TUTORIAL - 4

Ans 1. $T(n) = 3T(n/2) + n^2$

$a = 3$, $b = 2$, $f(n) = n^2$

$n^{\log_b^a} = n^{\log_2 3}$

comparing $n^{\log_2 3}$ and n^2

$n^{\log_2 3} < n^2$ (case 3)

according to master theorem

$T(n) = O(n^2)$

Ans 3. $T(n) = T(n/2) + 2^n$

$a = 1$, $b = 2$

$n^{\log_2 1} = n^0 = 1$

$1 < 2^n$ (case 3)

∴ According to master's theorem $T(n) = O(2^n)$

Ans 2. $T(n) = 4T(n/2) + n^2$

$a = 4$, $b = 2$

$n^{\log_b^a} = n^{\log_2 4} = n^2 = f(n)$

∴ according to Master's theorem $T(n) = O(n^2 \log n)$

Ans 4. $T(n) = 2^n T(n/2) + n^n$

∴ Master's theorem is not applicable as a function of n.

Ans 5. $T(n) = 16T(n/4) + n$

$a = 16$, $b = 4$, $f(n) = n$

$n^{\log_b^a} = n^{\log_4^{16}} = n^2$

$n^2 > f(n)$ (case 1)

$T(n) = O(n^2)$

Ans 6.

$T(n) = 2T(n/2) + n \log n$

$a = 2, \quad b = 2, \quad f(n) = n \log n$

$n \log_b^a = n^{\log_2^2} = n$

Now $f(n) > n$

$\therefore$ According to master's $T(n) = \Theta(n \log n)$

Ans 7. $T(n) = 2T(n/2) + n/\log n$

$a = 2, \quad b = 2, \quad f(n) = n/\log n$

$n^{\log_b^a} = n^{\log_2^2} = n$

$n > f(n)$

$\therefore$ According to master's theorem $T(n) = \Theta(n)$

Ans 8. $T(n) = 2T(n/4) + n^{0.5}$

$a = 2, \quad b = 4, \quad f(n) = n^{0.5}$

$n^{\log_b^a} = n^{\log_4^2} = n^{0.5}$

$n^{0.5} \leq f(n)$

$\therefore$ According to master's theorem $T(n) = \Theta(n^{0.5})$

Ans 9. $T(n) = 0.5T(n/2) + 1/n$

$\therefore$ Master's not applicable as $a < 1$

Ans 10. $T(n) = 16T(n/4) + n!$

$a = 16, \quad b = 4, \quad f(n) = n!$

$n^{\log_b^a} = n^{\log_4^{16}} = n^2$

$\therefore$ According to Master's theorem,

$T(n) = \Theta(n!)$

Ans 12 $T(n) = sqrt(n) * (n/2) + \log n$

$\therefore$ Master's not applicable as a is not constant.

Ans 13 $T(n) = 3T(n/2) + n$

$a = 3, \quad b = 2, \quad f(n) = n$

$n^{\log_b a} = n^{\log_2 3} = n^{1.58}$

$n^{1.58} > f(n)$

$\therefore$ According to master's theorem, $T(n) = O(n^{\log_2 3})$

Ans 14. $T(n) = 3T(n/3) + \sqrt{n}$

$a = 3, \quad b = 3, \quad f(n) = \sqrt{n}$

$n^{\log_b a} = n^{\log_3 3} = n$

$n > \sqrt{n}$

According to master's theorem, $T(n) = O(n)$

Ans 15 $T(n) = 4T(n/2) + cn$

$a = 4, \quad b = 2, \quad f(n) = c * n$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$n^2 > c * n$

$\therefore$ According to Master's theorem, $T(n) = \Theta(n^2)$

Ans 16. $T(n) = 3T(n/4) + n \log n$

$a = 3, \quad b = 4, \quad f(n) = n \log n$

$n^{\log_b a} = n^{\log_4 3} = n^{0.79}$

$n^{0.79} < n^{\log n}$

$\therefore$ According to Master's theorem, $T(n) = O(n \log n)$

Ans 17

$T(n) = 3T(n/3) + n/2$

$a = 3, \quad b = 3, \quad f(n) = n/2$

$n^{\log_a b} = n^{\log_3 3} = n$

$O(n) = O(n/2)$

∴ Acc. to Master's Theorem –

$T(n) = O(n \log n)$

Ans 18. $T(n) = 6T(n/3) + n^2 \log n$

$a = 6, \quad b = 3, \quad f(n) = n^2 \log n$

$\log_b^a = n^{\log_3^6} = n^{1.63}$

$n^{1.63} < n^2 \log n$

∴ According to master's theorem –

$T(n) = O(n^2 \log n)$

Ans 19. $T(n) = 4T(n/2) + n/\log n$

$a = 4, \quad b = 2, \quad f(n) = n/\log n$

$n^2 > n/\log n$

∴ Acc. to masee's theorem

$T(n) = O(n^2)$

Ans 20. $T(n) = 64T(n/8) - n^2 \log n$

Master's theorem is not applicable as $f(n)$ is not increasing function.

Ans 21

$T(n) = 7T(n/3) + n^2$

$a = 7, \quad b = 3, \quad f(n) = n^2$

$n^{\log_b^a} = n^{\log_3^7} = n^{1.7}$

$n^{1.7} < n^2$

∴ According to master's theorem, $T(n) = \Theta(n^2)$

Ans 22 $T(n) = T(n/2) + n(2 - \cos n)$

Master's theorem is not applicable since, regularity condition is isolated in case 3.