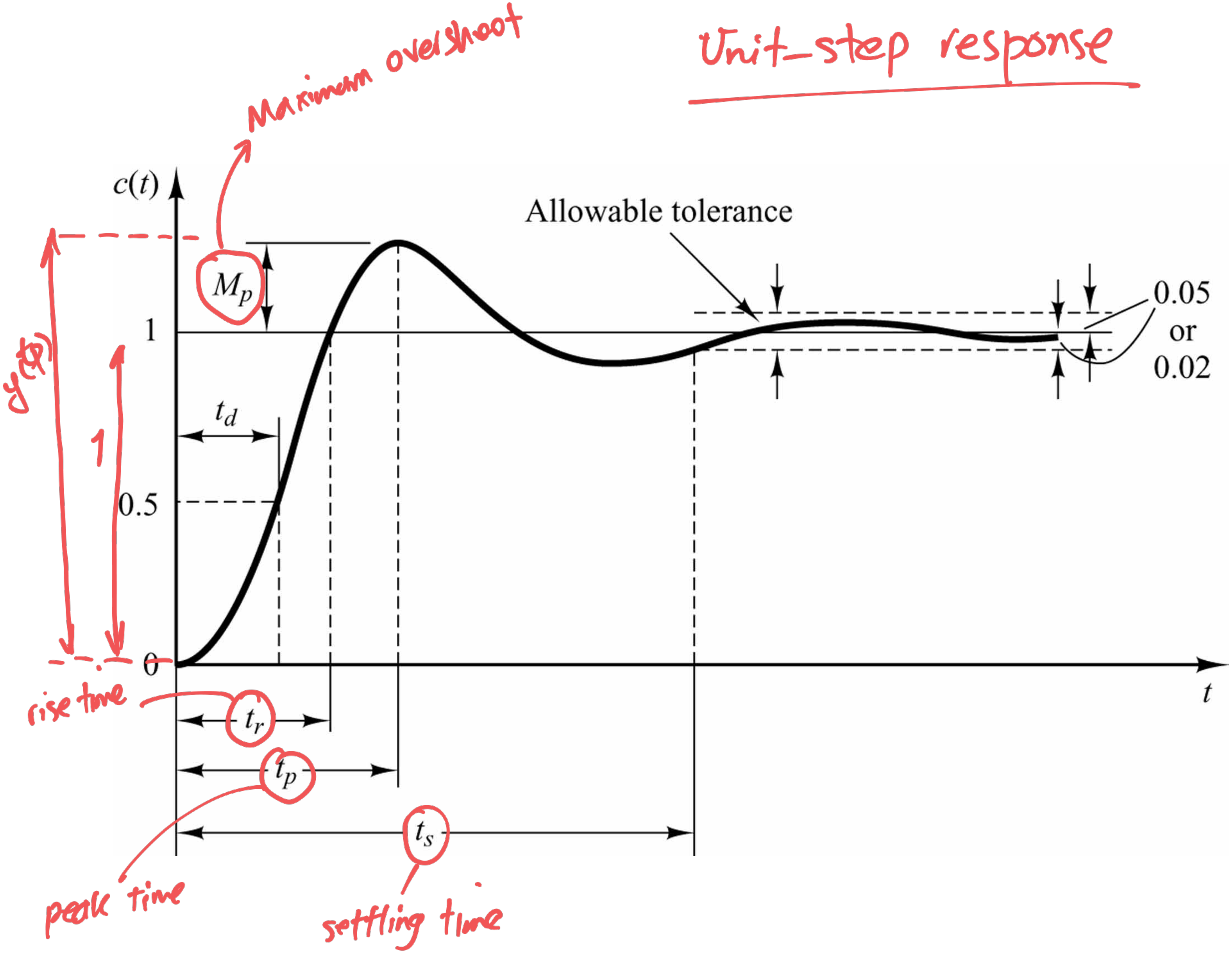
**Why are we doing transient analysis?**
**Transient Response** gives us the **performance** of the system i.e. how it reacts to different inputs over time. For example, how fast it reaches and settles at the steady-state response.

