Written HW #1

1. $$\begin{pmatrix}1\\4\\2\end{pmatrix}x_1 + \begin{pmatrix}-2\\0\\1\end{pmatrix}x_2 + \begin{pmatrix}-3\\1\\-1\end{pmatrix}x_3 + \begin{pmatrix}0\\1\\1\end{pmatrix}x_4 = \begin{pmatrix}14\\22\\-5\end{pmatrix}$$

$x_1 - 2x_2 - 3x_3 = 14$

$4x_1 + x_3 + x_4 = 22$

$2x_1 + x_2 - x_3 + x_4 = -5$

$2x_1 - x_2 + 2x_3 = 27$

$3x_2 + 8x_3 = -1$

$x_2 = -\frac{1}{3} - \frac{8}{3}x_3$

$2x_1 + \frac{1}{3} + \frac{8}{3}x_3 + 2x_3 = 27$

$x_1 = \frac{40}{3} - \frac{7}{3}x_3$

$x_4 = -\frac{94}{3} + \frac{25}{3}x_3$

$x_3 = k$

$x_1 = \frac{1}{3}(40-7k) \quad x_2 = \frac{1}{3}(-1-8k)$

$x_4 = \frac{1}{3}(-94+25k)$

$$\boxed{\begin{pmatrix}x_1\\x_2\\x_3\\x_4\end{pmatrix} = \begin{pmatrix}\frac{1}{3}(40-7k)\\ \frac{1}{3}(-1-8k)\\ k\\ \frac{1}{3}(-94+25k)\end{pmatrix} \text{ where } k \in \mathbb{R}}$$

2. $p(x) = ax^3 + bx^2 + cx + d$

$p(0) = 1 \quad d = 1$

$p(x) = ax^3 + bx^2 + cx + 1$

$p(-1) = 1 \Rightarrow -a + b - c + 1 = 1 \Rightarrow a - b + c = 0$

$p(1) = 4 \Rightarrow a + b + c + 1 = 4 \Rightarrow a + b + c = 3$

$p(2) = 4 \Rightarrow 8a + 4b + 2c + 1 = 4 \Rightarrow 4a + 2b + c = \frac{3}{2}$

$2b = 3 \Rightarrow b = \frac{3}{2}$

$a - \frac{3}{2} + c = 0 \Rightarrow a + c = \frac{3}{2}$

$4a + c = -\frac{3}{2}$

$3a = -\frac{6}{2} \Rightarrow a = -2$

$-2 + c = \frac{3}{2} \Rightarrow c = \frac{7}{2}$

3. $$\begin{pmatrix}a+b & a-b\\ c+d & c-d\end{pmatrix} = \begin{pmatrix}a+c & b+d\\ a-c & b-d\end{pmatrix}$$

$a + b = a + c \qquad c + d = a - c$

$b = c \qquad a = 2c + d$

$a - b = b + d \qquad c - d = b - d$

$a = 2b + d \qquad c = b$

$$\begin{pmatrix}a & b\\ c & d\end{pmatrix} = \begin{pmatrix}2b+d & b\\ b & d\end{pmatrix}$$

$$\begin{pmatrix}a & b\\ c & d\end{pmatrix} = b\begin{bmatrix}2 & 1\\ 1 & 0\end{bmatrix} + d\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$\begin{pmatrix}a & b\\ c & d\end{pmatrix} = b\begin{bmatrix}2 & 1\\ 1 & 0\end{bmatrix} + d\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

Written HW #1 Cont.

4. a) Let $R = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

$$R^2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0(0)+1(0) & 0(1)+1(0) \\ 0(0)+0(0) & 0(1)+0(0) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

b) $(I-M)(I+M+M^2) = I + M + M^2 - M - M^2 - M^3 = I - M^3 = I$

$M^3 = 0 \Rightarrow I - 0 = I$

$(I-M)^{-1} = I + M + M^2$

c) $(I-R)(I+R) = I + R - R - R^2 = I - R^2 = I - 0 = I \leftarrow$ bc $R^2 = 0$

$(I-R)^{-1} = I + R$

5. $v_1 \neq v_2 \quad Av_1 = Av_2$

let $v = v_1 - v_2 \neq 0$

$Av = A(v_1 - v_2)$

$Av = Av_1 - Av_2$

$Av = 0$

$Av_1 = Av_2$

$v = v_1 - v_2 \neq 0$ such that $Av = 0$

6. $M_1 = \left(\begin{array}{c|c} A & x \\ \hline 0 & B \end{array}\right) \quad M_2 = \left(\begin{array}{c|c} A^{-1} & -A^{-1}xB^{-1} \\ \hline 0 & B^{-1} \end{array}\right)$

$$M_1 \cdot M_2 = \left(\begin{array}{c|c} A & x \\ \hline 0 & B \end{array}\right) \cdot \left(\begin{array}{c|c} A^{-1} & -A^{-1}xB^{-1} \\ \hline 0 & B^{-1} \end{array}\right)$$

$$= \begin{pmatrix} AA^{-1} & -AA^{-1}(B)^{-1} + xB^{-1} \\ 0 & BB^{-1} \end{pmatrix}$$

$$= \begin{pmatrix} I_2 & -xB^{-1} + xB^{-1} \\ 0 & I_2 \end{pmatrix}$$

$$= \begin{pmatrix} I_2 & 0 \\ 0 & I_2 \end{pmatrix} = I_4$$

$$M_2 M_1 = \left(\begin{array}{c|c} A^{-1} & -A^{-1}xB^{-1} \\ \hline 0 & B^{-1} \end{array}\right)\left(\begin{array}{c|c} A & x \\ \hline 0 & B \end{array}\right)$$

$$= \begin{pmatrix} A^{-1}A & A^{-1}x - A^{-1}xB^{-1}B \\ 0 & B^{-1}B \end{pmatrix}$$

$$= \begin{pmatrix} I_2 & A^{-1}x - A^{-1}x \\ 0 & I_2 \end{pmatrix} = \begin{pmatrix} I_2 & 0 \\ 0 & I_2 \end{pmatrix} = I_4$$

$M_1 M_2 = M_2 M_1 = I_4$

$\Rightarrow M_1$ is inverse of M_2

Cont. on back $\rightarrow$

Written HW #1 Cont.

#6 cont.

$$\begin{pmatrix} 1 & 0 & 2 & -1 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 3 & -2 \\ 0 & 0 & -1 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^{-1} & -\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 2 & -1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}^{-1} \\ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}^{-1} \end{pmatrix}$$

$$= \begin{pmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} & -\begin{bmatrix} 2 & -1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} & \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \cdot & \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & -1 & -1 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 3 \end{pmatrix}$$