Written HW 4

1.

$$\left[\begin{array}{ccc|cc} 3 & 1 & -1 & 3 & 1 \\ -2 & 0 & -2 & -2 & 0 \\ -3 & -3 & 1 & -3 & -3 \end{array}\right]$$

$$3\cancel{(0)}(1) + 1(-2)(-3) + -1(-2)(-3)$$

$$-\left(-1\cancel{(0)}(-3) + 3(-2)(-3) + 1(-2)(1)\right)$$

$$-18 \qquad +3$$

$$= -16$$

2.

$$A = \begin{bmatrix} 3 & 1 & -1 \\ -2 & 0 & -2 \\ -3 & -3 & 1 \end{bmatrix} \qquad |A| = \det A = -16$$

$$\begin{bmatrix} +\begin{vmatrix} 0 & -2 \\ -3 & 1 \end{vmatrix} & -\begin{vmatrix} -2 & -2 \\ -3 & 1 \end{vmatrix} & +\begin{vmatrix} -2 & 0 \\ -3 & -3 \end{vmatrix} \\ -\begin{vmatrix} 1 & -1 \\ 0 & -2 \end{vmatrix} & +\begin{vmatrix} 3 & -1 \\ -3 & 1 \end{vmatrix} & -\begin{vmatrix} 3 & 1 \\ -3 & -3 \end{vmatrix} \\ +\begin{vmatrix} 1 & -1 \\ 0 & -2 \end{vmatrix} & -\begin{vmatrix} 3 & -1 \\ -2 & -2 \end{vmatrix} & +\begin{vmatrix} 3 & 1 \\ -2 & 0 \end{vmatrix} \end{bmatrix} = \begin{bmatrix} -(0+6) & -(-2-6) & -6+0 \\ -(-2+0) & 3-3 & -(-9+3) \\ 2+0 & -(-6-2) & 0-(-2) \end{bmatrix} = \begin{bmatrix} -6 & 8 & -6 \\ 2 & 0 & 6 \\ 2 & 8 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -6 & 2 & -2 \\ 8 & 0 & 8 \\ -6 & 6 & 2 \end{bmatrix} \cdot \frac{1}{-16} = \begin{bmatrix} 3/8 & -1/8 & 1/8 \\ -1/2 & 0 & -1/2 \\ -3/8 & -3/8 & -1/8 \end{bmatrix}$$

3.

$$\mathcal{E} = \left[\begin{array}{ccc|cc} 3 & 1 & 1 & 3 & 1 \\ -2 & 0 & -1 & -2 & 0 \\ -3 & -3 & c & -3 & -3 \end{array}\right]$$

$$\det \mathcal{E} = -1(-3) + (-2)(-3) - \left(c(-2) + (-1)(-3)(3) + 0\right)$$

$$\det \mathcal{E} = 2c + 6$$

$$2c + 6 = 0$$

$$c = \frac{-6}{2} = -3$$

for any $c \neq -3$, $\{a_1, a_2\}$ span $\mathbb{R}^3$

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4. $$\varepsilon = \begin{bmatrix} 3-\lambda & 1 & -1 \\ -2 & 0-\lambda & -2 \\ -3 & -3 & -\lambda \end{bmatrix} \begin{matrix} 3-\lambda & 1 \\ -2 & -\lambda \\ -3 & -3 \end{matrix}$$

$$\det \varepsilon = (3-\lambda)(-\lambda)(\lambda) + 1(-2)(-3) + -1(-2)(-3)$$
$$-(1(2)(\lambda) \quad + (3-\lambda)(-2)(-3) + -1(-\lambda)(-3))$$

$$-(\lambda-4)(\lambda-2)(\lambda+2) = 0$$
$$\lambda_1 = -2 \quad \lambda_2 = 2 \quad \lambda = 4$$

