MA 2071 - *Matrices & Linear Algebra I* C-term Spring 2020
Sections C01 - C09

Date: February 26, 2020

Name: ____________________________

Quiz 6

Use of the textbook or notes is not allowed. No electronic devices or calculators are allowed. To get credit, you must show **ALL** of your work, unless otherwise stated in the problem. Please do not cheat. *"The first and worst of all frauds is to cheat one's self."*

Read each question carefully and follow the directions stated in each question.

1. (6 points) Let $\vec{v}_1 = \begin{bmatrix} 1/\sqrt{2} \\ 0 \\ -1/\sqrt{2} \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 1/2 \\ \sqrt{2}/2 \\ 1/2 \end{bmatrix}$, and $\vec{v}_3 = \begin{bmatrix} 1/2 \\ -\sqrt{2}/2 \\ 1/2 \end{bmatrix}$. Determine whether $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is an orthonormal set.

   Answer: direct computation shows $\vec{v}_i \cdot \vec{v}_j = 0$ if $i \neq j$, and $||\vec{v}_i|| = 1$ for every i, so the set is orthonormal. (1 point for each of the six conditions checked.)

2. (4 points) Let $\vec{x} = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}$, $\vec{y}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, and $\vec{y}_2 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$. If $W = \text{span}\{\vec{y}_1, \vec{y}_2\}$, find the closest vector in W to $\vec{x}$.

   Answer: Check first that $\vec{y}_1$ and $\vec{y}_2$ are orthogonal by checking if $\vec{y}_1 \cdot \vec{y}_2 = 0$ (worth 1 point). Since they are, we can use the orthogonal projection formula to find the closest vector. The closest vector is $\text{proj}_W \vec{x} = \dfrac{\vec{x} \cdot \vec{y}_1}{\vec{y}_1 \cdot \vec{y}_1}\vec{y}_1 + \dfrac{\vec{x} \cdot \vec{y}_2}{\vec{y}_2 \cdot \vec{y}_2}\vec{y}_2 = \dfrac{4}{3}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + 1\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 4/3 \\ 7/3 \\ 1/3 \end{bmatrix}$ (worth 3 points).

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Read each question carefully and follow the directions stated in each question.

1. (6 points) Let $\vec{v}_1 = \begin{bmatrix} 1/\sqrt{3} \\ -1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 1/\sqrt{6} \\ 2/\sqrt{6} \\ 1/\sqrt{6} \end{bmatrix}$, and $\vec{v}_3 = \begin{bmatrix} -1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix}$. Determine whether $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is an orthonormal set.

   Answer: direct computation shows $\vec{v}_i \cdot \vec{v}_j = 0$ if $i \neq j$, and $||\vec{v}_i|| = 1$ for every i, so the set is orthonormal. (1 point for each of the six conditions checked.)

2. (4 points) Let $\vec{x} = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}$, $\vec{y}_1 = \begin{bmatrix} 0 \\ 0 \\ -5 \end{bmatrix}$, and $\vec{y}_2 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$. If $W = \text{span}\{\vec{y}_1, \vec{y}_2\}$, find the closest vector in W to $\vec{x}$.

   Answer: Check first that $\vec{y}_1$ and $\vec{y}_2$ are orthogonal by checking if $\vec{y}_1 \cdot \vec{y}_2 = 0$ (worth 1 point). Since they are, we can use the orthogonal projection formula to find the closest vector. The closest vector is $\text{proj}_W \vec{x} = \frac{\vec{x} \cdot \vec{y}_1}{\vec{y}_1 \cdot \vec{y}_1}\vec{y}_1 + \frac{\vec{x} \cdot \vec{y}_2}{\vec{y}_2 \cdot \vec{y}_2}\vec{y}_2 = 0 \begin{bmatrix} 0 \\ 0 \\ -5 \end{bmatrix} + \frac{6}{5} \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 12/5 \\ 6/5 \\ 0 \end{bmatrix}$ (worth 3 points).

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Read each question carefully and follow the directions stated in each question.

1. (6 points) Let $\vec{v}_1 = \begin{bmatrix} -2 \\ 1 \\ 3 \\ 0 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 0 \\ -3 \\ 1 \\ -6 \end{bmatrix}$, and $\vec{v}_3 = \begin{bmatrix} -2 \\ -4 \\ 0 \\ 2 \end{bmatrix}$. Determine whether the set $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is (a) orthogonal and (b) orthonormal.

   Answer: direct computation shows $\vec{v}_i \cdot \vec{v}_j = 0$ if $i \neq j$, so the set is orthogonal (3 points). Direct computation also shows no vector has length one. Since not all vectors have length 1, the set is not orthonormal. (3 points)

2. (4 points) Let $\vec{x} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$, $\vec{y}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, and $\vec{y}_2 = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$. If $W = \text{span}\{\vec{y}_1, \vec{y}_2\}$, find the closest vector in W to $\vec{x}$. (If dealing with annoying fractions, you do not need to simplify your answer.)

   Answer: Check first that $\vec{y}_1$ and $\vec{y}_2$ are orthogonal by checking if $\vec{y}_1 \cdot \vec{y}_2 = 0$ (worth 1 point). Since they are, we can use the orthogonal projection formula to find the closest vector. The closest vector is $\text{proj}_W \vec{x} = \frac{\vec{x} \cdot \vec{y}_1}{\vec{y}_1 \cdot \vec{y}_1}\vec{y}_1 + \frac{\vec{x} \cdot \vec{y}_2}{\vec{y}_2 \cdot \vec{y}_2}\vec{y}_2 = \frac{5}{14}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + \frac{3}{5}\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} (5/14) + (6/5) \\ (10/14) - (3/5) \\ 15/14 \end{bmatrix} = \begin{bmatrix} 109/70 \\ 4/35 \\ 15/14 \end{bmatrix}$ (worth 3 points, students may stop at the second to last step).