Final preparation

The final will contain **10 questions**. This final preparation contains **20 questions** similar in style to the final questions.

Throughout this exam, let $A = [a_1, a_2, a_3, a_4] = \begin{bmatrix} 1 & 5 & 1 & 2 \\ 2 & 11 & 5 & 0 \\ 3 & -1 & 4 & 0 \end{bmatrix}$

(1) Write the augmented matrix for the matrix equation $Ax = [3, -1, 2]^\top$.

(2) Use row reduction to find a parametric solution for $Ax = [3, -1, 2]^\top$.

(3) Does $Ax = y$ have a solution for any $y = [y_1, y_2, y_3]$? Why, or why not?

(4) What is the dimension of the nullspace of A? Why?

(5) Find a linear dependence between the columns of A or explain why none exists.

(6) Do there exist distinct vectors v_1, v_2 such that $Av_1 = Av_2$? Why, or why not?

$$\text{Let } B = [b_1, b_2, b_3] = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

(7) Find determinant of B. What is the rank of B?

(8) Write the characteristic polynomial and compute all eigenvalues.

(9) Are the columns of B linearly independent? Why, or why not?

(10) Does B^{-1} exist? Explain why.

(11) Find an eigenvector for each eigenvalue of B

(12) Diagonalize B, i.e. Find the invertible matrix P and diagonal matrix D satisfying $B = PDP^{-1}$. (**Note:** This matrix is more challenging than what you would be given on the exam - it's good practice however!)

(13) Explain why $B^{-1} = PD^{-1}P^{-1}$.

(14) Find a closed form for $B^n \begin{bmatrix} 2 \\ -6 \\ 2 \end{bmatrix}$. for any n.

$$\text{Let } C = [c_1, c_2, c_3] = \begin{bmatrix} 1 & 2 & 1 \\ 1 & -1 & 2 \\ -1 & 3 & 1 \end{bmatrix}$$

(15) Compute $c_1 \cdot c_2$ and $c_1 \cdot c_3$. Without performing the computation, explain why c_1 is orthogonal to $c_2 + c_3$.

(16) Compute the vectors d_1 and d_2 of length 1 pointing in the directions of c_1 and $c_2 + c_3$ respectively.

(17) Find a vector d_3 which makes $\{d_1, d_2, d_3\}$ into an orthonormal basis of $\mathbb{R}^3$.

(18) Write c_2 as a linear combination of the vectors d_1, d_2 and d_3.

(19) Compute the projection of c_1 onto $\mathrm{Span}(c_2)$, and the projection onto $\mathrm{Span}(c_2, c_3)$.

(20) Find the point z in the plane spanned by c_1 and c_3 closest to c_2. What is the distance from z to c_2?