Written assignment 3

1. $\begin{pmatrix} 1 & 4 & 2 \\ 3 & -1 & 0 \\ 1 & 1 & 2 \end{pmatrix}^{-1}$

$$\left[\begin{array}{ccc|ccc} 1 & 4 & 2 & 1 & 0 & 0 \\ 3 & -1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 2 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 3 & -1 & 0 & 0 & 1 & 0 \\ 0 & 13/3 & 2 & 1 & -1/3 & 0 \\ 0 & 0 & 18/13 & -4/13 & -3/13 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/9 & 1/3 & -1/9 \\ 0 & 1 & 0 & 1/3 & 0 & -1/3 \\ 0 & 0 & 1 & -2/9 & -1/6 & 13/18 \end{array}\right]$$

$$= \begin{bmatrix} \frac{1}{9} & 1/3 & -1/9 \\ \frac{1}{3} & 0 & -1/3 \\ -2/9 & -1/6 & 13/18 \end{bmatrix}$$

2. a) $\begin{vmatrix} -7-\lambda & 12 & -18 \\ -18 & 23-\lambda & -30 \\ -8 & 8 & -9-\lambda \end{vmatrix} = 0$

$\Rightarrow -(\lambda+1)(\lambda-3)(\lambda-5) = 0$

$\lambda_1 = -1,\ \lambda_2 = 3,\ \lambda_3 = 5$

$Bv_2 = -1 \cdot v_2$

So v_2 is eigen vector of $\lambda_1 = -1$

Also $B \cdot v_1 = 5 \cdot v_1$

So v_1 is eigen vector of $\lambda_3 = 5$

b) $(0,-4,-4) = \alpha(1,1,0) + \beta(1,2,1)$

$\alpha + \beta = 0$

$\alpha + 2\beta = -4$

$\beta = -4$

So $\beta = -4$ and $\alpha = 4$

then $B^{10}v_3 = B^{10}(\alpha v_1 + \beta v_2) = B^{10}(4v_1 + (-4)v_2)$

$B^{10}v_3 = 4B^{10}v_1 + (-4)B^{10}v_2 = 4(5)^{10}v_1 + (-4)(-1)^{10}v_2$

$B^{10}v_3 = 4 \cdot 5^{10}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + (-4)\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$

$B^{10}v_3 = \begin{pmatrix} 4 \cdot 5^{10} - 4 \\ 4 \cdot 5^{10} - 8 \\ -4 \end{pmatrix}$

3. $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ $\quad |A - \lambda I| = 0 \Rightarrow (\lambda^2 + 1) = 0$

$\lambda = \pm i$

$\lambda_1 = i$

$A - \lambda_1 I = \begin{bmatrix} -i & 1 \\ -1 & i \end{bmatrix}$

$\left[\begin{array}{cc|c} -i & 1 & 0 \\ -1 & -i & 0 \end{array}\right]$ $\quad R_1 \to \frac{1}{(-i)} R_1$

$\left[\begin{array}{cc|c} 1 & i & 0 \\ -1 & -i & 0 \end{array}\right]$ $\quad R_2 \to R_2 + R_1$

$\left[\begin{array}{cc|c} 1 & i & 0 \\ 0 & 0 & 0 \end{array}\right] \Rightarrow \begin{bmatrix} 1 & i \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$x_1 + ix_2 = 0 \Rightarrow x_1 = -ix_2$

$\vec{v}_1 = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -ix_2 \\ x_2 \end{bmatrix} = \begin{bmatrix} -i \\ 1 \end{bmatrix} x_2$

$\vec{v}_1 = \begin{bmatrix} -i \\ 1 \end{bmatrix}$

$\lambda_1 = i$ is $\begin{bmatrix} -i \\ 1 \end{bmatrix}$

$\lambda_2 = -i$

$A - \lambda_2 i = \begin{bmatrix} i & 1 \\ -1 & i \end{bmatrix}$

$\left[\begin{array}{cc|c} i & 1 & 0 \\ -1 & i & 0 \end{array}\right]$ $\quad R_1 \to \frac{R_1}{i}$

$\left[\begin{array}{cc|c} 1 & -i & 0 \\ -1 & i & 0 \end{array}\right]$ $\quad R_2 \to R_1 + R_2$

$\begin{bmatrix} 1 & -i \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$x_1 - ix_2 = 0 \Rightarrow x_1 = ix_2$

$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} ix_2 \\ x_2 \end{bmatrix} = \begin{bmatrix} i \\ 1 \end{bmatrix} x_2$

$v_2 = \begin{bmatrix} i \\ 1 \end{bmatrix}$

$\lambda_2 = -1$ is $\begin{bmatrix} i \\ 1 \end{bmatrix}$

Written Assignment 3 Cont.

4. $D = \begin{bmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{bmatrix}$ ⇐ Let D be a 3×3 diagonal matrix w/ diagonal entries a_1, a_2, a_3

eigenvalue of D is a_1, a_2, and a_3

$(D - a_1 I)x = 0 \Rightarrow D a_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$(D - a_2 I)x = 0 \Rightarrow D a_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$(D - a_3 I)x = 0 \Rightarrow D a_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

5. $E = \begin{bmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \\ a_4 & b_4 & c_4 & d_4 \end{bmatrix}$ Such that $a_1 + b_1 + c_1 + d_1 = a_2 + b_2 + c_2 + d_2 = a_3 + b_3 + c_3 + d_3 = a_4 + b_4 + c_4 + d_4$

$$\begin{bmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \\ a_4 & b_4 & c_4 & d_4 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} a_1 + b_1 + c_1 + d_1 \\ a_2 + b_2 + c_2 + d_2 \\ a_3 + b_3 + c_3 + d_3 \\ a_4 + b_4 + c_4 + d_4 \end{bmatrix}$$

$$= \begin{bmatrix} x \\ x \\ x \\ x \end{bmatrix} = x \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

Let $\alpha = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$, then α is a non-zero vector such that $E\alpha = x\alpha$

$\therefore$ $\alpha = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ is an eigen vector of E and the corresponding value is x.

6. A is a matrix of odd order n, then $\det(-A) = (-1)^n \det A = -\det A$ since n is odd.

$\therefore$ $\det(-A) = -\det A$ if A is an odd order matrix