Assignment WW-Diagonalise

1. (1 point) Find the eigenvalues $\lambda_1 < \lambda_2$ and associated unit eigenvectors $\vec{u}_1, \vec{u}_2$ of the symmetric matrix

$$A = \begin{bmatrix} -7 & 16 \\ 16 & 17 \end{bmatrix}.$$

The smaller eigenvalue $\lambda_1 =$ _____ has associated unit eigenvector $\vec{u}_1 = \begin{bmatrix} \_\_\_\_\_ \\ \_\_\_\_\_ \end{bmatrix}$.

The larger eigenvalue $\lambda_2 =$ _____ has associated unit eigenvector $\vec{u}_2 = \begin{bmatrix} \_\_\_\_\_ \\ \_\_\_\_\_ \end{bmatrix}$.

Note: The eigenvectors above form an orthonormal eigenbasis for A.

Correct Answers:

- -15
- $\begin{bmatrix} -0.894427 \\ 0.447214 \end{bmatrix}$
- 25
- $\begin{bmatrix} -0.447214 \\ -0.894427 \end{bmatrix}$

2. (1 point) Find a 2×2 matrix A such that

$$\begin{bmatrix} -3 \\ 2 \end{bmatrix}, \quad \text{and} \quad \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

are eigenvectors of A with eigenvalues 4 and -7, respectively.

$A = \begin{bmatrix} \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ \end{bmatrix}$

Correct Answers:

- $\begin{bmatrix} 4 & 0 \\ -7.33333 & -7 \end{bmatrix}$

3. (1 point) Find the eigenvalues $\lambda_1 < \lambda_2$ and associated orthonormal eigenvectors of the symmetric matrix

$$A = \begin{bmatrix} 5 & 0 & 0 & 3 \\ 0 & 5 & 3 & 0 \\ 0 & 3 & 5 & 0 \\ 3 & 0 & 0 & 5 \end{bmatrix}.$$

$\lambda_1 =$ _____ has associated orthonormal eigenvectors $\begin{bmatrix} \_\_\_\_\_ \\ \_\_\_\_\_ \\ \_\_\_\_\_ \\ \_\_\_\_\_ \end{bmatrix}, \begin{bmatrix} \_\_\_ \\ \_\_\_ \\ \_\_\_ \\ \_\_\_ \end{bmatrix}$.

$\lambda_2 =$ _____ has associated orthonormal eigenvectors $\begin{bmatrix} \_\_\_\_\_ \\ \_\_\_\_\_ \\ \_\_\_\_\_ \\ \_\_\_\_\_ \end{bmatrix}, \begin{bmatrix} \_\_\_ \\ \_\_\_ \\ \_\_\_ \\ \_\_\_ \end{bmatrix}$.

Note: The eigenvectors above form an orthonormal eigenbasis for A.

Correct Answers:

- 2
- [[0.5],[0.5],[-0.5],[-0.5]], $\begin{bmatrix} -0.5 \\ 0.5 \\ -0.5 \\ 0.5 \end{bmatrix}$
- 8
- [[0.5],[0.5],[0.5],[0.5]], $\begin{bmatrix} 0.5 \\ -0.5 \\ -0.5 \\ 0.5 \end{bmatrix}$