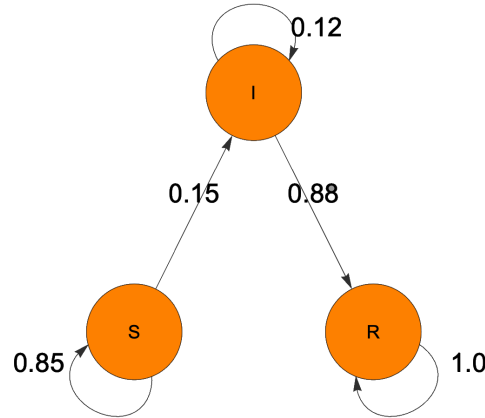


PANDEMIC WITH SIR MODEL

Covid19 is changing you and me, and the entire world. SIR, can you tell me where we are heading to? What does our future life look like? All the answer shall rely on the reliable prediction on the population. This project will guide you to go through one of scientific population models under pandemic influence, so called SIR model.

The simplest SIR model can be given by a directed graph, which reflects weekly transition of the population density among three classes: S represents susceptible, I represents infected, and R represents recovered. For instance, in every week, 85% of susceptible remain susceptible, while 15% turns to infected.



- (1) Complete the SIR transition matrix below derived from the diagram:

$$T = \begin{bmatrix} .85 & * & * \\ .15 & .12 & * \\ 0 & * & 1.0 \end{bmatrix}$$

- (2) Did you observe that each column of SIR matrix has its sum as 1? Such a matrix is called column stochastic matrix. Is it accidental or a common fact for SIR matrix?

- (3) The SIR density at time 0 is $x_0 = \begin{bmatrix} .9 \\ .07 \\ .03 \end{bmatrix}$, which means that 90% of current population is S, 7% is I, 3% is R. Find out x_1 , the SIR density after one week.

- (4) Find all eigenvalues $\lambda_1, \lambda_2, \lambda_3$ and corresponding eigenvectors v_1, v_2, v_3 of T .

- (5) It's the time to show off your diagonalize skill. What is the SIR density after 12 weeks? Hint: Use the fact that

$$T^n = V D^n V^{-1}, \text{ for all } n$$

where D is 3 by 3 diagonal matrix with eigenvalues on its diagonal.

- (6) After long long time, will SIR density stabilize to some level? It means that what $\lim_{n \rightarrow \infty} x_n$ is. Stop computing and find out the answer from your answers to the previous questions.
- (7) Have you observed any more secret of this model? Tell me. For instance,
 - I find a class of matrices having eigenvalue 1 and eigenvector ...
 - I find the first component of density function x_n is decreasing as n is getting larger ...
 - After reading the above data, would you recommend "close our campus"?
 - It is not right model for covid19, since there is no "D"ead mode. So we shall change ...