

Kirchhoff's Laws

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$$\textcircled{1} I = \text{current (amps)}$$

$$V = \text{voltage (V)}$$

$$R = \text{resistance (ohms)}$$

a) Kirchhoff's current law/Junction rule

$$\sum I_{\text{entering}} - \sum I_{\text{leaving}} = 0$$

at any given junction

b) Kirchhoff's voltage law/loop rule

$$\sum V = 0, \text{ around any loop}$$

c) Ohm's law

$$I = \frac{V}{R}, \text{ at every resistor}$$

②

$$V_1 = I_1 R_{11} + I_2 R_{12} + \dots + I_n R_{1n}$$

$$V_2 = I_1 R_{21} + I_2 R_{22} + \dots + I_n R_{2n}$$

\vdots

$$V_n = I_1 R_{n1} + I_2 R_{n2} + \dots + I_n R_{nn}$$

$$\begin{bmatrix} I_1 R_{11} & I_2 R_{12} & \dots & I_n R_{1n} \\ I_1 R_{21} & I_2 R_{22} & \dots & I_n R_{2n} \\ \vdots & \vdots & \ddots & \vdots \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix}$$

③

$$2) 40 = 12I_1 - 7I_2 - 4I_4$$

$$30 = 15I_2 - 7I_1 - 6I_3$$

$$20 = 14I_3 - 6I_2 - 5I_4$$

$$-10 = 13I_4 - 4I_1 - 5I_3$$

$$\begin{bmatrix} 12 & -7 & 0 & -4 & 40 \\ -7 & 15 & -6 & 0 & 30 \\ 0 & -6 & 14 & -5 & 20 \\ -4 & 0 & -5 & 13 & -10 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -\frac{7}{12} & 0 & -\frac{1}{3} & \frac{10}{3} \\ -7 & 15 & -6 & 0 & 30 \\ 0 & -6 & 14 & -5 & 20 \\ -4 & 0 & -5 & 13 & -10 \end{bmatrix} \xrightarrow{+7h_1} \begin{bmatrix} 1 & -\frac{7}{12} & 0 & -\frac{1}{3} & \frac{10}{3} \\ 0 & \frac{131}{12} & -6 & -\frac{7}{3} & \frac{160}{3} \\ 0 & -6 & 14 & -5 & 20 \\ -4 & 0 & -5 & 13 & -10 \end{bmatrix}$$

$$\xrightarrow{+4h_1} \begin{bmatrix} 1 & -\frac{7}{12} & 0 & -\frac{1}{3} & \frac{10}{3} \\ 0 & \frac{131}{12} & -6 & -\frac{7}{3} & \frac{160}{3} \\ 0 & -6 & 14 & -5 & 20 \\ 0 & -\frac{7}{3} & -5 & \frac{35}{3} & \frac{10}{3} \end{bmatrix} \xrightarrow{\frac{12}{131}h_2} \begin{bmatrix} 1 & -\frac{7}{12} & 0 & -\frac{1}{3} & \frac{10}{3} \\ 0 & 1 & -\frac{72}{131} & -\frac{28}{131} & \frac{640}{131} \\ 0 & -6 & 14 & -5 & 20 \\ 0 & -\frac{7}{3} & -5 & \frac{35}{3} & \frac{10}{3} \end{bmatrix}$$

$$\xrightarrow{+6h_2} \begin{bmatrix} 1 & -\frac{7}{12} & 0 & -\frac{1}{3} & \frac{10}{3} \\ 0 & 1 & -\frac{72}{131} & -\frac{28}{131} & \frac{640}{131} \\ 0 & -6 & 14 & -5 & 20 \\ 0 & -\frac{7}{3} & -5 & \frac{35}{3} & \frac{10}{3} \end{bmatrix} \xrightarrow{+6h_2+h_3} \begin{bmatrix} 1 & -\frac{7}{12} & 0 & -\frac{1}{3} & \frac{10}{3} \\ 0 & 1 & -\frac{72}{131} & -\frac{28}{131} & \frac{640}{131} \\ 0 & 0 & \frac{142}{131} & -\frac{803}{131} & \frac{6460}{131} \\ 0 & -\frac{7}{3} & -5 & \frac{35}{3} & \frac{10}{3} \end{bmatrix}$$

$$\xrightarrow{+\frac{12}{131}h_2} \begin{bmatrix} 1 & -\frac{7}{12} & 0 & -\frac{1}{3} & \frac{10}{3} \\ 0 & 1 & -\frac{72}{131} & -\frac{28}{131} & \frac{640}{131} \\ 0 & 0 & \frac{142}{131} & -\frac{803}{131} & \frac{6460}{131} \\ 0 & -\frac{7}{3} & -5 & \frac{35}{3} & \frac{10}{3} \end{bmatrix} \xrightarrow{+\frac{12}{131}h_2} \begin{bmatrix} 1 & -\frac{7}{12} & 0 & -\frac{1}{3} & \frac{10}{3} \\ 0 & 1 & -\frac{72}{131} & -\frac{28}{131} & \frac{640}{131} \\ 0 & 0 & \frac{142}{131} & -\frac{803}{131} & \frac{6460}{131} \\ 0 & 0 & -\frac{623}{131} & \frac{1463}{131} & \frac{1432}{131} \end{bmatrix}$$

