Joint Foreground Sampler

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Consider the case with two diffuse foregrounds, Free-Free and Synchrotron, each modeled using a foreground template. Then, the data measurement equation looks like:

$$d_\nu = A \left\{ \vec{s} + \sum_{j=1}^{2} b_j f_j(\nu) \vec{f}_j \right\} + n_\nu \tag{1}$$

We solve the linear equation $\mathcal{A}^{-1}x = b$ and invert $\mathcal{A}^{-1}$ where:

$$\mathcal{A}^{-1} = \begin{bmatrix} \mathrm{S}^{-1} + \mathrm{A}^{\mathrm{T}}\mathrm{N}^{-1}\mathrm{A} & \mathrm{A}^{\mathrm{T}}\mathrm{N}^{-1}\mathrm{F} \\ \mathrm{F}^{\mathrm{T}}\mathrm{N}^{-1}\mathrm{A} & \mathrm{F}^{\mathrm{T}}\mathrm{N}^{-1}\mathrm{F} \end{bmatrix} \tag{2}$$

and the right-hand side b is:

$$b = \begin{bmatrix} \sum_\nu \mathrm{A}^{\mathrm{T}}\mathrm{N}_\nu^{-1}\mathrm{d} + \mathrm{C}^{1/2}\omega_0 + \sum_\nu \mathrm{A}_\nu^{\mathrm{T}}\mathrm{N}_\nu^{-1/2}\omega_\nu \\ \sum_\nu \mathrm{f_j}(\nu)\mathrm{f_j^T}\mathrm{d} + \sum_\nu \mathrm{f_j}(\nu)\mathrm{f_j^T}\mathrm{N}^{-1/2}\omega_\nu \end{bmatrix} \tag{3}$$

The element blocks of Eq. 2 are calculated as follows:

$$\mathrm{F}^{\mathrm{T}}\mathrm{N}^{-1}\mathrm{F} \equiv \sum_\nu f_j(\nu) f_j^T N_\nu^{-1} f_k f_k(\nu) \tag{4}$$

$$\mathrm{A}^{\mathrm{T}}\mathrm{N}^{-1}\mathrm{F} \equiv \sum_\nu A_\nu^T N_\nu^{-1} f_j(\nu) f_j \tag{5}$$

and $\mathrm{F}^{\mathrm{T}}\mathrm{N}^{-1}\mathrm{A}$ is the **complex conjugate transpose** of $\mathrm{A}^{\mathrm{T}}\mathrm{N}^{-1}\mathrm{F}$.

0.1 Shapes

The shape of A^{-1} is:

$$\begin{bmatrix} (n_{\mathrm{alm}}, n_{\mathrm{alm}}) & (n_{\mathrm{alm}}, n_{\mathrm{comp}}) \\ (n_{\mathrm{comp}}, n_{\mathrm{alm}}) & (n_{\mathrm{comp}}, n_{\mathrm{comp}}) \end{bmatrix} \tag{6}$$

For $\mathrm{F}^{\mathrm{T}}\mathrm{N}^{-1}\mathrm{F}$, we want the equivalent of $F^{\alpha\beta\gamma}N^{-1}F_{\delta\beta\alpha} \to (FN^{-1}F)^\gamma_\delta$, where δ is the number of components, β is the number of frequencies, α is the number of pixels.