CMB Gibbs sampling and extension to joint foreground sampling

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1 CMB Gibbs Sampling

Assume the data measurement equation to be:

$$d_k = \mathbf{A_k s} + \mathbf{n}_k \tag{1}$$

where k runs over the freq. bands considered, **s** is the CMB signal vector, **n** is uncorrelated noise, and **A** is the convolved beam.

Sample the joint posterior $\mathcal{P}(s, C_\ell | d)$ conditionally, as

$$s \hookleftarrow \mathcal{P}(s|C_\ell, d) \tag{2}$$

$$C_\ell \hookleftarrow \mathcal{P}(C_\ell|s, d) = \mathcal{P}(C_\ell|s) \tag{3}$$

1.1 Sampling $\mathcal{P}(C_\ell|s)$

$$C_\ell^{i+1} = \frac{\sigma_\ell}{\rho_\ell^2} \tag{4}$$

where σ_ℓ is constructed as $\sigma_\ell = \sum_{m=-\ell}^{\ell} |a_{\ell,m}|^2$, and $\rho_\ell^2 = \sum_{j=1}^{2\ell-1} |\rho_\ell^j|^2$, where ρ_ℓ^j is drawn from a standard normal Gaussian.

1.2 Sampling $\mathcal{P}(s|C_\ell, d)$

The map sampling process is performed as a conjugate gradient (CG) step, to invert the equation:

$$(1 + C^{1/2} A^T \mathbf{Y^T} N^{-1} \mathbf{Y} A C^{1/2})(C^{-1/2} s) = \underbrace{C^{1/2} A^T \mathbf{Y^T} N^{-1} d}_{\text{Wiener Filter}} + \underbrace{C^{1/2} A^T \mathbf{Y^T} N^{-1/2} \omega_1 + \omega_2}_{\text{Fluctuations}} \tag{5}$$

- **Change all map2alm calls to** `jaxbind.get_healpix_sht`
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1.3 Results

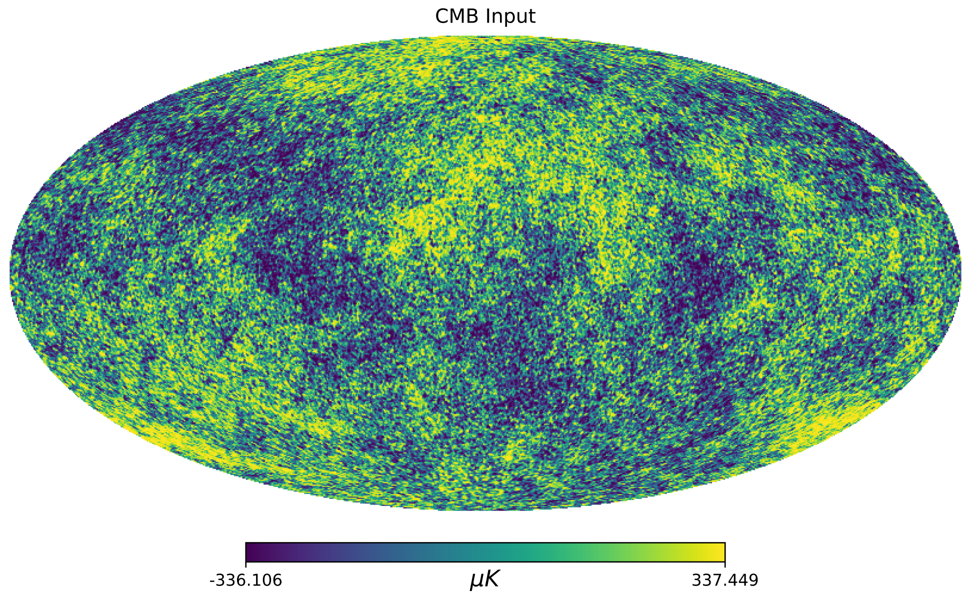

Figure 1: CMB ground truth used to construct data.

