MA2071 - Assignment I

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This assignment assesses the material covered in Modules 1-8, corresponding to Sections 1.1-1.6 and 2.1-2.3 of the textbook. Write full and complete solutions, using full sentences where appropriate. Explain all row operations when computing an RREF. Answer the questions asked.

Questions 1-5 are worth 20 points each. The bonus question is worth 10 points (but points on this assignment are capped at 100).

Recommended Deadline: April 10th

Final Deadline: April 24th.

1. Find all solutions to the vector equation

$$\begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} x_1 + \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix} x_2 + \begin{pmatrix} -3 \\ 1 \\ -1 \end{pmatrix} x_3 + \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} x_4 = \begin{pmatrix} 14 \\ 22 \\ -5 \end{pmatrix}.$$

2. Let $p(x)$ be the cubic polynomial which satisfies

$$p(-1) = p(0) = 1, \;\; p(1) = p(2) = 4\,.$$

Find the coefficients of $p(x)$.

3. Find all matrices which satisfy the matrix equation

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix}.$$

(Hint: Carry out the matrix multiplication on each side, solve the system of equations you get by setting entries in the same position of the resulting matrix equal. Your answer should be in the form of a matrix with entries which are linear combinations of two free variables.)

4.
   - Give an example of a non-zero 2×2 matrix R which satisfies $R^2 = \mathbf{0}$.
   - Suppose that M is an $n \times n$ matrix satisfying $M^3 = 0$. Show that $(I - M)^{-1} = (I + M + M^2)$.
   - Since $R^2 = 0$ implies $R^3 = 0$, use the previous part to find the inverse of $(I - R)$.

5. Let A be an $m \times n$ matrix. If these exist two distinct vectors v_1 and v_2 such that $Av_1 = Av_2$ then there exists a non-zero vector v such that $Av = 0$. Explain in your own words why this must be true.

6. (Bonus) Let A, B be invertible 2×2 matrices. Let X be any 2×2 matrix, and let $\mathbf{0}$ be the 2×2 zero matrix.

$$M_1 = \left(\begin{array}{c|c} A & X \\ \hline \mathbf{0} & B \end{array}\right), \;\; M_2 = \left(\begin{array}{c|c} A^{-1} & -A^{-1}XB^{-1} \\ \hline \mathbf{0} & B^{-1} \end{array}\right)$$

Show that M_1 is the inverse of M_2. (Hint: Have a look a section 2.4 of the textbook.) Hence find the inverse of the matrix

$$\begin{pmatrix} 1 & 0 & 2 & -1 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 3 & -2 \\ 0 & 0 & -1 & 1 \end{pmatrix}$$