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Tutorial - 1

Ans - 1

i) Big O(n)

$f(x) = O(g(x))$

if $f(x) \leq g(x) \times C \ \forall x \geq x_0$

for some constant, $C > 0$

$g(x)$ is 'tight upper bound of $f(x)$.

e.g. $f(x) = x^2 + x$

$g(x) = n^3$

$x^2 + x \leq c * x^3$

$x^2 + x = O(x^3)$

ii) Big-Omega (Ω)

when $f(n) = \Omega(g(n))$ means $(g(n))$ is "tight" lowerbound of $f(n)$ i.e. $f(n)$ can go beyond $g(n)$ i.e. $f(n) = \Omega g(n)$

if. $f(x) \geq c.g(x)$

$\forall \ x_2 > x_0$ & $c =$ constant > 0

e.g. $f(x) = x^3 + 4x^2$

i.e $f(n) \geq c * g(n)$

$x^3 + 4x^2 = \Omega(x^2)$

iii) <u>Big-Theta (Θ)</u>

When $f(x) = \Theta(g(x))$ gives the tight upper bound & lowerbound both. i.e. $f(x) = \Theta(g(x))$. if $c_1 * g(x_1) \leq f(x) \leq c_2 * g(x_2)$ for all $x \geq \max(x_1, x_2)$, some constant.

$c_1 > 0$ & $c_2 > 0$. i.e.

$f(x)$ can never go beyond $c_2 g(x)$ & will never come down of $c_2(g(x)$ & will never come down of $c_2 g(x)$

e.g. $3x+2 = \Theta(x)$ as $3x+1 \geq 3x$ &

$3x+2 \leq 4x$ for x, $c_1 = 3$, $c_2 = 4$ & $n_0 = ?$

iv) <u>Small $O(\theta)$</u>

When $f(x) = o\,g(x)$ gives the upper bound if $f(x) = o\,g(n)$

if $f(x) < c * g(x) \ \forall x > x_0$ & $x > 0$.

Ex. $f(x) = x^3$, $g(x) = x^3$

$f(x) < c * g(x)$

$x^2 = o(x^3)$

v) <u>Small omega (ω)</u>

It gives the lower bound ie. $f(x) = \omega(g(x))$

Where $g(x)$ is lower bound of $f(x)$.

if $f(x) > c * g(x) \ \forall x > n_0$ & $c > 0$.

Ans 2

for $i = 1, 2, 3, 4 \ldots n$ times

i.e series is a GP

So, $a = 1$, $r = 2$

k^{th} value of GP

$t_k = a r^{k-1}$

$t_k = 1(2)^{k-1}$

$2n = 2^k$

$\log_2 (2n) = k \log 2$

$\log_2 2 + \log_2 n^2 k$

$\log_2 n + 1 = k$

So, Time Complexity $T(n) = O(\log_2 n)$

Ans 3.

$T(n) = 3T(n-1)$ — ①

$T(n) = 1$

Put $n = n-1$ in eqn ①

$T(n-1) = 3T(n-1-1)$ — ②

Put eqn ② in eqn ①

$T(n) = 3(3T(n-2))$

$T(n) = 9 \cdot T(n-2)$ — ③

Put $n = n-2$ in eqn ①

$T(n-2) = 3T(n-3)$

Put in eqn (3)

$T(n) = 27\, T(n-3)$ — (4)

So,

$T(k) = 3^k\, T(n-k)$ —(5)

for k^{th} then,

Let $n-k=1$ (

$k = n-1$, put in eqn ⑤

$T(n) = 3^{n-1}\, T(1)$

$T(n) = 3^{n-1}$

$T(n) = O(3^n)$

Ans 4

$T(n) = 2T(n-1) - 1$ —(1)

Put $n = n-1$

$T(n-1) = 2T(n-2) - 1$ —(2)

Put in eqn (1)

$T(n) = 2(2T(n-2)-1)-1$

$= 4T(n-2) - 2 - 1$ —(3)

Put $n = n-2$ in eqn (1)

$T(n-2) = 2T(n-3) - 1$

Put in eqn (1)

$T(n) = 8T(n-3) - 4 - 2 - 1$ —(4)