5. $\lambda_1 = -2$

$$\begin{bmatrix} 5 & 1 & -1 \\ -2 & 2 & -2 \\ -3 & -3 & 3 \end{bmatrix} \cdot \frac{1}{5} \Rightarrow \left[\begin{array}{ccc|c} 1 & \frac{1}{5} & \frac{-1}{5} & 0 \\ -2 & 2 & -2 & 0 \\ -3 & -3 & 3 & 0 \end{array}\right] R_2 \to R_2 - (-2)R_1 \Rightarrow \left[\begin{array}{ccc|c} 1 & \frac{1}{5} & \frac{-1}{5} & 0 \\ 0 & \frac{12}{5} & \frac{-12}{5} & 0 \\ -3 & -3 & 3 & 0 \end{array}\right] \Rightarrow$$

$$R_3 \to R_3 - (-3)R_1 \left[\begin{array}{ccc|c} 1 & \frac{1}{5} & -\frac{1}{5} & 0 \\ 0 & \frac{12}{5} & \frac{-12}{5} & 0 \\ 0 & \frac{-12}{5} & 12/5 & 0 \end{array}\right] \cdot \frac{5}{12} \Rightarrow \left[\begin{array}{ccc|c} 1 & \frac{1}{5} & \frac{-1}{5} & 0 \\ 0 & 1 & -1 & 0 \\ 0 & \frac{-12}{5} & \frac{12}{5} & 0 \end{array}\right] \cdot \frac{12}{5} \Rightarrow$$

$$\left[\begin{array}{ccc|c} 1 & 1/5 & -1/5 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow R_1 \to R - \frac{R_2}{5} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad \begin{matrix} x_1 = 0 \\ x_2 - x_3 = 0 \\ x_2 = x_3 \end{matrix}$$

for $\lambda_1 = -2 \Rightarrow v_1 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$

$\lambda_1 = 2$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ -2 & -2 & -2 & 0 \\ -3 & -3 & -1 & 0 \end{array}\right] \Rightarrow R_2 \to R_2 - (-2)(R_1) \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 0 & -4 & 0 \\ -3 & -3 & -1 & 0 \end{array}\right] \Rightarrow R_3 \to R_3 - (-3)R_1$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 0 & -4 & 0 \\ 0 & 0 & -4 & 0 \end{array}\right] \cdot \frac{-1}{4} \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 4 & 0 \end{array}\right] \Rightarrow R_3 - (-4)R_2 \to R_3 \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow$$

$$R_1 \to R_1 - (-1)R_2 \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad \begin{matrix} x_1 + x_2 = 0 \\ x_3 = 0 \end{matrix} \quad \begin{matrix} x_1 = -x_2 \\ x_2 = x_2 \\ x_3 = 0 \end{matrix}$$

$$v_2 = \begin{pmatrix} -x_2 \\ x_2 \\ 0 \end{pmatrix} = x_2 \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \text{ Let } x_2 = 1$$

for $\lambda_2 = 2 \Rightarrow v_2 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$

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5. $\lambda_3 = 4$

$$\begin{bmatrix} -1 & 1 & -1 \\ -2 & -4 & -2 \\ -3 & -3 & -3 \end{bmatrix} (-1) \Rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ -2 & -4 & -2 & 0 \\ -3 & -3 & -3 & 0 \end{array}\right] \Rightarrow R_2 \to R_2 - (-2)R_1 \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & -6 & 0 & 0 \\ -3 & -3 & -3 & 0 \end{array}\right] \Rightarrow$$

$$R_3 \to R_3 - (-3)R_1 \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & -6 & 0 & 0 \\ 0 & -6 & 0 & 0 \end{array}\right] \cdot -\frac{1}{6} \Rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -6 & 0 & 0 \end{array}\right] \Rightarrow R_3 \to R_3 - (-6)R_2$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow R_1 \to R_1 - (-1)R_2 \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1 + x_3 = 0$

$x_2 = 0$

$x_1 = -x_3$

$V_3 = \begin{pmatrix} -x_3 \\ 0 \\ x_3 \end{pmatrix}$

$x_3 \times \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ Let $x_3 = 1 \Rightarrow$ for $\lambda_3 = 4 \Rightarrow V_3 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$

6. $A = \begin{bmatrix} 3 & 1 & -1 \\ -2 & 0 & -2 \\ -3 & -3 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$A^2 = A \cdot A = \text{diag}(1,1,1) \cdot \text{diag}(1,1,1) = \text{diag}(1,1,1)$

$A^4 = A^2 \cdot A^2 = \text{diag}(1,1,1) \cdot \text{diag}(1,1,1) = \text{diag}(1,1,1)$

$A^5 = A^4 \cdot A = \text{diag}(1,1,1) \cdot \text{diag}(1,1,1) = \text{diag}(1,1,1)$

$A^5 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$