Dynamic response of a simply supported beam subject to a transient concentrated load at the center

Ananya Bijaya

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Problem Description

A simply supported beam, as shown in figure 1, is loaded with a concentrated load at the center; the load is removed after the beam attains the deformed configuration. The response of the beam is required to be calculated for 5 seconds.

Given:

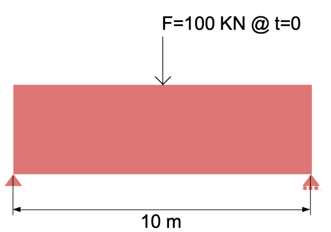

Figure 1: Schematic of the problem

L=10 m
A=$400 \times 400 \quad mm^2$
E=200 GPa
$\rho = 7860 \ kg/m^3$
F(t=0)=100 KN

The response of the beam can written in the PDE form as:

$$EI\frac{\partial^4 w(x,t)}{\partial x^4} + \rho A \frac{\partial^2 w(x,t)}{\partial t^2} = 0 \tag{1}$$

Boundary Conditions

$$w(x=0,t) = 0 \tag{2}$$
$$w(x=10,t) = 0 \tag{3}$$
$$\frac{\partial^2 w(x=0,t)}{\partial x^2} = 0 \tag{4}$$
$$\frac{\partial^2 w(x=10,t)}{\partial x^2} = 0 \tag{5}$$

Inital Values

$$u(x,t=0) = \frac{Wx}{12EI}\left(\frac{3L^2}{4} - x^2\right) \tag{6}$$

$$\frac{\partial w(x,t=0)}{\partial t} = 0 \tag{7}$$

Solution

$For \quad 1 \leq i \leq n \quad and \quad 1 \leq m \leq T$

where: i denotes space and m denotes time

$$\rho A \frac{\partial^2 w}{\partial t^2} = -EI \frac{\partial^4 w(x,t)}{\partial x^4}$$

$$\frac{\partial^2 w}{\partial t^2} = \frac{w_i^{m+1} - 2w_i^m + w_i^{m-1}}{\Delta t^2}$$

$$\frac{\partial^4 w}{\partial x^4} = \frac{w_{i-2}^m - 4w_{i-1}^4 + 6w_i^m - 4w_{i+1}^m + w_{i+2}^m}{\Delta x^4}$$

$$\frac{w_i^{m+1} - 2w_i^m + w_i^{m-1}}{\Delta t^2} = -\frac{EI}{\Delta x^4 \rho A}\left(w_{m-2}^m - 4w_{i-1}^m + 6w_i^m - 4w_{i+1}^m + w_{i+2}^m\right)$$

$$w_i^{m+1} = -\frac{EI\Delta t^2}{\Delta x^4 \rho A}\left(w_{m-2}^m - 4w_{i-1}^m + 6w_i^m - 4w_{i+1}^m + w_{i+2}^m\right) + 2w_i^m - w_i^{m-1} \tag{8}$$

For first time step; Putting m=2

$$\left.\frac{\partial w}{\partial t}\right|_{m=1} = \frac{w_i^1 - w_i^0}{\Delta t}$$

$$w_i^0 = w_i^1 - \Delta t \left.\frac{\partial w}{\partial t}\right|_{m=1} = w_i^1$$

Introducing boundary values in the space derivative in order to get response at i=2 and i=n-1

$$\left.\frac{\partial^2 w}{\partial x^2}\right|_i = \frac{w_{i+1}^m - 2w_i^m + w_{i-1}^m}{\Delta x^2}$$

Putting i=1

$$w_0^m = \Delta x^2 \left.\frac{\partial^2 w}{\partial x^2}\right|_{i=1} + 2w_1^m - w_2^m = 2w_1^m - w_2^m \tag{9}$$

Putting i=n

$$w_{n+1}^m = \Delta x^2 \left.\frac{\partial^2 w}{\partial x^2}\right|_{i=n} + 2w_n^m - w_{n-1}^m = 2w_n^m - w_{n-1}^m \tag{10}$$

For obtaing stable solution $\frac{K\Delta t^2}{\Delta x^4}$ is taken less than 0.25

$$K = \frac{EI}{\rho A} = 3.39 \times 10^5$$

$$\Delta t = 4.0e^{-05} \quad \Delta x = 0.217$$

$$\frac{K\Delta t^2}{\Delta x^4} = 0.2431$$

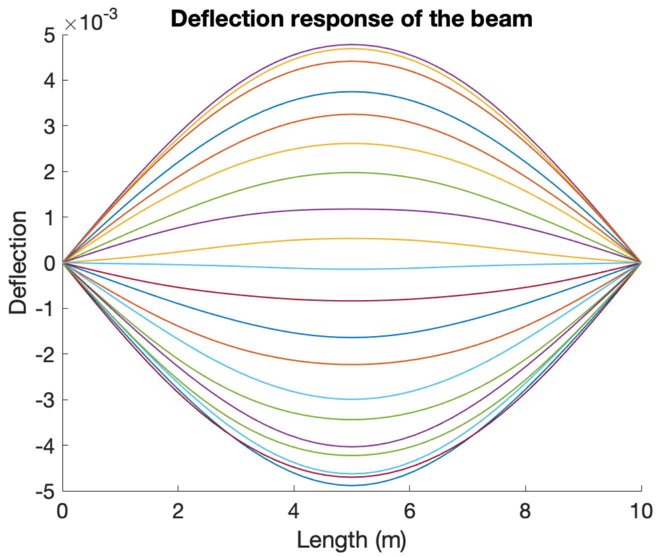

Figure 2: Response plotted at 0.2 sec intervals