So,

$T(n) = 2^k\, T(n-k) - 2^{k-1} - 2^{k-2} \ldots 2$

k^{th} term

Let $n-k=1$

$k = n-1$

$T(n) = 2^{n-1}\, T(1) - 2^k \left(1/2 + 1/2^2 + \ldots 1/2^n\right)$

$= 2^{n-1} - 2^{n-1}\left(1/2 + 1/2^2 + \ldots 1/2^{n-1}\right)$

$a = 1/2$, $r = 1/2$

So, $T(n) = 2^{n-1}\left(1 - \left(\frac{1/2\,(1-(1/2)^{n-1})}{1-1/2}\right)\right)$

$= 2^{n-1}\left(1 - 1 + (1/2)^{n-1}\right)$

$= 2^{n-1}/2^{n-1}$

$T(n) = O(1)$

Ans 5. ~~for i=1~~

$i = 1, 2, 3, 4, 5 \ldots$

$s = 1 + 3 + 6 + 10 + 15 \ldots$

sum of $s = 1 + 3 + 6 + 10 + \ldots x$ —(1)

Also, $s = 1 + 3 + 6 + 10 + \ldots T_{n-1} + T_n$ —(2)

$0 = 1 + 2 + 3 + 4 + \ldots n - T_n$

$T_k = 1 + 2 + 3 + 4 + \ldots k$

$T_k = 1/2\, k(k+1)$

for k, $1 + 2 + 3 + \ldots k <= x$

$\frac{k(k+1)}{2} <= x$

$\frac{k^2 + k}{2} <= n$

$O(k^2) \leq n$

$T(n) = O(\sqrt{n})$

Ans-6 $i^2 = n$

$i = \sqrt{n}$

$i = 1, 2, 3, 4 \ldots \sqrt{n}$

$\sum_{i=1} 1 + 2 + 3 + 4 + \ldots + \sqrt{n}$

$T(n) = \frac{\sqrt{n} * (\sqrt{n} + 1)}{2}$

$T(n) = \frac{(n * \sqrt{n})}{2}$

$T(n) = O(n)$

Ans7. Since, for $n = k^2$

$k = 1, 2, 4, 8, \ldots n$

series in GP

So, $a = 1$, $r = 2$

$$= \frac{a(r^n - 1)}{r - 1}$$

$$= \frac{1(2^r - 1)}{1}$$

$n = 2^k - 1$

$n + 1 = 2^k$

$\log_2(n) = k$

i	j	k
1	$\log(n)$	$\log(n) * \log(n)$
2	$\log(n)$	$\log(n) * \log(n)$
⋮	⋮	⋮
n	$\log(n)$	$\log(n) * \log(n)$

$T = O(n * \log n * \log n)$

$= O(n \log^2(n))$

Ans-8 for($i = 1$ to n)

we get $j = n$ times every time

$i * j = n^3$

k^{th}, Now, $T(n) = n^2 + T(n-3)$

$T(n-3) = (n^2 - 3)^2 + T(n-6)$

$T(n-6) = n^3 - 6)^2 + T(n-9)$

and $T(1) = 1$

Now, put these value in $T(n)$

$T(n) = n^2 + (n-3)^2 + (n-6)^2 + \dots + 1$

Let $n - 3k = 1$

$k = (n-1)/3$ Total terms = $k+1$

$T(n) = n^2 + (n-3)^2 + (n-6)^2 + \dots + 1$

$T(n) = kn^2$

$T(n) = (k-1)/3\, n^2$

So, $T(n) = O(n^3)$

Ans 9 for $i=1$ $j = 1+2+\dots+(n \geq j+i)$

$i=2$ $j = 1+3+5+\dots (n \geq j+i)$

$i=3$ $j = 1+4+7+\dots (n \geq j+i)$

n^{th} term is of AP is

$T(n) = a + d * m$

$T(m) = 1 + d * m$

for $i=1$ $(n-1)/1$

$i=2$ $(n-1)/2$

$i = n-1$

we get,

$T(n) = i_1 j_1 + i_2 j_2 + \dots + i_{n-1} j_{n-1}$

$= \left(\frac{n-1}{2}\right) + \left(\frac{n-2}{2}\right) + \dots 1$

$= n + n/2 + n/3 + \dots n/n-1 - n + 1$

$= n[1 + 1/2 + 1/3 + \dots + 1/n + 1] - n + 1$

$= n \times \log n - n + 1$

$T(n) = O(n \log n)$

Ans10.

As given as n^k & c^m

Relationship b/w m^k & c^m is

$n^k = O(c^m)$

$m^k \leq a(c^m)$

$\forall\ n \geq n_0$ & constant, $a > 0$

for $n_0 = 1$, $c = 2$

$1^k = a^2$

$n_0 = 1$ & $c = 2$