MA 2071 - *Matrices & Linear Algebra I* C-term Spring 2020
Sections C01 - C09

Date: January 22, 2020

Name: ______________________________

Quiz 1

Use of the textbook or notes is not allowed. No electronic devices or calculators are allowed. To get credit, you must show **ALL** of your work, unless otherwise stated in the problem. Please do not cheat. *"The first and worst of all frauds is to cheat one's self."*

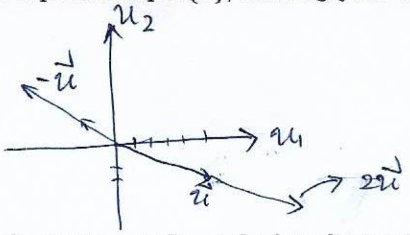
Read each question carefully and follow the directions stated in each question.

1. Consider the vector $\vec{u} = \begin{bmatrix} 5 \\ -2 \end{bmatrix}$.

   (a) Can you build the vector $\vec{w} = \begin{bmatrix} 1/5 \\ -1/2 \end{bmatrix}$ out of only $\vec{u}$? Justify your answer. (1 point)

   No, sine $\vec{w}$ cannot be written as $c\vec{u}$ where c is some scalar!

   (b) Recall that span$\{\vec{u}\}$ is the collection of all possible vectors that can be built out of just $\vec{u}$. Graph the geometric object that represents span$\{\vec{u}\}$, labeling your axes and making sure all vectors start at the origin. (2 points)



2. (a) Use either *Gaussian elimination* or *Gauss Jordan elimination* to solve the following system of equations. Clearly state which method you are using. (3 points)

$$\begin{cases} 2x_1 + 5x_2 = 1 \\ -x_1 - x_2 = 1 \end{cases}$$

$$\left[\begin{array}{cc|c} 2 & 5 & 1 \\ -1 & -1 & 1 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 5/2 & \frac{1}{2} \\ -1 & -1 & 1 \end{array}\right] \tfrac{1}{2}R_1 \sim \left[\begin{array}{cc|c} 1 & 5/2 & 1/2 \\ 0 & 3/2 & 3/2 \end{array}\right] R_1+R_2 \sim \left[\begin{array}{cc|c} 1 & 5/2 & 1/2 \\ 0 & 1 & 1 \end{array}\right] \tfrac{2}{3}R_2$$

$$\sim \left[\begin{array}{cc|c} 1 & 0 & -2 \\ 0 & 1 & 1 \end{array}\right] R_1 - \tfrac{5}{2}R_2$$

So, $x_1 = -2$ and $x_2 = 1$

using Gauss Jordan elimination.

   (b) Suppose you are trying to solve a system of equations in matrix form, and, after row reducing the system, you obtain the augmented matrix:

$$\left(\begin{array}{ccccc|c} 1 & 6 & 0 & 3 & 0 & 0 \\ 0 & 0 & 1 & -8 & 0 & 5 \\ 0 & 0 & 0 & 0 & 1 & 7 \end{array}\right).$$

State the solution(s) of the original system, and clearly label any free variables. (4 points)

$x_5 = 7$

$x_3 = 5 + 8x_4$

$x_1 = -6x_2 - 3x_4$

x_2 and x_4 are free variable

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -6x_2 - 3x_4 \\ x_2 \\ 5+8x_4 \\ x_4 \\ 7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 5 \\ 0 \\ 7 \end{bmatrix} + \begin{bmatrix} -6 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} x_2 + \begin{bmatrix} -3 \\ 0 \\ 8 \\ 1 \\ 0 \end{bmatrix} x_4$$

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Read each question carefully and follow the directions stated in each question.

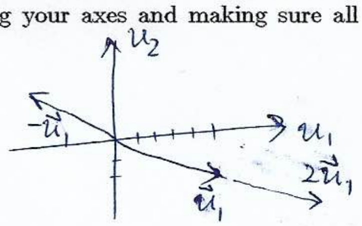
1. Consider the vector $\vec{u} = \begin{bmatrix} 5 \\ -2 \end{bmatrix}$.