# Transient Response Specifications

Unit-step response



1) Rise Time: the first time $y(t)$ reaches final value $= \underline{1}$

$$y(t) = \cancel{1} - \boxed{\underbrace{\frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}}}_{0,\ t \to \infty} \underbrace{\sin(\omega_d t + \beta)}_{\to 0 \text{ when } \omega_d t + \beta = \pi}} = \cancel{1}$$

$$\beta = \cos^{-1} \xi$$

$$\Rightarrow \boxed{t_r = \frac{\pi - \beta}{\omega_d}} \rightarrow \text{related to both } \omega_n, \xi$$

2) peak time: Time for the largest peak (first peak)

$$\frac{dy}{dt} = 0 \Rightarrow \text{solve for } t \Rightarrow \omega_d t_p = \pi$$

$$\Rightarrow \boxed{t_p = \frac{\pi}{\omega_d}} \rightarrow \omega_n, \xi$$

$$\omega_d = \omega_n \sqrt{1-\xi^2}$$

3) Maximum Overshoot: The amount $y(t)$ goes over $\underline{1}$

$$M_p = y(t_p) - 1 \Rightarrow M_p = e^{-\pi\alpha/\omega_d}$$

$\alpha = \xi\omega_n$

$\omega_d = \omega_n\sqrt{1-\xi^2}$

$$\Rightarrow \boxed{M_p = e^{-\pi\xi/\sqrt{1-\xi^2}}}$$

related only to $\xi$

4) Settling time: the time after which $y(t)$ remains within $\varepsilon$ of $\underline{1}$

$\varepsilon$: 2% , 5%

$$|y(t) - 1| < \begin{cases} 0.05 \\ 0.02 \\ 0.01 \end{cases}$$

$$\frac{1}{\alpha} = \tau$$

$$t_s = \begin{cases} 3\tau \leftarrow \frac{3}{\alpha} = \frac{3}{\xi\omega_n} & (5\%) \\ 4\tau \leftarrow \frac{4}{\alpha} = \frac{4}{\xi\omega_n} & (2\%) \\ 5\tau \leftarrow 5/\alpha = 5/\xi\omega_n & (1\%) \end{cases}$$

## Exercise:

$$G(s) = \frac{25}{\boxed{s^2+6s+25}}$$

$t_r = ?$ $\quad t_p = ?$ $\quad M_p ?$

$t_s = ?$ (2%)

$$s^2 + \underbrace{2\xi\omega_n}_{6} s + \underbrace{\omega_n^2}_{25}$$

$\omega_n = 5$

$\xi = 0.6 \rightarrow \beta = \cos^{-1}\xi = 0.93$

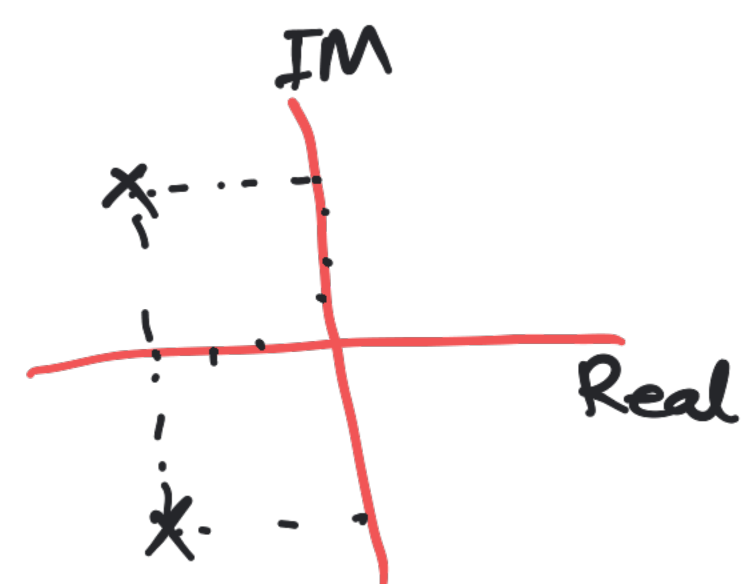
$\omega_d = \omega_n\sqrt{1-\xi^2} = 4$

$$t_r = \frac{\pi-\beta}{\omega_d} = \frac{\pi - 0.93}{4} = \boxed{0.55\ s}$$

$$t_p = \frac{\pi}{\omega_d} = \frac{3.14}{4} = \boxed{0.785\ s}$$

$$M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} = 0.095 \rightarrow \boxed{9.5\%}$$

$$t_s = \frac{4}{\xi\omega_n} = \frac{4}{(0.6)(5)} = \boxed{1.33\ s \quad 2\%}$$




$$P_{1,2} = -\alpha \pm j\omega_d = -\xi\omega_n \pm j\,\omega_n\sqrt{1-\xi^2} = -3 \pm j4$$

# One last thing!

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**Why are we doing transient analysis?**

**Transient Response** gives us the **performance** of the system i.e. how it reacts to different inputs over time. For example, how fast it reaches and settles at the steady-state response.

