Kirchhoff's Laws

Monday, April 20, 2020 4:04 PM

1) I = current (amps)

V = voltage (V)

R = resistance (ohms)

a) Kirchoff's current law/junction rule

$\Sigma I_{Entering} - \Sigma I_{leaving} = 0$

at any given junction

b) Kirchoff's voltage law/loop rule

$\Sigma V = 0$, around any loop

c) Ohm's law

$I = \frac{V}{R}$, at every resistor

(2)

$$V_1 = I_1 R_{11} + I_2 R_{12} \cdots + I_n R_{1n}$$

$$V_2 = I_1 R_{21} + I_2 R_{22} + \cdots I_n R_{2n}$$

$$\vdots$$

$$V_n = I_1 R_{n1} + I_2 R_{n2} + \cdots I_n R_{nn}$$

$$\begin{bmatrix} I_1 R_{11} & I_2 R_{12} \cdots & I_n R_{1n} \\ I_1 R_{21} & I_2 R_{22} \cdots & I_n R_{2n} \\ \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix}$$

(3)

$$40 = 12I_1 - 7I_2 - 4I_4$$

$$30 = 15I_2 - 7I_1 - 6I_3$$

$$20 = 14I_3 - 6I_2 - 5I_4$$

$$-10 = 13I_4 - 4I_1 - 5I_3$$

$$\left[\begin{array}{cccc|c} 12 & -7 & 0 & -4 & 40 \\ -7 & 15 & -6 & 0 & 30 \\ 0 & -6 & 14 & -5 & 20 \\ -4 & 0 & -5 & 13 & -10 \end{array}\right]$$

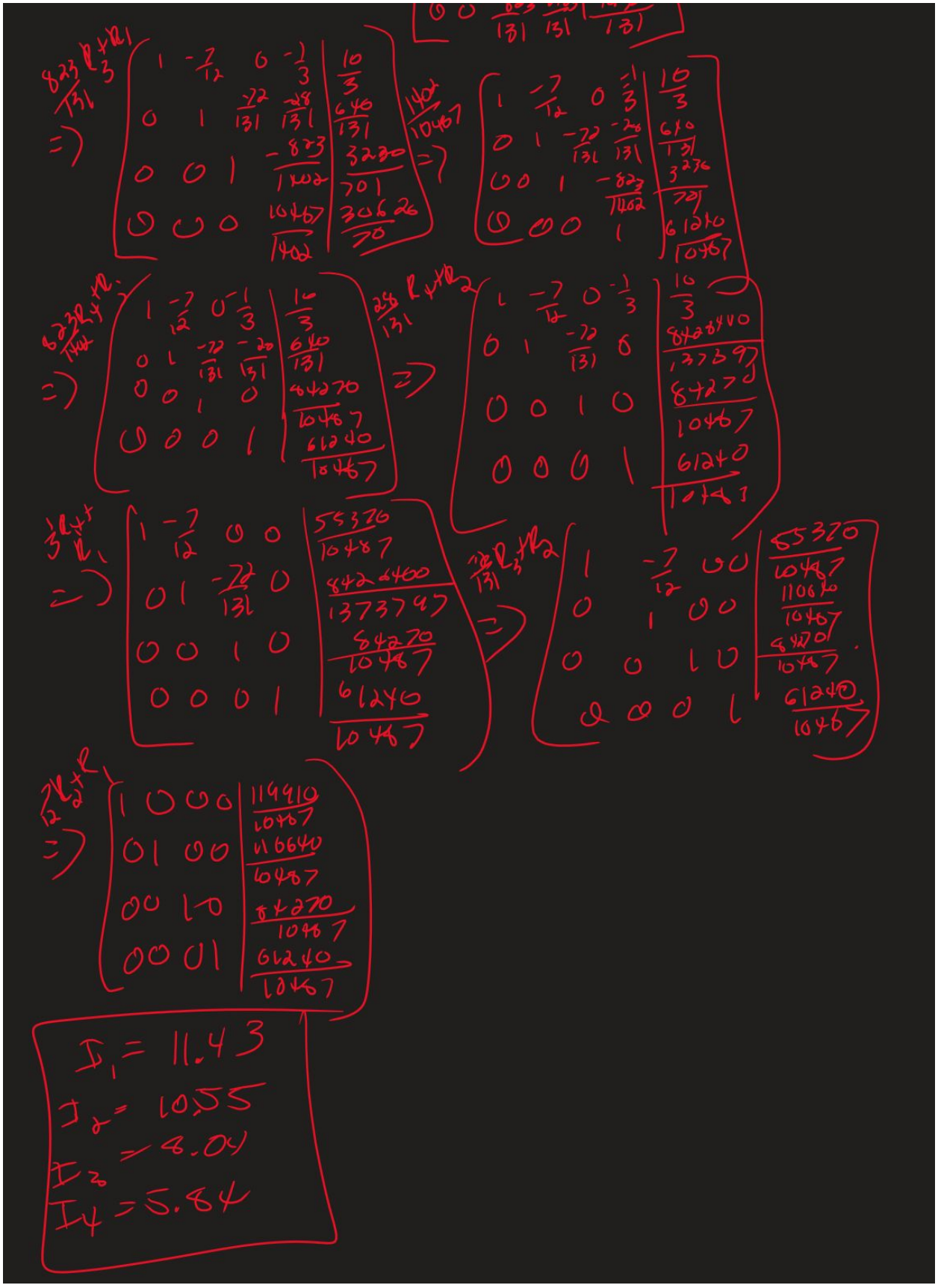
$$\left[\begin{array}{cccc|c} 1 & -\frac{7}{12} & 0 & -\frac{1}{3} & \frac{10}{3} \\ -7 & 15 & -6 & 0 & 30 \\ 0 & -6 & 14 & 5 & 20 \\ -4 & 0 & -5 & 13 & -10 \end{array}\right] \xrightarrow{7h_1+h_2} \Rightarrow \left[\begin{array}{cccc|c} 1 & -\frac{7}{12} & 0 & -\frac{1}{3} & \frac{10}{3} \\ 0 & \frac{131}{12} & -6 & -\frac{7}{3} & \frac{160}{3} \\ 0 & -6 & 14 & -5 & 20 \\ -4 & 0 & -5 & 13 & -10 \end{array}\right]$$

