MA2071 - Assignment II

PADRAIG Ó CATHÁIN & QINGSHUO SONG
WPI

April 16, 2020

This assignment assesses the material covered in Modules 6-10. Write full and complete solutions, using full sentences where appropriate. Explain all row operations when computing an RREF. Answer the questions asked.

Questions 1-5 are worth 20 points each. The bonus question is worth 10 points (but points on this assignment are capped at 100).

Recommended Deadline: April 24th
Final Deadline: May 1st.

1. Compute the inverse of the matrix

$$A = \begin{pmatrix} 1 & 4 & 2 \\ 3 & -1 & 0 \\ 1 & 1 & 2 \end{pmatrix}$$

   Hence solve the linear system $Ax = b$ where $b = [1, 2, 3]^\top$.

2. Consider the following matrix and vectors.

$$B = \begin{pmatrix} -7 & 12 & -18 \\ -18 & 23 & -30 \\ -8 & 8 & -9 \end{pmatrix}, \quad v_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \quad v_2 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}.$$

   (a) Show that v_1 and v_2 are eigenvectors of B and find the corresponding eigenvalues.

   (b) Express $v_3 = [0, -4, -4]^\top$ as a linear combination of v_1 and v_2, and hence compute $B^{10}v_3$.

3. Compute the characteristic polynomial of the matrix

$$C = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

   Hence show that C has no non-zero eigenvectors with real entries. Find the complex-values eigenvectors of C.

4. Let D be a 3×3 diagonal matrix with diagonal entries $\lambda_1, \ldots, \lambda_3$, which are distinct real numbers. Describe in your own words the eigenvalues and eigenvectors of D.

5. Suppose that E is a 4×4 matrix in which every row sums to the constant k. Explain why $u = [1, 1, 1, 1]^\top$ is an eigenvector of E, and give the corresponding eigenvalue.

6. **Bonus.** Describe all matrices which satisfy $\det(-A) = -\det(A)$.