**Why do we care about Closed-Loop Poles?**

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**Why do we care about Closed-Loop Poles?**

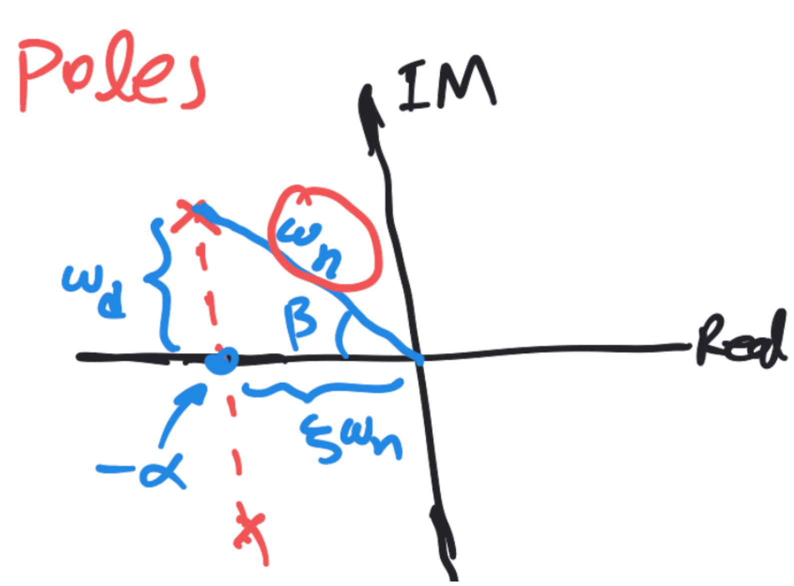
There are multiple reasons (Poles are shown on the s-plane):

1) We can conclude about **stability/convergence** without solving for y(t)

   Any Pole on the RHP means that the system is unstable

   Poles on the imaginary axis means that the system is oscillatory

   Poles on the LHP means that the system is stable (Convergent).