$$\begin{array}{l} 823 R_3 \\ 131 \\ \hline \Rightarrow \end{array} \left[\begin{array}{cccc|c} 1 & -\frac{7}{12} & 0 & -\frac{1}{3} & \frac{10}{3} \\ 0 & 1 & \frac{72}{131} & \frac{28}{131} & \frac{640}{131} \\ 0 & 0 & 1 & -\frac{823}{1402} & \frac{3220}{701} \\ 0 & 0 & 0 & \frac{10467}{1402} & \frac{30626}{70} \end{array} \right] \xrightarrow{1402} \left[\begin{array}{cccc|c} 1 & -\frac{7}{12} & 0 & -\frac{1}{3} & \frac{10}{3} \\ 0 & 1 & \frac{72}{131} & \frac{28}{131} & \frac{640}{131} \\ 0 & 0 & 1 & -\frac{823}{1402} & \frac{3220}{701} \\ 0 & 0 & 0 & 1 & \frac{61240}{10467} \end{array} \right]$$

$$\begin{array}{l} 6232 R_3 \\ 1402 \\ \hline \Rightarrow \end{array} \left[\begin{array}{cccc|c} 1 & -\frac{7}{12} & 0 & -\frac{1}{3} & \frac{10}{3} \\ 0 & 1 & \frac{72}{131} & \frac{28}{131} & \frac{640}{131} \\ 0 & 0 & 1 & 0 & \frac{44270}{10467} \\ 0 & 0 & 0 & 1 & \frac{61240}{10467} \end{array} \right] \xrightarrow{28} \left[\begin{array}{cccc|c} 1 & -\frac{7}{12} & 0 & -\frac{1}{3} & \frac{10}{3} \\ 0 & 1 & \frac{72}{131} & 0 & \frac{8428410}{137397} \\ 0 & 0 & 1 & 0 & \frac{84270}{10467} \\ 0 & 0 & 0 & 1 & \frac{61240}{10467} \end{array} \right]$$

$$\begin{array}{l} 3 R_1 \\ 131 \\ \hline \Rightarrow \end{array} \left[\begin{array}{cccc|c} 1 & -\frac{7}{12} & 0 & 0 & \frac{55370}{10467} \\ 0 & 1 & \frac{72}{131} & 0 & \frac{8424400}{1373797} \\ 0 & 0 & 1 & 0 & \frac{84270}{10467} \\ 0 & 0 & 0 & 1 & \frac{61240}{10467} \end{array} \right] \xrightarrow{131} \left[\begin{array}{cccc|c} 1 & -\frac{7}{12} & 0 & 0 & \frac{55370}{10467} \\ 0 & 1 & 0 & 0 & \frac{110640}{10467} \\ 0 & 0 & 1 & 0 & \frac{84270}{10467} \\ 0 & 0 & 0 & 1 & \frac{61240}{10467} \end{array} \right]$$

$$\begin{array}{l} 2 R_2 \\ 12 \\ \hline \Rightarrow \end{array} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & \frac{114910}{10467} \\ 0 & 1 & 0 & 0 & \frac{110640}{10467} \\ 0 & 0 & 1 & 0 & \frac{84270}{10467} \\ 0 & 0 & 0 & 1 & \frac{61240}{10467} \end{array} \right]$$

$$\begin{array}{l} I_1 = 11.43 \\ I_2 = 10.55 \\ I_3 = 8.04 \\ I_4 = 5.84 \end{array}$$

$$(8) \quad 50 = 9I_1 - I_2 - I_4 - 4I_5$$

$$-30 = 7I_2 - I_1 - 2I_3 - 3I_5$$

$$20 = 10I_3 - 2I_2 - 3I_5 - 3I_4$$

$$-40 = 7I_4 - 3I_3 - I_1 - 2I_5$$

$$0 = 12I_5 - 4I_1 - 3I_2 - 3I_3 - 2I_4$$

$$\left[\begin{array}{ccccc|c} 9 & -1 & 0 & -1 & -4 & 50 \\ -1 & 7 & -2 & 0 & -3 & -30 \\ 6 & -2 & 10 & -3 & -3 & 20 \\ -1 & 0 & -3 & 7 & -2 & -40 \\ -4 & -3 & -3 & -2 & 12 & 0 \end{array} \right]$$

$$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 0 & 3019 \\ 0 & 1 & 0 & 0 & 0 & 255 \\ 0 & 0 & 1 & 0 & 0 & -3309 \\ 0 & 0 & 0 & 1 & 0 & 683 \\ 0 & 0 & 0 & 0 & 1 & -722 \end{array} \right]$$

$$I_1 = \frac{3019}{755} \quad I_2 = \frac{-3309}{755}$$

$$I_3 = \frac{-683}{755} \quad I_4 = \frac{-4382}{755}$$

$$I_5 = \frac{-722}{755}$$