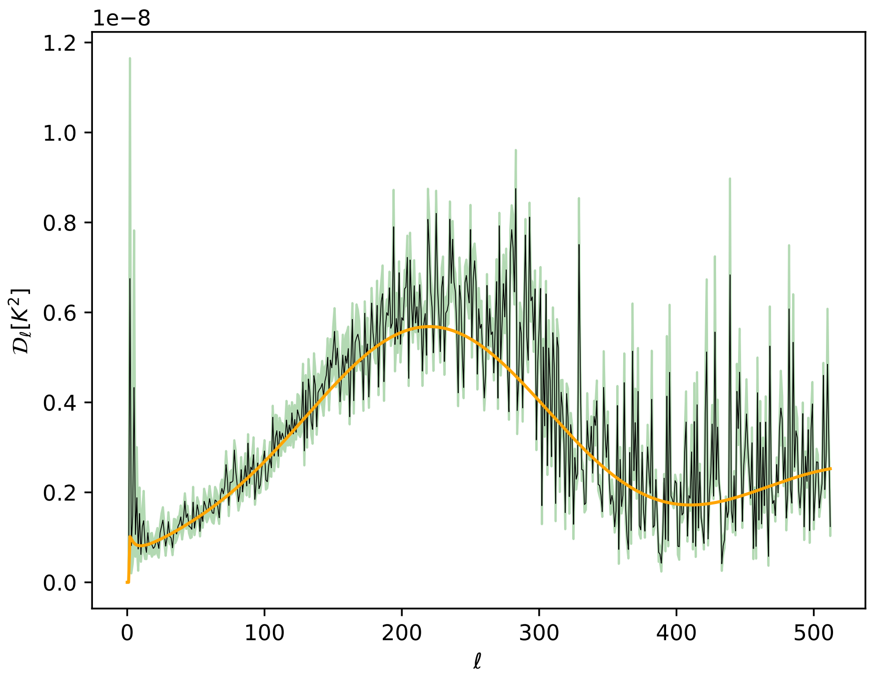

Figure 2: Power spectrum mean of 10 samples with the 2σ interval

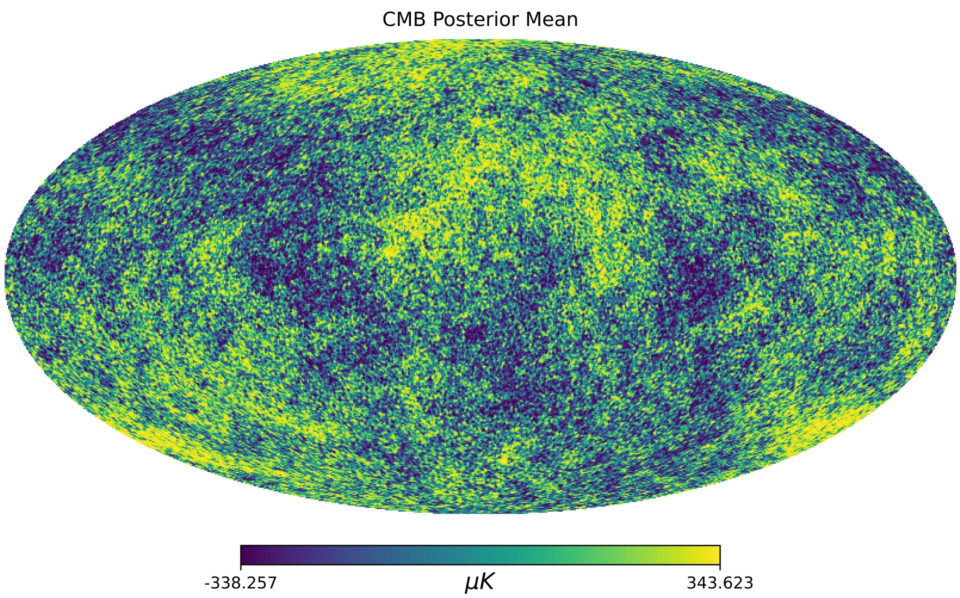

Figure 3: Posterior mean of 10 samples.

2 Joint Foreground Sampling

Incorporating foregrounds, the data measurement equation looks like:

$$d_\nu = A\left(\mathbf{s} + \underbrace{\sum_{i=1}^{M} a_{\nu,i}\ \mathbf{t_i}}_{\text{Spatial Template}} + \underbrace{\sum_{j=1}^{N} b_j f_j(\nu)\mathbf{f_j}}_{\text{Spatial template w/ global freq. scaling}} + \underbrace{\sum_{k=1}^{K} c_k\ \mathbf{g_k}(\nu,\theta_k)}_{\text{Spatial template w/ pixelwise freq. scaling}}\right) + \mathbf{n}_\nu \tag{6}$$

All amplitude-type degrees of freedom are sampled from the conditional joint Gaussian $\mathcal{P}(\mathbf{s}, a_{\nu,i}, b_j, c_k | C_\ell, \theta_k, d)$. This 4-component distribution has a mean $\hat{x}$ and posterior covariance matrix $\mathcal{A}$.

$$\mathbf{x} = (\mathbf{s}, a_{\nu,i}, b_j, c_k)^T \tag{7}$$

$$\mathcal{A}^{-1} = \begin{bmatrix} S^{-1} + A^T N^{-1} A & A^T N^{-1} T & A^T N^{-1} F & A^T N^{-1} G \\ T^T N^1 A & T^T N^1 T & T^T N^1 F & T^T N^1 G \\ F^T N^{-1} A & F^T N^{-1} T & F^T N^{-1} F & F^T N^{-1} G \\ G^T N^{-1} A & G^T N^{-1} T & G^T N^{-1} F & G^T N^{-1} G \end{bmatrix} \tag{8}$$

Similar to the CMB Gibbs sampler, one constructs a right hand vector comprising of the Wiener filter mean and random fluctations:

$$b = \begin{bmatrix} \sum_\nu A^T N_\nu^{-1} d + c^{1/2}\omega_0 + \sum_\nu A^T N_\nu^{-1/2}\omega_\nu \\ t_{\nu,j}^T N_\nu^{-1} d + t_{\nu,j}^T N_\nu^{-1/2}\omega_\nu \\ \sum_\nu f_j(\nu) f_j^T N^{-1} d + \sum_\nu f_j(\nu) f_j^T N^{-1/2}\omega_\nu \\ \sum_\nu g_k(\nu,\theta_k) N_\nu^{-1} d + \sum_\nu g_k(\nu,\theta_k) N_\nu^{-1/2}\omega_\nu \end{bmatrix} \tag{9}$$

and solve for $\mathcal{A}^{-1}\mathbf{x} = \mathbf{b}$.

A priori not saying that the different ℓ modes are correlated, that is why the posterior power spectrum fluctates so much between modes.