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1. $\left[\begin{array}{cccc|c} 0 & -1 & 0 & 6 & 3 \\ -1 & 3 & 1 & -1 & 2 \\ -1 & -1 & -1 & -1 & 2 \end{array}\right]$

2. $\left[\begin{array}{cccc|c} 1 & -1 & 0 & 6 & 3 \\ -1 & 3 & 1 & -1 & 2 \\ -1 & -1 & -1 & -1 & 2 \end{array}\right]$ swap 1st & 2nd row $\rightarrow \left[\begin{array}{cccc|c} -1 & 3 & 1 & -1 & 2 \\ 0 & -1 & 0 & 6 & 3 \\ -1 & -1 & -1 & -1 & 2 \end{array}\right] \cdot -1 \rightarrow$

$\left[\begin{array}{cccc|c} 1 & -3 & -1 & 1 & -2 \\ 0 & -1 & 0 & 6 & 3 \\ -1 & -1 & -1 & -1 & 2 \end{array}\right]$ $R_3 \to R_3 + R_1 \rightarrow \left[\begin{array}{cccc|c} 1 & -3 & -1 & 1 & -2 \\ 0 & -1 & 0 & 6 & 3 \\ 0 & -4 & -2 & 0 & 0 \end{array}\right] \cdot -1 \rightarrow \left[\begin{array}{cccc|c} 1 & -3 & -1 & 1 & -2 \\ 0 & 1 & 0 & -6 & -3 \\ 0 & -4 & -2 & 0 & 0 \end{array}\right]$

$R_3 \to 4R_2 + R_3 \rightarrow \left[\begin{array}{cccc|c} 1 & -3 & -1 & 1 & -2 \\ 0 & 1 & 0 & -6 & -3 \\ 0 & 0 & -2 & -24 & -12 \end{array}\right] \cdot (-.5) \rightarrow \left[\begin{array}{cccc|c} 1 & -3 & -1 & 1 & -2 \\ 0 & 1 & 0 & -6 & -3 \\ 0 & 0 & 1 & 12 & 6 \end{array}\right]$ $R_1 \to R_3 + R_1 \rightarrow$

$\left[\begin{array}{cccc|c} 1 & -3 & 0 & 13 & 4 \\ 0 & 1 & 0 & -6 & -3 \\ 0 & 0 & 1 & 12 & 6 \end{array}\right]$ $R_1 \to 3R_2 + R_1 \rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & -5 & -5 \\ 0 & 1 & 0 & -6 & -3 \\ 0 & 0 & 1 & 12 & 6 \end{array}\right]$

$$\begin{bmatrix} -5 \\ -3 \\ 6 \\ 0 \end{bmatrix} + s \begin{bmatrix} 5 \\ 6 \\ -12 \\ 1 \end{bmatrix}$$

3. The dimension of the null space is 1, by (3) and the Rank-nullity theorem. A bases for the null space is given by $\begin{bmatrix} 5 \\ 6 \\ -12 \\ 1 \end{bmatrix}$.

4. Yes, for example $A \begin{bmatrix} 5 \\ 6 \\ -12 \\ 1 \end{bmatrix} = A \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

There exist 2 null vectors if and only if Null space of A is nonzero

5. $\begin{bmatrix} -2 & 3 & 0 \\ 0 & 1 & 0 \\ 3 & -3 & 1 \end{bmatrix} \begin{matrix} -2 & 3 \\ 0 & 1 \\ 3 & -3 \end{matrix}$

The rank is 3 because $\det(B) \neq 0$.

$$-2(1)(1) + 3(0)(3) + 0(0)(-3)$$
$$-\left(3(1)(0) + -3(0)(-2) + (1)(0)(-3)\right)$$
$$-2$$

6. $\det \begin{bmatrix} -2-\lambda & 3 & 0 \\ 0 & 1-\lambda & 0 \\ 3 & -3 & 1-\lambda \end{bmatrix} = -\lambda^3 + 3\lambda - 2 = -(\lambda-1)(\lambda^2+\lambda-2)$

$= \boxed{-(\lambda-1)(\lambda+2)(\lambda-1)}$

eigenvalues are $\lambda_1 = 1$ $\lambda_2 = -2$

$\lambda_1 = 1$

7. $\left[\begin{array}{ccc|c} -3 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 3 & -3 & 0 & 0 \end{array}\right] \xrightarrow{\cdot -\frac{1}{3}} \left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 3 & -3 & 0 & 0 \end{array}\right] \xrightarrow{R_3 - R_1 \to R_3} \left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$x_1 - x_2 = 0$
$x_1 = x_2$
$x_2 = x_2$
$x_3 = x_3$

$x = \begin{bmatrix} x_2 \\ x_2 \\ x_3 \end{bmatrix}$ $\quad x_2 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ $\quad \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

$\lambda_2 = -2$

$\left[\begin{array}{ccc|c} 0 & 3 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 3 & -3 & 3 & 0 \end{array}\right]$ Swap $R_3 \,\&\, R_1 \to \left[\begin{array}{ccc|c} 3 & -3 & 3 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 3 & 0 & 0 \end{array}\right] \xrightarrow{\cdot \frac{1}{3}} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 3 & 0 & 0 \end{array}\right] \xrightarrow{\cdot \frac{1}{3}}$

$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 3 & 0 & 0 \end{array}\right]$ $R_3 \to R_3 - 3R_2 \to \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$ $R_1 - (-1)R_2 \to R_1 \to \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$x_1 + x_3 = 0$
$x_2 = 0$

$x_1 = -x_3$
$x_2 = 0$
$x_3 = x_3$

$x = \begin{bmatrix} -x_3 \\ 0 \\ x_3 \end{bmatrix}$ $\quad v = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ $\quad \left\{ \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right\}$

8. $B = \begin{bmatrix} -2 & 3 & 0 \\ 0 & 1 & 0 \\ 3 & -3 & 1 \end{bmatrix}$

$P = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \quad D = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$B = PDP^{-1} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}^{-1}$$

9. $B^{2020} x = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$

$$\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$$

$$B^{2020} \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} = B^{2020} \left(\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right)$$

$$= (1)^{2020} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + 0^{2020} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + (-1)^{2020} \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

10. linear algebra can help me in my career because as a robotics major i'll have to solve many circuit problems. Solving the circuits using simultaneus equations in Matrix form will help me compute the solution faster.