# $2^{nd}$-order System

**We have already derived the equation for Closed-Loop Poles of the system.**

$$\boxed{s_{1,2} = P_{1,2} = -\alpha \pm j\omega_d}$$

Poles

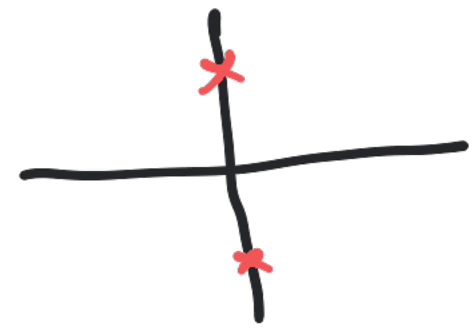
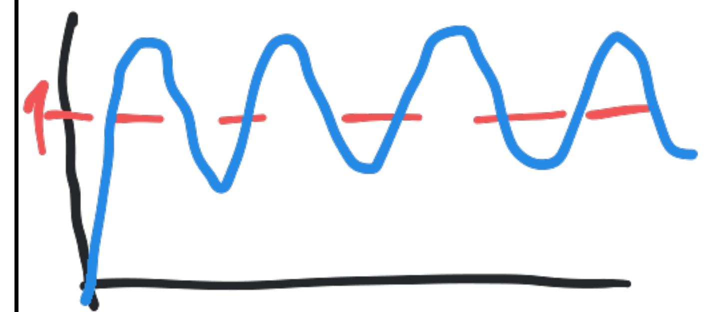
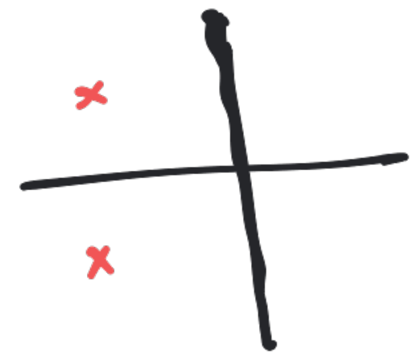
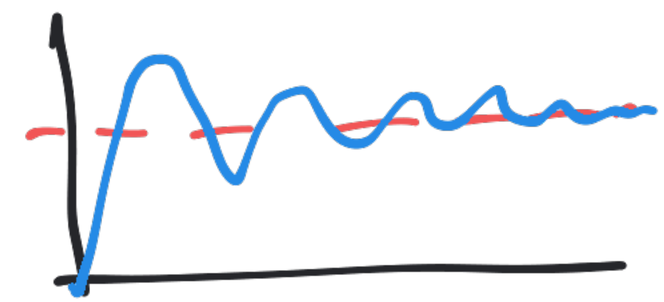
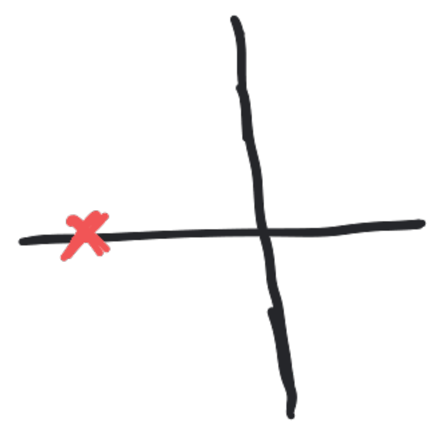
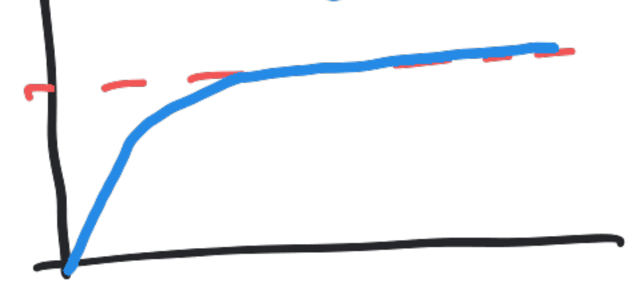
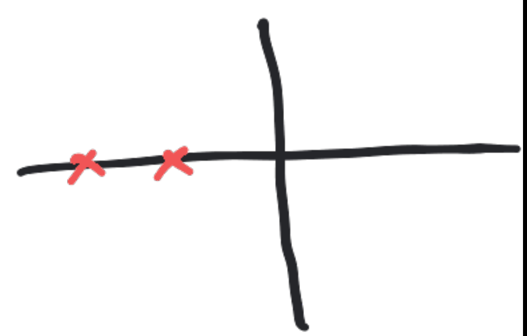
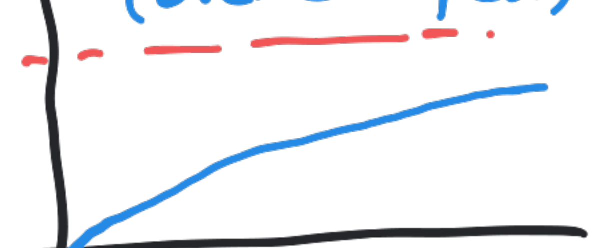
$$\boxed{P_{1,2} = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}}$$