   (a) Can you build the vector $\vec{w} = \begin{bmatrix} -20 \\ 8 \end{bmatrix}$ out of only $\vec{u}$? Justify your answer. (1 point)

   Yes, since $\vec{w} = -4\vec{u}$

   (b) Recall that span$\{\vec{u}\}$ is the collection of all possible vectors that can be built out of just $\vec{u}$. Graph the geometric object that represents span$\{\vec{u}\}$, labeling your axes and making sure all vectors start at the origin. (2 points)

   let $\vec{u} = \begin{bmatrix} 5 \\ -2 \end{bmatrix}$



2. (a) Use either *Gaussian elimination* or *Gauss Jordan elimination* to solve the following system of equations. Clearly state which method you are using. (3 points)

$$\begin{cases} x_1 + 3x_2 = 1 \\ 3x_1 + x_2 = 1 \end{cases}$$

$$\left[\begin{array}{cc|c} 1 & 3 & 1 \\ 3 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 3 & 1 \\ 0 & -8 & -2 \end{array}\right]_{-3R_1+R_2} \sim \left[\begin{array}{cc|c} 1 & 3 & 1 \\ 0 & 1 & \frac{1}{4} \end{array}\right]_{(-\frac{1}{8})R_2} \sim \left[\begin{array}{cc|c} 1 & 0 & \frac{1}{4} \\ 0 & 1 & \frac{1}{4} \end{array}\right]^{R_1-3R_2}$$

So, $x_1 = \frac{1}{4}$ and $x_2 = \frac{1}{4}$ using Gauss Jordan elimination.

   (b) Suppose you are trying to solve a system of equations in matrix form, and, after row reducing the system, you obtain the augmented matrix:

$$\left(\begin{array}{ccccc|c} 1 & 6 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right).$$

   State the solution(s) of the original system, and clearly label any free variables. (4 points)

$x_5 = 7$

$x_1 = -6x_2 - 3x_4$ where x_2, x_3, x_4 are free. variable.

So, $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -6x_2 - 3x_4 \\ x_2 \\ x_3 \\ x_4 \\ 7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 7 \end{bmatrix} + \begin{bmatrix} -6 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} x_2 + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} x_3 + \begin{bmatrix} -3 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} x_4.$

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Read each question carefully and follow the directions stated in each question.

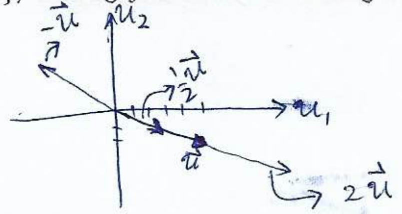
1. Consider the vector $\vec{u} = \begin{bmatrix} 5 \\ -2 \end{bmatrix}$.

   (a) Can you build the vector $\vec{w} = \begin{bmatrix} 25 \\ 4 \end{bmatrix}$ out of only $\vec{u}$? Justify your answer. (1 point)

   No, since there is no scalar c such that $\vec{w} = c\vec{u}$.

   (b) Recall that span$\{\vec{u}\}$ is the collection of all possible vectors that can be built out of just $\vec{u}$. Graph the geometric object that represents span$\{\vec{u}\}$, labeling your axes and making sure all vectors start at the origin. (2 points)

   let $\vec{u} = \begin{bmatrix} 5 \\ -2 \end{bmatrix}$



2. (a) Use either *Gaussian elimination* or *Gauss Jordan elimination* to solve the following system of equations. Clearly state which method you are using. (3 points)

$$\begin{cases} 2x_1 + 2x_2 = 1 \\ -x_1 - x_2 = 1 \end{cases}$$

$$\left[\begin{array}{cc|c} 2 & 2 & 1 \\ -1 & -1 & 1 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 1 & 1/2 \\ -1 & -1 & 1 \end{array}\right] : \tfrac{1}{2}R_1 \sim \left[\begin{array}{cc|c} 1 & 1 & 1/2 \\ 0 & 0 & 3/2 \end{array}\right] R_1 + R_2$$

The system has no solution by Gaussian elimination.

   (b) Suppose you are trying to solve a system of equations in matrix form, and, after row reducing the system, you obtain the augmented matrix:

$$\left(\begin{array}{ccccc|c} 1 & 6 & 0 & 3 & 0 & 0 \\ 0 & 0 & 1 & -8 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right).$$

State the solution(s) of the original system, and clearly label any free variables. (4 points)

x_2, x_4 and x_5 are free.

$x_3 = 5 + 8x_4$

$x_1 = -6x_2 - 3x_4$

$$\Rightarrow \vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -6x_2 - 3x_4 \\ x_2 \\ 5 + 8x_4 \\ x_4 \\ x_5 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 5 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -6 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} x_2 + \begin{bmatrix} -3 \\ 0 \\ 8 \\ 1 \\ 0 \end{bmatrix} x_4 + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} x_5}_{\text{Soln.}}$$