$$\xrightarrow{4h_1+h_4} \Rightarrow \left[\begin{array}{cccc|c} 1 & -\frac{7}{12} & 0 & -\frac{1}{3} & \frac{10}{3} \\ 0 & \frac{131}{12} & -6 & -\frac{7}{3} & \frac{160}{3} \\ 0 & -6 & 14 & -5 & 20 \\ 0 & -\frac{7}{3} & -5 & \frac{35}{3} & \frac{10}{3} \end{array}\right] \xrightarrow{\frac{12}{131}h_2} \Rightarrow \left[\begin{array}{cccc|c} 1 & -\frac{7}{12} & 0 & -\frac{1}{3} & \frac{10}{3} \\ 0 & 1 & -\frac{72}{131} & -\frac{28}{131} & \frac{640}{131} \\ 0 & -6 & 14 & -5 & 20 \\ 0 & -\frac{7}{3} & -5 & \frac{35}{3} & \frac{10}{3} \end{array}\right]$$

$$\xrightarrow{\frac{12}{131}h_2} \Rightarrow \left[\begin{array}{cccc|c} 1 & -\frac{7}{12} & 0 & -\frac{1}{3} & \frac{10}{3} \\ 0 & 1 & -\frac{72}{131} & -\frac{28}{131} & \frac{640}{131} \\ 0 & -6 & 14 & -5 & 20 \\ 0 & -\frac{7}{3} & -5 & \frac{35}{3} & \frac{10}{3} \end{array}\right] \xrightarrow{6h_2+h_3} \Rightarrow \left[\begin{array}{cccc|c} 1 & -\frac{7}{12} & 0 & -\frac{1}{3} & \frac{10}{3} \\ 0 & 1 & -\frac{72}{131} & -\frac{28}{131} & \frac{640}{131} \\ 0 & 0 & \frac{1402}{131} & -\frac{803}{131} & \frac{6460}{131} \\ 0 & -\frac{7}{3} & -5 & \frac{35}{3} & \frac{10}{3} \end{array}\right]$$

$$\xrightarrow{\frac{7}{3}h_2+h_4} \Rightarrow \left[\begin{array}{cccc|c} 1 & -\frac{7}{12} & 0 & -\frac{1}{3} & \frac{10}{3} \\ 0 & 1 & -\frac{72}{131} & -\frac{28}{131} & \frac{640}{131} \\ 0 & 0 & \frac{1402}{131} & -\frac{803}{131} & \frac{6460}{131} \\ 0 & 0 & -\frac{803}{131} & \frac{1460}{131} & \frac{10730}{131} \end{array}\right] \xrightarrow{\frac{131}{1402}h_3} \Rightarrow \left[\begin{array}{cccc|c} 1 & -\frac{7}{12} & 0 & -\frac{1}{3} & \frac{10}{3} \\ 0 & 1 & -\frac{72}{131} & -\frac{28}{131} & \frac{640}{131} \\ 0 & 0 & 1 & -\frac{803}{1402} & \frac{3230}{701} \\ 0 & 0 & -\frac{803}{131} & \frac{1460}{131} & \frac{10730}{131} \end{array}\right]$$

$$\xrightarrow{\frac{803}{131}h_3+h_4}$$


$I_1 = 11.43$
$I_2 = 10.55$
$I_3 = 8.04$
$I_4 = 5.84$

(8) $50 = 9I_1 - 1I_2 - I_4 - 4I_5$

$-30 = 7I_2 - I_1 - 2I_3 - 3I_5$

$20 = 10I_3 - 2I_2 - 3I_5 - 3I_4$

$-40 = 7I_4 - 3I_3 - I_1 - 2I_5$

$0 = 12I_5 - 4I_1 - 3I_2 - 3I_3 - 2I_4$

$$\left[\begin{array}{ccccc|c} 9 & -1 & 0 & -1 & -4 & 50 \\ -1 & 7 & -2 & 0 & -3 & -30 \\ 0 & -2 & 10 & -3 & -3 & 20 \\ -1 & 0 & -3 & 7 & -2 & -40 \\ -4 & -3 & -3 & -2 & 12 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 0 & \frac{3049}{755} \\ 0 & 1 & 0 & 0 & 0 & \frac{-3309}{755} \\ 0 & 0 & 1 & 0 & 0 & \frac{-683}{755} \\ 0 & 0 & 0 & 1 & 0 & \frac{-4382}{755} \\ 0 & 0 & 0 & 0 & 1 & \frac{-727}{755} \end{array}\right]$$

$I_1 = \frac{3049}{755}$ $I_2 = \frac{-3309}{755}$

$I_3 = \frac{-683}{755}$ $I_4 = \frac{-4382}{755}$

$I_5 = \frac{-727}{755}$