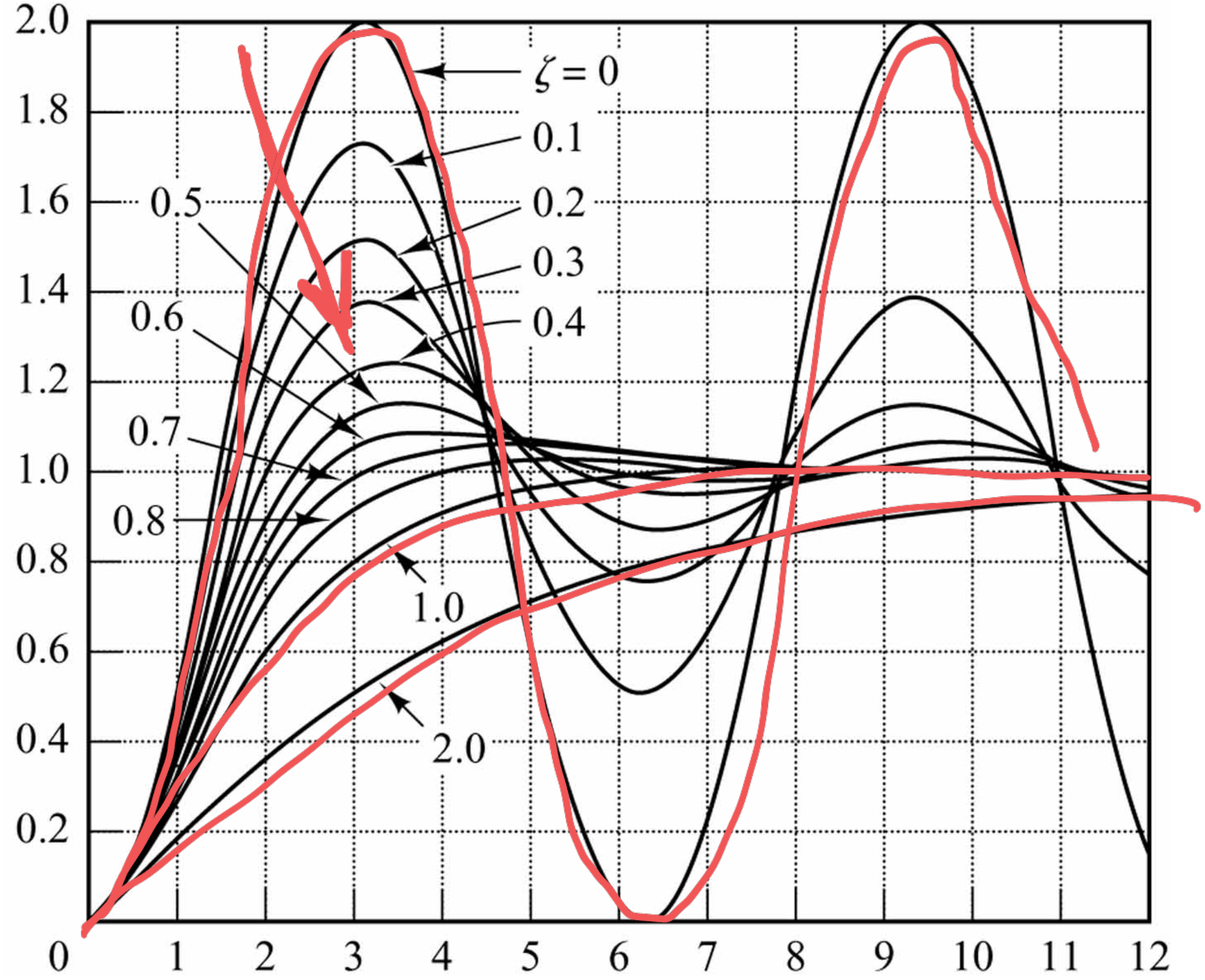

**As can be seen, <u>Damping Ratio</u> affects the <u>location of the Poles on the s-plane</u>.**

**<u>Location of Poles (root locus)</u> affects the <u>transient response of the system</u>.**

**Therefore, <u>Damping Ratio</u> directly affects the <u>transient response of the system</u>.**

# There are four possible cases in the 2nd-order System Response

| Damping Ratio | Poles $P_{1,2} = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$ | Poles on the s-Plane | How it affects the output | Converge? |
|---|---|---|---|---|
| $\xi=0$ | $P_{1,2} = \pm j\omega_n$ undamped |  |  | NO! Osilation |
| $0<\xi<1$ | $P_{1,2} = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$ |  |  | Yes |
| $\xi=1$ | $P_{1,2} = -\omega_n \pm 0$ Repeated real poles |  | critically damped  | Yes No oscil |
| $\xi>1$ | $P_{1,2} = -\xi\omega_n \pm \omega_n\sqrt{\xi^2-1}$ Real distinct poles |  | No oscillation (over damped)  | Yes But very slowly |



2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
1
2
3
4
5
6
7
8
9
10
11
12
ζ = 0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
1.0
2.0

$$Y(s) = \frac{K}{\boxed{s^2 - 6s + 5}}$$

$$(s-1)(s-5) = 0$$

$$\Rightarrow s = +1 \ \& \ s = +5$$

$$\underbrace{y(t) = C_1 e^{5t} + C_2 e^{t}}_{\text{unstalbe}}$$

$$Y(s) = \frac{K}{\boxed{s^2 + 6s + 5}}$$

$$(s+5)(s+1) = 0$$

$$\Rightarrow s = -5 \ \& \ s = -1$$

$$\underbrace{y(t) = C_1 e^{-5t} + C_2 e^{-t}}_{\text{stable}}$$

# Routh's stability Criterion :